Multiscaling Comparative Analysis of Time Series and a Discussion on "Earthquake Conversations" in California

Nicola Scafetta[1] and Bruce J. West[1,2]

[1]Physics Department, Duke University, Durham, North Carolina 27708, USA

[2]Mathematics Division, Army Research Office, Research Triangle Park, North Carolina 27709, USA

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Time series are characterized by complex memory and/or distribution patterns. In this Letter we show that stochastic models characterized by different statistics may equally well reproduce some pattern of a time series. In particular, we discuss the difference between Lévy-walk and fractal Gaussian intermittent signals and show that the adoption of complementary scaling analysis techniques may be useful to distinguish the two cases. Finally, we apply this methodology to the earthquake occurrences in California and suggest the possibility that earthquake occurrences are described by a *colored* ("long-range correlated") generalized Poisson model.

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Herein we introduce a method of multiscaling comparative analysis (MSCA) for the study of intermittent signals. We show that to distinguish between fractal Gaussian intermittent noise and Lévy-walk intermittent noise the scaling results obtained using diffusion entropy analysis (DEA) should be compared with those obtained from both finite variance scaling methods (FVSM) and probability distribution functions (PDFs) [1,2]. Finally, we apply MSCA to the seismic data of California and suggest that, instead of being described by a statistics according to which the waiting times between Omori's earthquake clusters are uncorrelated from one another, as the traditional generalized Poisson model [3,4] or a recent Lévy-walk-like model assume [4], the data may also be characterized by fractal Gaussian intercluster $1/f$ long-range correlations that may disclose the *earthquake conversations* recently suggested by Stein [3].

Hurst *et al.*, in their pioneering work [5], introduced the notion of *rescaled range analysis* of a time series that takes the scaling form of $(R/S)(t) \propto t^H$ (H is now called the Hurst exponent). This stimulated Mandelbrot to introduce the concept of fractional Brownian motion (FBM) [6]. In a random walk context the value $H = 0.5$ indicates uncorrelated noise, $0 < H < 0.5$ indicates antipersistent noise, and $0.5 < H < 1$ indicates persistent or long-range correlated noise [6]. Alternative scaling methods applied to a time series $\{\xi_i\}$, where $i = 1, 2, \ldots$, focus on the autocorrelation function $C(t) \propto \langle \xi_i \xi_{i+t} \rangle \propto t^{2H-2}$ on the power spectrum representation $P_S(f) \propto f^{1-2H}$ [7] and on the evaluation of the variance of the diffusion generated by $\{\xi_i\}$ [8] ($V(t) \propto t^{2H}$). All such scaling methods are related to the original Hurst's analysis and yield his H exponent for sufficiently long time series. These techniques, referred to by us as FVSM, assume a finite variance and according to the central limit theorem [9] the underlying statistics are Gaussian.

Recently, Scafetta *et al.* [1] introduced a complementary scaling analysis, the DEA, that focuses on the scaling exponent δ evaluated through the Shannon entropy $S(t)$ of the diffusion generated by the fluctuations $\{\xi_i\}$ of a time series [1,2]. Here, the PDF of the diffusion process, $p(x, t)$, is evaluated by means of the subtrajectories $x_n(t) = \sum_{i=0}^{t} \xi_{i+n}$ with $n = 0, 1, \ldots$. The PDF scaling property for an anomalous diffusion takes the form $p(x, t) = t^{-\delta} F(xt^{-\delta})$, and its entropy increases in time as $S(t) = -\int_{-\infty}^{\infty} p(x, t) \ln[p(x, t)]dx = A + \delta \ln(t)$, where A is a constant. One can also examine the scaling properties of the second moment for the same process using the FVSM. One version of FVSM is the standard deviation analysis (SDA) [2] which is based on the evaluation of the standard deviation $D(t)$ of the same variable x and PDF $p(x, t)$, and yields $D(t) = \sqrt{\langle x^2; t \rangle - \langle x; t \rangle^2} \propto t^H$ [2,6].

Note that the entropy $S(t)$ does not require the variance of the PDF $p(x, t)$ to be finite [2]. The existence of scaling for a process with a diverging second moment implies that DEA is *complementary* to and not simply an *alternative* to FVSM. So, the scaling exponent δ is conceptually different from the Hurst exponent H measured by means of the FVSM. This suggests that the scaling exponents δ and H may fulfill multiple relations according to the process under study and, therefore, the combined use of DEA, SDA, and PDF analysis may increase our understanding of complex phenomena through a MSCA.

Herein we focus on the statistics of intermittent noises. The simplest way to represent intermittent noise $\{\xi_i\}$ is through a dichotomous representation in which the value $\xi = 1$ indicates the occurrence of an event and the value $\xi = 0$ represents no event [10–13]. An intermittent noise is characterized by the correlation properties of the waiting time sequence $\{\tau_j\}$ between consecutive events and by its waiting time distribution $\psi(\tau)$. There are two basic distinct forms of intermittent noises as follows:

(i) *Fractal Gaussian intermittent noise* is characterized by a long-range correlated waiting time sequence, $\langle \tau_i \tau_{i+t} \rangle \propto t^{2H-2}$, and by a finite variance waiting time distribution $\psi(\tau)$ whose form may be, for example, that of a Gaussian, exponential, or Poisson distribution. The diffusion generated by a fractal Gaussian intermittent noise is a particular type of FBM and satisfies the asymptotic scaling relation between indices

$$\delta = H. \tag{1}$$

We refer to (1) as the *fractal Gaussian diffusion relation.* If the long-range correlations of $\{\tau_i\}$ are destroyed via shuffling, the new intermittent sequence is characterized by the value $H = \delta = 0.5$ of random time series. Figure 1(a) shows the scaling properties of a computer generated fractal Gaussian intermittent noise with an exponential waiting time distribution and $H = \delta = 0.75$.

(ii) *Lévy-walk intermittent noise* is characterized by an uncorrelated waiting time sequence, $\langle \tau_i \tau_j \rangle \propto \delta_{ij}$, and a Lévy or an inverse power law waiting time distribution $\psi(\tau) \propto (T + \tau)^{-\mu}$, with $2 < \mu < 3$ that ensures that although the first moment of τ is finite, the second moment diverges. The presence of a Lévy-walk process in a given time series can be detected [2,10] by means of the following asymptotic relation among the three exponents:

$$0.5 < \delta = (3 - 2H)^{-1} = (\mu - 1)^{-1} < 1. \tag{2}$$

We refer to (2) as the *Lévy-walk diffusion relation.* Interesting applications of this type of noise, introduced in Ref. [14], have been found in several Lévy phenomena including the distribution of solar flares [10–13]. In the particular case in which the sequence $\{\tau_i\}$ is correlated the scaling exponents δ and H are larger than the values predicted by Eq. (2). Figure 1(b) shows the scaling properties of a computer generated random Lévy-walk intermittent noise with $\mu = 2.5$ that has $H = 0.75$ and $\delta = 0.67$.

We stress that the Lévy-walk relation (2) is fulfilled if the waiting times $\{\tau_i\}$ are uncorrelated, in which case any shuffling of $\{\tau_i\}$ would not alter the scaling exponents H and δ. In fact, the superdiffusion scaling exponents $0.5 < \delta < H < 1$ of a Lévy-walk intermittent noise are related to the fatness of the waiting time inverse power law tail, as measured by the exponent μ. Contrary to a fractal Gaussian intermittent noise, this Lévy scaling does not

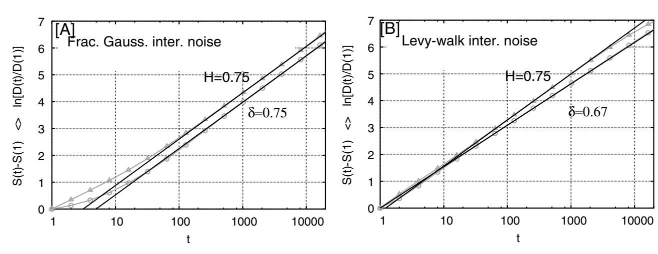

FIG. 1. DEA end SDA of (a) a fractal Gaussian intermittent noise with $\psi(\tau) \propto \exp(-\tau/\gamma)$ with $\gamma = 25$ and $H = \delta = 0.75$; the fractal Gaussian relation (1) of equal exponents is fulfilled. (b) A Lévy-walk intermittent noise with $\psi(\tau) \propto \tau^{-\mu}$ and $\mu = 2.5$; note the bifurcation between $H = 0.75$ and $\delta = 0.67$ caused by the Lévy-walk relation (2).

imply a temporal correlation, or a historical memory, among events because the occurrence of future events is independent of the frequency of past events.

We also observe that there exist particular intermittent sequences obtained by mixing Lévy and Gaussian noises [12], with a *Lévy memory beyond memory* [15] or by substituting an event with a cluster of events [4]. In these cases the asymptotic properties of the scaling exponent H and δ are expected to depend on the component, Gaussian or Lévy, with the strongest persistence.

The relations (1) and (2), and the correlation or shuffling effects indicate that the DEA should be jointly used with the FVSM and/or PDFs. The adoption of a single technique can lead to a misinterpretation of the characteristics of a phenomenon, because Lévy-walk intermittent noise can be confused with fractal Gaussian intermittent noise, and uncorrelated noise of one kind of statistics can be mistaken for correlated noise with another kind of statistics. Figures 1(a) and 1(b) clearly show that the determination of only one of the two exponents H and δ is not sufficient to conclude whether a phenomenon is characterized by a Lévy-walk intermittent statistics or by a fractal Gaussian intermittent statistics. So, we suggest a MSCA by combining complementary techniques.

Recently, DEA has been applied by Mega *et al.* [4] to study the time distribution of earthquakes in Southern California (20°–45° N latitude and 100°–125° W longitude) from 1976 to 2002. The catalog [16] is complete for local events with magnitude $M \geq 3$ since 1932, for $M \geq 1.8$ since 1981, and for $M \geq 0$ since 1984. The time intervals between large earthquakes were studied in Ref. [4] by setting a temporal variable $\xi(t) = 1$ at the occurrence of an earthquake with a magnitude larger than a given threshold M_t, and by setting $\xi(t) = 0$ when no earthquake of the specified magnitude occurs. We refer to $\{\tau_i\}$ as the waiting time sequence between consecutive earthquakes with $M \geq M_t$. So, the intermittent sequence $\xi(t)$ was analyzed by means of the DEA and the measured scaling exponent was $\delta = 0.94 \pm 0.01$. The authors of Ref. [4] concluded that the time intervals, $\tau^{[m]}$, between two consecutive Omori earthquake clusters [17] is modeled by an inverse power law $\psi(\tau^{[m]}) \propto (\tau^{[m]})^{-\mu}$ with an exponent $\mu = 2.06$ calculated via Eq. (2). This calculation was based on the traditional assumption [4] that the waiting times between such clusters are uncorrelated ($\langle \tau_i^{[m]} \tau_j^{[m]} \rangle = \delta_{ij}$, implying that the observed superdiffusion is induced by a Lévy walk between the Omori clusters. Finally, Mega *et al.* [4] showed that a synthetic sequence produced with Omori's uncorrelated clusters, $\langle \tau_i^{[m]} \tau_j^{[m]} \rangle = \delta_{ij}$, temporally distributed according to an inverse power law $\psi(\tau^{[m]}) \propto (\tau^{[m]})^{-\mu}$ with $\mu = 2.06$, generates a superdiffusive process with $\delta = 0.94$.

However, the authors of Ref. [4] did not make the important distinction between Lévy-walk and fractal Gaussian intermittent noises such as we did above. We

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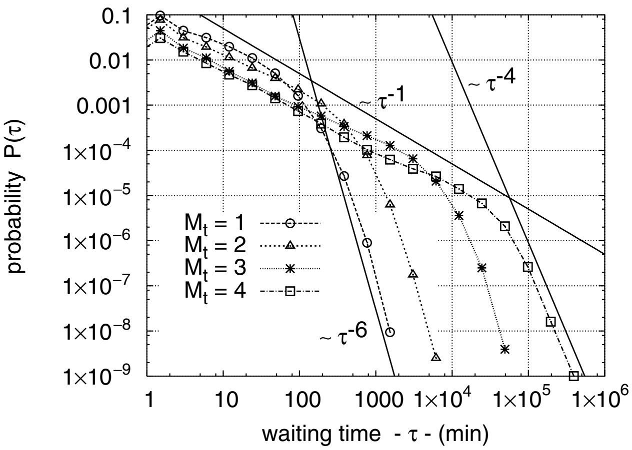

FIG. 2. PDF of the waiting times τ_i of earthquakes with a magnitude $M \geq M_t = 1, 2, 3$, and 4. The initial $P(\tau) \propto 1/\tau$ is Omori's law [17].

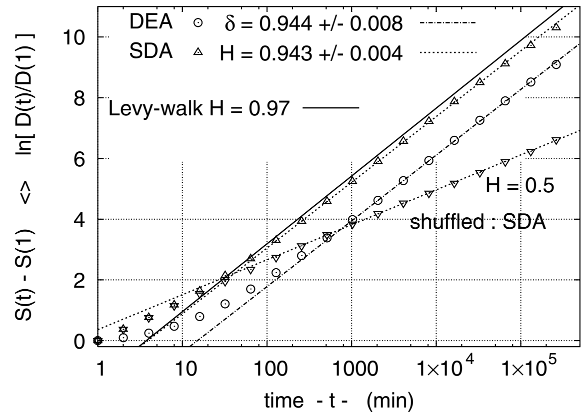

FIG. 3. DEA and SDA of the intermittent time signal $\xi(t)$ for magnitude $M \geq M_t = 1$. The data are fitted with scaling exponents $\delta = 0.944 \pm 0.008$ and $H = 0.943 \pm 0.004$. The uppermost solid line with $H = 0.97$ corresponds to the expectation of H if the Levy-walk condition (2) holds true.

showed that a scaling exponent in the range $0.5 < \delta < 1$ can be associated with either a correlated fractal Gaussian intermittent noise or with an uncorrelated Lévy-walk noise. Consequently, we apply a MSCA to determine which of the two statistics better describes the data.

Figure 2 shows the waiting time PDFs between earthquakes using four magnitude thresholds $M_t = 1, 2, 3$, and 4. The PDFs show an initial Omori law [17] [$P(\tau) \propto 1/\tau$], but the PDF tails present a large inverse power law exponent $\mu > 4$ and may even approach an exponential (or Poisson) distribution asymptotically. An Omori cluster is determined by correlated aftershocks [17] and lasts for a time that increases with the magnitude threshold. If the waiting time distribution between Omori's clusters were an inverse power law with $\mu = 2.06$ it might be expected that by increasing the magnitude threshold, most of the aftershocks could be cut off and the tail of the distribution could converge to an inverse power law with $\mu = 2.06$. This does not seem to happen. Therefore, such a $\mu = 2.06$ inverse power law, if it is real, cannot be observed in this way.

Figure 3 compares the DEA and SDA applied to the intermittent sequence $\xi(t)$ of earthquakes with $M \geq 1$. Different magnitude thresholds give similar results. If these data corresponded to a random intermittent Lévy walk and if the curves shown in the figure corresponded to the asymptotic regime, the condition (2) interrelating the exponents should hold. A rigorous DEA fit in the range $[2^7{:}2^{15}]$ gives $\delta = 0.944 \pm 0.008$ implying a Lévy walk $H = 0.97 \pm 0.005$. Instead, the same-range SDA fit gives $H = 0.943 \pm 0.004$. The error analysis seems to confirm better the Gaussian relation of equal scaling exponents (1) because δ and H overlap within the statistical error as in Fig. 1(a), while the difference between the measured H and the Lévy walk H is statistically significant ($p < 0.01$). By shuffling the earthquake waiting time intervals $\{\tau_i\}$ we get $H = 0.5$. Finally, by directly applying DEA and SDA to the waiting time series $\{\tau_i\}$, we again get $\delta = H = 0.94$. These findings suggest that the data do not fulfill the Lévy-walk relation (2) and that it might be more likely that the Californian earthquakes are long-range temporal correlated according to the persistence of a fractal Gaussian intermittent noise with $H \approx 1$ known as $1/f$ or *pink* noise [7].

The curve with circles in Fig. 4 shows the DEA applied to a synthetic earthquake catalog obtained by *coloring* a kind of generalized Poisson model for earthquakes. First we generated several Omori clusters exactly as done in Ref. [4], that is, by assuming that the number of earthquakes in a cluster follows an inverse power law distribution with an exponent equal to 2.5 and that the events within the same cluster are temporally distributed

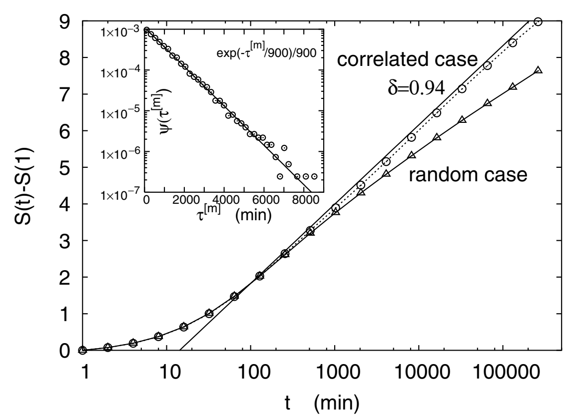

FIG. 4. DEA applied to a $1/f$ long-range correlated generalized Poisson model (circles) for earthquakes for $M \geq 1$. The intercluster waiting time PDF $\psi(\tau^{[m]})$ is exponential and $\langle \tau_i^{[m]} \tau_{i+t}^{[m]} \rangle \propto t^{2H-2}$ with $H \approx 1$. The curve with triangles refers to the case in which the intercluster waiting time sequence $\{\tau^{[m]}\}$ is randomized such that $\langle \tau_i^{[m]} \tau_j^{[m]} \rangle \propto \delta_{ij}$.

according to Omori's law, that is, an inverse power law with exponent equal to $p = 1$. We generated a total number of events equal to the total number of earthquakes in the catalog with a magnitude threshold $M_t = 1$. However, contrary to what was done in Ref. [4], we do not randomly ($\langle \tau_i^{[m]} \tau_j^{[m]} \rangle \propto \delta_{ij}$) position these clusters according to an inverse power law intercluster waiting time distribution $\psi(\tau^{[m]}) \propto 1/\tau^{\mu}$ with $\mu = 2.06$. Instead, we distribute the clusters according to a $1/f$ fractal Gaussian intermittent noise $\{\tau_j^{[m]}\}$, that is, with $\langle \tau_i^{[m]} \tau_{i+t}^{[m]} \rangle \propto t^{2H-2}$ and $H \approx 1$. The intercluster waiting time distribution $\psi(\tau^{[m]}) \propto \exp(-\tau^{[m]}/\gamma)$ shown in the inset of Fig. 4 could be substituted with any other distribution with finite variance. Figure 4 shows that the model is able to reproduce the same superdiffusion pattern shown by the data. Finally, the curve with triangles shows the reduction of long-range persistency of a synthetic catalog obtained with the same clusters of above but temporally distributed after having shuffled, to randomize, the same intercluster waiting time sequence $\{\tau_j^{[m]}\}$.

However, Eqs. (1) and (2) are fulfilled only asymptotically where the central limit theorem for Gaussian processes or its generalization for Lévy processes apply [9]. There might be the possibility that Fig. 3 as well as Fig. 2 in Ref. [4] do not show the asymptotic limit but a transition regime that is strongly superdiffusive ($\delta \approx 1$) because of the Omori intracluster correlations. A long transition regime is also evident in the curve with triangles shown in Fig. 4 that refers to an uncorrelated intercluster waiting time sequence and should asymptotically converge to $\delta = 0.5$. In fact, also a generalized Poisson model or an ETAS [18] model with appropiate parameters may generate a synthetic catalog showing superdiffusive properties similar to the real data within a time range [19]. However, we observe that such a result may depend too strongly on the particular parameters used in the models.

In conclusion, we have discussed some of the difficulties that can be encountered in interpreting intermittent sequences and shown that models with alternative statistics can reproduce some pattern of a time series equally well. This fact suggests the need of an analysis involving complementary tests. In particular, we showed how to distinguish fractal Gaussian intermittent noise from Lévy-walk intermittent noise using MSCA. This methodology has an important application in the analysis of phenomena having intermittent signals because different statistics imply different dynamics. Our analysis supports the idea that earthquakes generate strain diffusion, whose propagation over hundreds of kilometers induces remote seismic activity [3,20]. This propagation according to our statistical analysis produces correlations in the time intervals between earthquake clusters. In fact, the thesis that earthquakes are assembled into uncorrelated Omori clusters, $\langle \tau_i^{[m]} \tau_j^{[m]} \rangle = \delta_{ij}$, as both the standard *generalized Poisson* model [4] and the Lévy-walk model [4] require, seems unrealistic. We suggest that it is more plausible that earthquake clusters are $1/f$ long-range correlated and, perhaps, they are subclusters of a larger Omori cluster. In fact, a $1/f$ noise can be generated by the superposition of relaxation processes within a wide range of energies [7] that may well describe the coexistent stress alterations caused by old and recent, as well as large and small shocks. Thus, the $1/f$ long-range intercluster correlations may imply that earthquake occurrences may strongly depend on the geological history of a vast region.

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