Large-Scale Magnetic Field Generation by α Effect Driven by Collective Neutrino-Plasma Interaction

V. B. Semikoz*

The Institute of the Terrestrial Magnetism, the Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences, IZMIRAN, Troitsk, Moscow region, 142190, Russia

D. D. Sokoloff†

Department of Physics, Moscow State University, 119899, Moscow, Russia

(Received 3 July 2003; revised manuscript received 22 January 2004; published 1 April 2004)

We suggest a new mechanism for the generation of a large-scale magnetic field in the hot plasma of the early Universe which is based on parity violation in weak interactions and depends neither on the helicity of matter turbulence resulting in the standard α effect nor on general rotation. The mechanism can result in a self-excitation of an (almost) uniform cosmological magnetic field.

DOI: 10.1103/PhysRevLett.92.131301 PACS numbers: 98.62.En, 98.80.Cq

The large-scale magnetic field **B** self-excitation in astrophysical bodies such as the Sun, stars, galaxies, etc., is usually connected with the so-called α effect, i.e., a specific term in the Faraday electromotive force $\mathcal{E}=\alpha\mathbf{B}$ connected with a violation of the mirror symmetry of a rotating stratified turbulence or convection: the number of right-handed vortices systematically differs from the number of left-handed vortices due to the Coriolis force action. In this sense, α is determined by the helicity of turbulent motions.

The differential rotation Ω usually participates with the α effect in the dynamo action; however, the α effect induced in a rigidly rotating turbulent body could lead to a dynamo action alone (so-called α^2 dynamo) while the differential rotation alone is unable to result in a dynamo action [1]. The α effect is induced by the Coriolis force which destroys the mirror symmetry of turbulent motions.

The α effect is impossible in electrodynamics of classical nonmoving media because of its mirror symmetry. On the other hand, the mirror asymmetry of matter happens at the level of particle physics and we can expect that an α effect could be based on this asymmetry. The aim of our Letter is to present such a mechanism based on parity violation in weak interactions.

We recall that the main problem of most particle physics mechanisms of the origin of seed fields is how to produce them coherently on cosmological (large) scales. There are many ways allowing the generation of seed *small-scale random magnetic fields* in the early Universe, e.g., at phase transitions [2], however, the following growth of the correlation length, e.g., in the inverse cascade with the merging of such small-scale fields [3], hardly could produce a substantial large-scale field at the present time [4]. We do not consider an evolution of correlated domains and the corresponding growth of correlation length considered, e.g., in the review [5], and concentrate here on the generation of a mean magnetic field (amplification of its strength) via an α effect if such a mean field has been already seeded somehow from small-scale magnetic fields.

Let us consider hot plasma of the early Universe after electroweak phase transition, $T \ll T_{EW} \simeq 10^5$ MeV, when we may use pointlike (Fermi) approximation for weak interactions and where at the beginning a weak random magnetic field has a small macroscopic scale comparing with the horizon, $\Lambda \ll l_H$, while within a domain of the volume $\sim\Lambda^3$ such a magnetic field can be uniform and directed along an arbitrary z axis, $\mathbf{B} = (0, 0, B)$. Obviously, this does not violate the isotropy of the Universe as a whole with many randomly oriented domains.

Within a domain with a uniform magnetic field obeying the WKB limit $|e|B \ll T^2$ the single quantum (spin) effect remains for electrons and positrons which populate the main Landau level only and contribute to the lepton gas magnetization, $M_j^{(\sigma)} = \mu_B\langle\bar{\psi}_\sigma\gamma_j\gamma_5\psi_\sigma\rangle = \delta_{jz}\mu_B(\mathrm{sgn}\,\sigma)n_{0\sigma} \sim (\mathrm{sgn}\,\sigma)B_j$ [6], where $\mu_B = |e|/2m_e$ is the Bohr magneton, $n_{0\sigma}$ is the number density at the main Landau level for the electrons and positrons ($\sigma = e^-$, $\bar{\sigma} = e^+$),

$$n_{0\sigma} \approx n_{0\bar{\sigma}} = \frac{|e|B}{2\pi^2}\int_0^\infty f_{\mathrm{eq}}^{(\sigma)}(\varepsilon_p)dp \simeq \frac{|e|BT\ln 2}{2\pi^2}. \qquad (1)$$

The magnetization $M_j^{(\sigma)}$ changes sign for electrons and positrons, sgn $\sigma = \pm 1$, effectively due to the opposite spin projections on the magnetic field at the main Landau levels.

For a small magnitude of magnetic fields we neglect the small polarization of other components: muons, tau-leptons, quarks, or nucleons. Obviously, densities (1) are very small in comparison with the total lepton densities $n_\sigma = \int[d^3p/(2\pi)^3]f_{\mathrm{eq}}^{(\sigma)}(\varepsilon_p) \approx 0.183T^3$, $n_{0\sigma} \ll n_\sigma$. Here $\sigma = e^\mp$; $f_{\mathrm{eq}}^{(\sigma)}(\varepsilon_p)$ is the Fermi distribution; e, $\varepsilon_p = \sqrt{p^2 + m_e^2}$, T are the lepton electric charge, the energy, and the temperature of lepton gas, correspondingly.

In a magnetized plasma the pseudovector $M_j^{(\sigma)}$ enters the weak interaction of the charged σ-fluid component with neutrinos (antineutrinos) through the axial part of the pointlike current $\times$ current interaction Hamiltonian,

$$V_\sigma^{(A)} = G_F M_j^{(\sigma)} \cdot \delta j_j^{(\nu)}/\mu_B,$$

where G_F is the Fermi constant, $\delta \mathbf{j}^{(\nu)} = \mathbf{j}_\nu - \mathbf{j}_{\bar{\nu}}$ is the neutrino current density asymmetry.

Such interaction provides a force $\mathbf{F}_\sigma^{\text{weak}}$ [see below Eq. (6)] that is additive to the Lorentz force $q_\sigma(\mathbf{E} + \mathbf{V}_\sigma \times \mathbf{B})$ acting in magnetohydrodynamics (MHD) plasma on charged particles of the kind σ and obviously depends on gradients of the interaction potential, $(F_\sigma^{\text{weak}})_i \sim -\partial_i V_\sigma^{(A)}$, or for an uniform magnetization within a domain ($M_j =$ constant) on derivatives of the neutrino current density asymmetry, $\partial_i \delta j_j^{(\nu)}$.

The electric field $\mathbf{E}$ being common for all charged particles is obtained multiplying the motion equations for each of the charged components by the corresponding electric charge q_σ with the following summation over components that leads to $\sum_\sigma q_\sigma^2 \mathbf{E}$ in the Lorentz force and the remarkable addition of $n_{0\sigma}$ in the weak electromotive force term for electron-positron components, $E_j^{\text{weak}} = \alpha B_j \sim (n_{0+} + n_{0-})$, due to the independence of the product $q_\sigma \partial_i V_\sigma^{(A)} \sim q_\sigma G_F M_j^{(\sigma)} \cdot \partial_i \delta j_j^{(\nu)}/\mu_B$ on the sign of the electric charge since $M_j^{(\sigma)} \sim \operatorname{sgn} \sigma$. Such a term violates parity and provides a new particle physics origin of α effect for magnetic field generation, $\partial_t \mathbf{B} = -\nabla \times \mathbf{E}^{\text{weak}}$.

The pair motion equation in the one-component MHD is derived after the summation of Euler equations for comoving electrons and positrons for which the standard (polar vector) electric field cancels since $q_\sigma = \pm|e|$. In contrast, the standard Lorentz force $|e|(\mathbf{V}_+ - \mathbf{V}_-) \times \mathbf{B} = \text{rot}\mathbf{B} \times \mathbf{B}/4\pi n_e$ arises while the weak force term $\sum_\sigma \partial_i V_\sigma^{(A)}$ depends in hot lepton plasma on the negligible difference of densities, $(n_{0+} - n_{0-})$ as well as for the neutrino axial vector potential $V^{(A)}$ describing a probe neutrino in the electron- positron plasma [7] when $\delta j_j^{(\nu)} \to k_j$ and the sum over σ leads to $V^{(A)} = G_F\sqrt{2}(n_{0+} - n_{0-})\mathbf{B}\cdot\mathbf{k}/Bk$.

Now we estimate the α effect originating in the early Universe by particle effects. Let us note that in an external large-scale magnetic field $\mathbf{B}$ a polarized equilibrium lepton plasma is characterized by the density matrix,

$$\hat{f}^{(\sigma)}(\varepsilon_p) = \frac{\delta_{\lambda'\lambda}}{2} f_{\text{eq}}^{(\sigma)}(\varepsilon_p) + \frac{(\vec{\sigma}\hat{\vec{b}})_{\lambda'\lambda}}{2} S_{\text{eq}}^{(\sigma)}(\varepsilon_p), \quad (2)$$

where $\vec{\sigma}$ is the Pauli matrix; $\hat{\mathbf{b}} = \mathbf{B}/B$ is the ort directed along the magnetic field; $f_{\text{eq}}^{(\sigma)}(\varepsilon_p)$ is the Fermi distribution; $S_{\text{eq}}^{(\sigma)}(\varepsilon_p) = -(|e|B/2\varepsilon_p) df_{\text{eq}}^{(\sigma)}(\varepsilon_p)/d\varepsilon_p$ is the spin equilibrium distribution that defines the number density at the main Landau level (1), $\int d^3p S_{\text{eq}}^{(\sigma)}(\varepsilon_p)/(2\pi)^3 = n_{0\sigma}$; $\lambda = \pm 1$ is the spin projection on magnetic field.

Then we start from the linearized relativistic kinetic equations (RKE) derived in the Vlasov approximation for a magnetized lepton plasma in the standard model (SM) of electroweak interactions after the summing over spin variables as given in Eq. (30) of Ref. [6],

$$\frac{\partial \delta f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t)}{\partial t} + \mathbf{v}\frac{\partial \delta f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t)}{\partial \mathbf{x}} + q_\sigma \mathbf{E}\frac{\partial f_{\text{eq}}^{(\sigma)}(\varepsilon_p)}{\partial \mathbf{p}} + [\mathbf{v} \times \mathbf{B}]\frac{\partial \delta f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t)}{\partial \mathbf{p}} + \mathbf{F}_{\text{weak}}^{(V)}\frac{\partial f_{\text{eq}}^{(\sigma)}(\varepsilon_p)}{\partial \mathbf{p}} + \mathbf{F}_{\text{weak}}^{(A)}\frac{\partial S_{\text{eq}}^{(\sigma)}(\varepsilon_p)}{\partial \mathbf{p}} = 0. \quad (3)$$

Here weak forces $\mathbf{F}_{\text{weak}}^{(V)}$, $\mathbf{F}_{\text{weak}}^{(A)}$ that appear due to the generalization in SM of the standard Boltzman equation have the form

$$\mathbf{F}_{\text{weak}}^{(V)} = (\operatorname{sgn}\sigma) G_F\sqrt{2}\sum_a c_a^{(V)}\left[-\nabla \delta n_{\nu_a} - \frac{\partial \delta \mathbf{j}_{\nu_a}}{\partial t} + \mathbf{v} \times \nabla \times \delta \mathbf{j}_{\nu_a}\right], \quad (4)$$

$$\mathbf{F}_{\text{weak}}^{(A)} = -(\operatorname{sgn}\sigma) G_F\sqrt{2}\sum_a c_a^{(A)}\left[-\frac{\partial \delta n_{\nu_a}\hat{\mathbf{b}}}{\partial t} - \mathbf{v} \times \nabla \times \delta n_{\nu_a}\hat{\mathbf{b}} + \frac{m_e}{\varepsilon_p}\nabla(\mathbf{a}(\mathbf{p}) \cdot \delta \mathbf{j}_{\nu_a})\right]; \quad (5)$$

$c_a^{(V)} = 2\xi \pm 0.5$, $c_a^{(A)} = \mp 0.5$ are the vector and axial couplings correspondingly (upper sign for electron neutrino) where subindex $a = e, \mu, \tau$ characterizes the kind of neutrino, $\xi = \sin^2\theta_W \simeq 0.23$ is the Weinberg parameter; $\delta j_{\nu_a}^\mu$ is the neutrino four-current density asymmetry,

$$j_{\nu_a,\bar{\nu}_a}^\mu(\mathbf{x}, t) \equiv (n_{\nu_a,\bar{\nu}_a}, \mathbf{j}_{\nu_a,\bar{\nu}_a}) = \int \frac{d^3k}{(2\pi)^3}\frac{k^\mu}{\varepsilon_k} f^{(\nu_a,\bar{\nu}_a)}(\mathbf{k}, \mathbf{x}, t)$$

is the neutrino (antineutrino) four-current density; $\delta n_{\nu_a} = n_{\nu_a} - n_{\bar{\nu}_a}$ is the neutrino density asymmetry that plays below an important role in the generation of magnetic field.

Finally $\mathbf{a}(\mathbf{p})$ in the last term in (5) is the three-vector component of the four-vector a_μ that is the analogue of the Pauli-Lubański four-vector $a_\mu(p) = Tr(\rho\gamma_5\gamma_\mu)/4m_e = (\vec{p}\,\vec{\zeta}/m_e; \boldsymbol{\zeta} + \vec{p}(\vec{p}\cdot\vec{\zeta})/m_e(\varepsilon_p + m_e))$ with the change of the spin $\vec{\zeta}$ to $\hat{\mathbf{b}}$.

Notice that we can substitute the total number density distribution function $f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t) = f_{\text{eq}}^{(\sigma)}(\varepsilon_p) + \delta f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t)$ normalized on the total density $n_\sigma = \int (d^3p/(2\pi)^3) f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t) \approx \int (d^3p/(2\pi)^3) f_{\text{eq}}^{(\sigma)}$ into all the terms of RKE in the first and second lines (3) restoring its standard Boltzman form. Equation (3) obeys the continuity equation, $\partial j_\mu^{(\sigma)}/\partial x_\mu = 0$, where

$j_\mu^{(\sigma)}(\mathbf{x}, t) = \int d^3p(p_\mu/\varepsilon_p)f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t)/(2\pi)^3$ is the lepton four-current density.

Then we can use the standard method [8] for transition from kinetic equations to the hydrodynamical ones. Multiplying RKE (3) by the momentum $\mathbf{p}$ and integrating it over d^3p with the use of the standard definitions of the fluid velocity $\mathbf{V}_\sigma = n_\sigma^{-1}\int d^3p\mathbf{v}f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t)/(2\pi)^3$ and the generalized momentum $\mathbf{P}_\sigma = n_\sigma^{-1}\int d^3p\mathbf{p}f^{(\sigma)}(\mathbf{p}, \mathbf{x}, t)/(2\pi)^3$ one obtains the Euler equation for the fluid species σ in plasma with the additive collision terms taken in the τ approximation,

$$\frac{d\mathbf{P}_\sigma}{dt} = -\nu_\sigma^{\rm em}\delta\mathbf{P}_\sigma - (\nu_{\sigma\nu} + \nu_{\sigma\bar{\nu}})\mathbf{P}_\sigma - \frac{\nabla p_\sigma}{n_\sigma} + q_\sigma(\mathbf{E} + [\mathbf{V}_\sigma \times \mathbf{B}]) + \mathbf{F}_\sigma^{\rm weak}. \tag{6}$$

Here $\sigma = e^-, \mu^-, \tau^-, q_u, q_d, \ldots$ ($\bar{\sigma} = e^+, \mu^+, \tau^+, \bar{q}_u, \bar{q}_d, \ldots$ for antiparticles) enumerates the plasma components; $\nu_\sigma^{\rm em}$ is the electromagnetic collision frequency which leads to the fast equilibrium in plasma and defines plasma conductivity; $\nu_{\sigma\nu}$, $\nu_{\sigma\bar{\nu}}$ are the weak collision frequencies providing the generation mechanism suggested in [9] for neutrino scattering off electrons and positrons before neutrino decoupling; p_σ is the fractional pressure.

We isolate in the weak ponderomotive force vector and axial parts, i.e., $\mathbf{F}_\sigma^{\rm weak} = \mathbf{F}_\sigma^{(V)} + \mathbf{F}_\sigma^{(A)}$. The first term $\mathbf{F}_\sigma^{(V)}$ coming from (4) was found by the independent (Lagrangian) method in [10] and is irrelevant to the magnetic field generation mechanism considered here. The axial vector force $\mathbf{F}_\sigma^{(A)}$ appearing from (5) due to the polarization of lepton gas in a small mean magnetic field $\mathbf{B}$, is

$$\mathbf{F}_\sigma^{(A)} = \frac{G_F\sqrt{2}\delta_{\sigma e}(\mathrm{sgn}\,\sigma)}{n_\sigma} \times \sum_{a=e,\mu,\tau} c_{\sigma\nu_a}^{(A)}\left[n_{0\sigma}\hat{\mathbf{b}}\frac{\partial\delta n_{\nu_a}}{\partial t} + N_{0\sigma}\nabla(\hat{\mathbf{b}}\cdot\delta\mathbf{j}_{\nu_a})\right]. \tag{7}$$

Finally the relativistic polarization term $N_{0\sigma}$,

$$N_{0\sigma} = \frac{n_{0\sigma}}{3} + \frac{4\pi|e|Bm_e}{9(2\pi)^3}\int_0^\infty f_{\rm eq}^{(\sigma)}(\varepsilon_p)dp\frac{\partial}{\partial p}[v(3 - v^2)], \tag{8}$$

in the nonrelativistic case tends to the lepton density at the main Landau level given by Eq. (1), $N_{0\sigma} \to n_{0\sigma}$. Obviously, the weak force (7) changes sign for positrons, $\sigma \to \bar{\sigma}$, due to the signature function.

For the hot plasma multiplying the Euler equation (6) by the electric charge q_σ, summing over σ and dividing by $\sum_\sigma q_\sigma^2 = Q^2$ we find the electric field $\mathbf{E}$ including all known polar vector terms plus the new axial vector $\mathbf{E} \sim \alpha\mathbf{B}$ originated by electron-positron polarizations which violates parity. This is similar to the derivation of $\mathbf{E}$ in [10] for unpolarized plasma, and, in particular, from the Lorentz force one obtains the term $-\sum_\sigma(q_\sigma^2/Q^2)\mathbf{V}_\sigma \times \mathbf{B}$ that obviously leads from the Maxwell equation $\partial_t\mathbf{B} = -\nabla \times \mathbf{E}$ to the dynamo effect in Faraday equation. Thus, we arrive to a governing equation for magnetic field evolution

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla \times \alpha\mathbf{B} + \eta\nabla^2\mathbf{B}, \tag{9}$$

where we omitted the dynamo term neglecting any macroscopic rotation in plasma of the early Universe.

Here we approximate the tensor α_{ij} coming in $\mathbf{E}$ from the axial vector force (7) by the first diagonal term:

$$\alpha = \frac{G_F}{2\sqrt{2}|e|B}\sum_a c_{e\nu_a}^{(A)}\left[\left(\frac{n_{0-} + n_{0+}}{n_e}\right)\frac{\partial\delta n_{\nu_a}}{\partial t}\right] \simeq \frac{\ln 2}{4\sqrt{2}\pi^2}\left(\frac{10^{-5}T}{m_p^2\lambda_{\rm fluid}^{(\nu)}}\right)\left(\frac{\delta n_\nu}{n_\nu}\right), \tag{10}$$

where densities $n_{0\pm}$ are given by Eq. (1), *equilibrium* densities obey $n_\nu/n_e = 0.5$, and we assume a scale of neutrino fluid inhomogeneity $t \sim \lambda_{\rm fluid}^{(\nu)}$ that is small compared with a large Λ scale of the mean magnetic field $\mathbf{B}$, $\lambda_{\rm fluid}^{(\nu)} \ll \Lambda$. Let us stress that the addition of positron and electron contributions in α stems from the change of the sign in the weak force (7).

The diffusion coefficient $\eta \approx (4\pi \times 137T)^{-1}$ is given by the relativistic plasma conductivity. We do not present in Eq. (9) standard terms like differential rotation, etc., which seem to be not very important in the problem under consideration.

This is our main result. We stress that Eq. (9) is the usual equation for mean magnetic field evolution (see, e.g., [11]) with α effect based on particle effects rather on the averaging of turbulent pulsations. It is well known (see, e.g., [1]) that Eq. (9) describes a self-excitation of a magnetic field with the spatial scale $\Lambda \approx \eta/\alpha$ and the growth rate $\alpha^2/4\eta$.

Let us estimate these values for the early Universe. For a small neutrino chemical potential μ_ν, $\xi_{\nu_a}(T) = \mu_{\nu_a}(T)/T \ll 1$, the neutrino asymmetry in the right-hand side of Eq. (10) is the algebraic sum following the sign of the axial coupling, $c_{e\nu_a}^{(A)} = \pm 0.5$,

$$\frac{\delta n_\nu}{n_\nu} \equiv \sum_a c_{e\nu_a}^{(A)}\frac{\delta n_{\nu_a}}{n_{\nu_a}} = \frac{2\pi^2}{9\zeta(3)}[\xi_{\nu_\mu}(T) + \xi_{\nu_\tau}(T) - \xi_{\nu_e}(T)]. \tag{11}$$

We take for crude estimations below $\xi_{\nu_\mu}(T) + \xi_{\nu_\tau}(T) - \xi_{\nu_e}(T) \approx -2\xi_{\nu_e}(T)$ because different chemical potentials almost compensate each other for high temperatures [12], i.e., $\xi_{\nu_e}(T) + \xi_{\nu_\mu}(T) + \xi_{\nu_\tau}(T) \approx 0$.

As a result, we arrive at the following estimate of the α coefficient (10):

$$\alpha = 2.8 \times 10^{-34}(T/\mathrm{MeV})^6[l_\nu(T)/\lambda_{\rm fluid}^{(\nu)}]|\xi_{\nu_e}|,$$

where a free parameter for our collisionless mechanism scale $\lambda_{\rm fluid}^{(\nu)}$ is normalized on the neutrino mean free path $l_\nu(T) = \Gamma_W^{-1}$ given by the weak rate $\Gamma_W = 5.54 \times 10^{-22}(T/\mathrm{MeV})^5$ MeV.

Substituting α into $\Lambda = \eta/\alpha$ we arrive now at the estimate

$$\frac{\Lambda}{l_H} = 1.6 \times 10^9 \left(\frac{T}{\text{MeV}}\right)^{-5} \left[\frac{\lambda_{\text{fluid}}^{(\nu)}}{l_\nu(T)}\right] (|\xi_{\nu_e}(T)|)^{-1}, \quad (12)$$

where $l_H(T) = (2H)^{-1}$ and $H = 4.46 \times 10^{-22}(T/\text{MeV})^2$ MeV is the Hubble parameter.

If the neutrino fluid inhomogeneity scale $\lambda_{\text{fluid}}^{(\nu)}$ is of the order $l_\nu(T_0) \sim 4\text{cm} \ll l_H(T_0) \sim 10^6$ cm, we have $\Lambda/l_H \geq 1$ at the beginning of the lepton era ($T = T_0 \sim 10^2$MeV, redshift $z \sim 3 \times 10^{11}$), or more correctly, accounting for the big bang nucleosynthesis (BBN) limit $|\xi_{\nu_e}| \lesssim 0.07$ [12] obtained for $T_{\text{BBN}} = 0.1$ MeV, the mean magnetic field will be uniform in the whole Universe, $\Lambda/l_H \geq 1$, at $T \sim 118$ MeV. If this neutrino parameter would be much smaller at high temperatures T_0, $|\xi_{\nu_e}(T_0)| \ll 0.07$, one can choose another free neutrino parameter $\lambda_{\text{fluid}}^{(\nu)} \ll l_\nu(T_0)$ in such a way that the ratio $\lambda_{\text{fluid}}^{(\nu)}/(l_\nu(T_0)|\xi_{\nu_e}(T_0)|)$ remains invariant and our conclusion about the tendency to a global uniform field is still valid. Note that for the neutrino gas the macroscopic parameter $\lambda_{\text{fluid}}^{(\nu)}$ varies in a wide region $T^{-1} \ll \lambda_{\text{fluid}}^{(\nu)} \leq l_\nu(T)$.

The magnetic field time evolution is given by

$$B(t) = B_{\max} \exp\left[\int_{t_{\max}}^{t} \frac{\alpha^2(t')}{4\eta(t')} dt'\right], \quad (13)$$

where $B_{\max}$ is some seed value at the instant $T_{\max} \ll T_{EW} \sim 100$ GeV (here we imbed the standard estimates of α^2 dynamo into the context of an expanding universe).

For $\lambda_{\text{fluid}}^{(\nu)}(T) \sim l_\nu(T)$ we can estimate the index in the exponent (13) substituting in the integrand the expansion time $t(T) = 3.84 \times 10^{21}(T/\text{MeV})^{-2}\text{MeV}^{-1}/\sqrt{g^*}$ with the effective number of degrees of freedom $g^* \sim 100$ at the temperatures $T \gtrsim 1.1$ GeV. Then using the above values of $\alpha(T)$ and $\eta(T)$ with the change of the variable $(T/2 \times 10^4 \text{ MeV}) \rightarrow x$ one finds the fast growth of the mean field (13) in hot plasma ($x \leq 1$) with a conservative estimate,

$$B(x) = B_{\max} \exp\left[25 \int_x^1 \left(\frac{\xi_{\nu_e}(x')}{0.07}\right)^2 x'^{10} dx'\right], \quad (14)$$

given by the upper limit $x_{\max} = 1$, $T_{\max} = 20$ GeV. Such upper limit defines entirely the magnetic field amplification due to the steep dependence on the temperature and still obeys the pointlike Fermi interaction we rely on. As in the case of magnetic field scales (12) the second free parameter $\lambda_{\text{fluid}}^{(\nu)}$ can be chosen much smaller, $\lambda_{\text{fluid}}^{(\nu)} \ll l_\nu(T)$, providing the invariant ratio $l_\nu(T)|\xi_{\nu_e}(T)|/\lambda_{\text{fluid}}^{(\nu)}$ for very small neutrino chemical potential $\xi_{\nu_e}(T) \ll 0.07$ and resulting in an enhancement of a small mean magnetic field $B_{\max} \ll T_{\max}^2/|e| \ll T_{EW}^2/|e|$ by collective neutrino-plasma interactions considered here in Eq. (14).

Note that the inflation mechanism (with a charged scalar field fluctuations at super-horizon scales) explains the origin of the mean field at cosmological scales; however, the value of this field is too small for seeding the galactic magnetic fields [13].

The amplification mechanism suggested in our Letter can improve this very low estimate by a substantial factor from Eq. (14).

Thus, while in the temperature region $T_{EW} \gg T \gg T_0 = 10^2$ MeV there are many small random magnetic field domains, a mean magnetic field turns out to be developed into the uniform *global* magnetic field. The global magnetic field can be small enough to preserve the observed isotropy of cosmological model [14] while strong enough to be interesting as a seed for galactic magnetic fields. This scenario was intensively discussed by experts in galactic magnetism [15], however, until now no viable origin for the global magnetic field has been suggested. We believe that the α^2 dynamo based on the α effect induced by particle physics solves this fundamental problem and opens a new and important option in galactic magnetism.

V. S. is grateful to RFBR for financial support under Grant No. 04-02-16386.

*Electronic address: semikoz@izmiran.rssi.ru
†Electronic address: sokoloff@dds.srcc.msu.su

[1] Ya. B. Zeldovich, A. A. Ruzmaikin, and D. D. Sokoloff, *Magnetic Fields in Astrophysics* (Gordon & Breach, New York, 1983); E. Parker, *Cosmological Magnetic Fields* (Oxford Univ. Press, Oxford, 1979); A. A. Ruzmaikin, A. A. Shukurov, and D. D. Sokoloff, *Magnetic Fields of Galaxies* (Kluwer, Dordrecht, 1988).
[2] T. Vachaspati, Phys. Lett. B **265**, 258 (1991); K. Enqvist and P. Olesen, *ibid.* **319**, 178 (1993); **329**, 195 (1994).
[3] A. Brandenburg, K. Enqvist, and P. Olesen, Phys. Rev. D **54**, 1291 (1996); M. Christensson, M. Hindmarsh, and A. Brandenburg, Phys. Rev. E **64**, 056405 (2001); M. Christensson, M. Hindmarsh, and A. Brandenburg, astro-ph/0201466.
[4] R. Beck, A. Brandenburg, D. Moss, A. Shukurov, and D. Sokoloff, Annu. Rev. Astron. Astrophys. **34**, 155 (1996).
[5] K. Enqvist, Int. J. Mod. Phys. D **7**, 331 (1998).
[6] V. N. Oraevsky and V. B. Semikoz, Astropart. Phys. **18**, 261 (2002); A. I. Rez and V. B. Semikoz, hep-ph/0201259.
[7] H. Nunokawa *et al.*, Nucl. Phys. **B501**, 17 (1997).
[8] E. M. Lifshits and L. P. Pitaevsky, *Physical Kinetics* (Pergamon Press, Oxford, New York, 1981).
[9] A. D. Dolgov and D. Grasso, Phys. Rev. Lett. **88**, 011301 (2002).
[10] A. J. Brizard, H. Murayama, and J. S. Wurtele, Phys. Rev. E **61**, 4410 (2000).
[11] F. Krause and K.-H. Rädler, *Mean-Field Electrodynamics and Dynamo Theory* (Pergamon Press, Oxford, 1980).
[12] A. D. Dolgov *et al.*, Nucl. Phys. **B632**, 363 (2002).
[13] M. Giovannini and M. Shaposhnikov, Phys. Rev. D **62**, 103512 (2000).
[14] Ya. B. Zeldovich, Sov. Phys. JETP **21**, 656 (1965).
[15] R. M. Kulsrud, Annu. Rev. Astron. Astrophys. **37**, 37 (1999).