

Phase-Controlled Force and Magnetization Oscillations in Superconducting Ballistic Nanowires

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The emergence of superconductivity-induced phase-controlled forces in the (10^{-2} – 10^{-1}) nN range and of magnetization oscillations in nanowire junctions is discussed. A giant magnetic response to applied weak magnetic fields is predicted in the ballistic Josephson junction formed by a superconducting tip and a surface, bridged by a normal-metal nanowire where Andreev states form.

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Formation, energetics, and mechanical properties of metallic nanowires (NWs) were predicted in early simulations [1], and they have been the subject of subsequent significant research endeavors [2]. For normal metals the oscillatory behavior of the elongation forces [1] with the size of a NW formed between a surface and a retracting tip has been shown to be correlated with a quantized staircase behavior of the electrical conductance [3–7]. However, the influence of superconductivity (SC) on the nanomechanical properties of such NWs has not been explored yet, and these effects are the focus of this Letter.

In normal metals the cohesive force in an atomic-scale contact can be estimated as $F_n \sim \varepsilon_F/\lambda_F$, where ε_F and λ_F are the Fermi energy and wavelength. The onset of SC introduces a new energy scale, i.e., the superconducting gap $\Delta \ll \varepsilon_F$, and a new length scale, i.e., the SC coherence length $\xi_0 = \hbar v_F/\pi\Delta$. On first sight, the resulting SC-induced forces are expected to be of the order of $F_{sc} \sim \Delta/\xi_0$, and when added to the aforementioned normal-metal forces (F_n , which are of the order of several nN), they are estimated to be below the atomic force microscopy (AFM) detection limit [8]. However, in the superconducting regime, under certain conditions all the transverse channels (N_\perp) supported by the junction will contribute coherently to the free energy, and when $N_\perp \gg 1$ the above consideration may result in a gross underestimation of F_{sc} . In particular, we predict that under favorable conditions, in a nanowire (of length L) connecting two superconducting electrodes, modeled here as a transparent short ($L \ll \xi_0$) superconducting-normal-superconducting (SNS) junction, a (measurable) force $F_{sc} \sim (L/\xi_0)(\varepsilon_F/\lambda_F)$ would be manifested—we refer to these forces as Andreev forces (AF); in long SNS junctions the AF are expected to be significantly smaller.

To calculate the SC-induced contribution to the total elongation force we model the junction, comprised of the superconducting tip and surface and of the NW bridging them, as an SNS junction (the NW remains normal due to the suppression of superconductivity in very small structures [9]). Andreev reflections occur at the SN boundaries

of the junction, and the dimensions of the NW are such that transport through it is ballistic. This holds for short SNS junctions, while in long ones impurity scattering may reduce the transmission probability.

The force equals the spatial derivative of the grand-canonical potential Ω , i.e., $F = -\partial\Omega/\partial L$. In superconducting junctions there are two contributions to the AF, $F_A = F_\perp + F_L$; F_\perp is related to the dependence of the number of quantized transverse channels, N_\perp , on the degree of elongation, and F_L originates from the dependence of the Andreev bound states on the length of the junction.

Short ($L \ll \xi_0$) junctions.—Consider first the SC-induced force oscillations in short junctions. Since here the Andreev states are independent of the mode index [10], $\Omega_s(\varphi, L) \approx N_\perp(L)\Omega_A(\varphi, L)$ where $\Omega_A(\varphi, L)$ corresponds to a single-channel SNS junction. In a cylindrical geometry $N_\perp(L) \approx \pi V/\lambda_F^2 L$, where V is the volume (which remains constant during the elongation process [1]).

In general, both bound [superscript (b) below] and scattering Andreev states contribute to $\Omega_A(\varphi, L)$; for our purpose only the bound states are important (see below), and thus $F_A^{(b)} = F_\perp^{(b)} + F_L^{(b)}$, where

$$F_\perp^{(b)} = \frac{N_\perp}{L} \Omega_A^{(b)}(\varphi, L), \quad F_L^{(b)} = -N_\perp \frac{\partial \Omega_A^{(b)}(\varphi, L)}{\partial L}. \quad (1)$$

The spectrum of Andreev bound states in a single-mode ballistic SNS junction of length L and transparency $D \leq 1$ is found from the equation (see, e.g., Ref. [11])

$$\cos\left(2 \arccos \frac{E}{\Delta} - 2 \frac{E}{\Delta_L}\right) = R + D \cos \varphi, \quad (2)$$

where $\Delta_L \equiv \hbar v_F/L$, $D + R = 1$, and φ is the superconducting phase difference. The distinction between different channels enters through Δ_L , and it is neglected for short junctions. A reduced NW transparency reflects the presence of interfacial barriers, such as those likely to

form at tip-surface contacts. From Eq. (2) we obtain in the limit $L \ll \xi_0$ the bound-state energies

$$E_0^{(+)} = -E_0^{(-)} \simeq \Delta W(\varphi) \left(1 - \frac{L\sqrt{D}}{\xi_0} \left| \sin \frac{\varphi}{2} \right| \right), \quad (3)$$

where $W(\varphi) = \sqrt{1 - D \sin^2 \frac{\varphi}{2}}$. Using Eq. (1), with Ω_A taken simply as the thermodynamic potential for a two-level ($E_0^{(+)}, E_0^{(-)}$) system, yields

$$F_{\perp}^{(b)} \simeq -2N_{\perp} \frac{T}{L} \ln \left\{ \frac{\cosh^2[W(\varphi)\Delta/2T]}{\cosh^2(\Delta/2T)} \right\}, \quad (4)$$

$$F_L^{(b)} \simeq 2N_{\perp} \frac{\Delta}{\xi_0} W(\varphi) \sqrt{D \sin^2 \frac{\varphi}{2}} \tanh \left(\frac{\Delta}{2T} W(\varphi) \right). \quad (5)$$

For low transparency junctions ($D \ll 1$) $F_{\perp}^{(b)}$ may be approximated as

$$F_{\perp}^{(b)} \simeq \frac{N_{\perp} \Delta}{L} D \sin^2 \frac{\varphi}{2} \tanh \left(\frac{\Delta}{2T} \right) \sim \frac{N_{\perp} \Delta}{L} D, \quad (6)$$

and since $F_L^{(b)} \sim N_{\perp} (\Delta/\xi_0) \sqrt{D}$ it is evident that $|F_{\perp}^{(b)}| \gg |F_L^{(b)}|$, provided that $D \gg (L/\xi_0)^2$. In contrast, when $D \ll (L/\xi_0)^2 \ll 1$, the $F_L^{(b)}$ contribution dominates.

For point contacts (i.e., extremely short junctions) the AF can be calculated [when $D \gg (L/\xi_0)^2$] by taking the limit $L/\xi_0 \rightarrow 0$ for the Andreev bound states [Eq. (3)]. In this case, continuum states do not affect the free energy (see, e.g., Ref. [11]), and, therefore, they do not contribute to the force. In contrast, for low transparency junctions the L/ξ_0 corrections to the bound-state energies determine the force oscillations. To this order the continuum states do contribute to the free energy, and they can change the dependence of the AF on the phase difference. Thus, Eq. (5) can be considered as an estimate of the AF in a short junction. The contribution of the continuum states ($F_A^{(c)}$) is extremely small, i.e., $F_A^{(c)}(D \ll 1) \ll (L/\xi_0)^3 (\varepsilon_F/\lambda_F)$.

The phase-dependent force in a superconducting quantum point contact ($D = 1$) is related to the quantized Josephson current J_s (see Ref. [10])

$$J_s = \frac{e}{\hbar} \left(-L \frac{\partial F_A^{(b)}}{\partial \varphi} \right) = N_{\perp} \frac{e\Delta}{\hbar} \sin \frac{\varphi}{2}. \quad (7)$$

The force oscillations (portrayed by the dependence of the force on the contact size) are determined by two distinct contributions: (i) a large phase-independent term (operative also in normal-metal NWs) of the order of $N_{\perp} \varepsilon_F/\lambda_F$ originating from incoherent contributions of all the conducting electrons to the thermodynamic potential [4–6], and (ii) a coherent SC-induced force [Eqs. (4)–(6)]. It is the latter, phase-dependent, term that is directly related to the quantized Josephson current.

The amplitude of the AF oscillations may be readily estimated as follows: for $D \sim 1$ and $L \ll \xi_0$ the amplitude of the Andreev force is of the order of $F_A^{(b)} \sim$

$N_{\perp} \Delta/L \sim L \varepsilon_F/\xi_0 \lambda_F \sim (L/\xi_0)$ [nN]; in the ballistic regime for a nontransition metal $\xi_0 \sim 10^{-5}$ – 10^{-4} cm. Using state-of-the-art instrumentation such forces (e.g., 10^{-2} – 10^{-1} nN) can be measured [8].

Long ($L \gg \xi_0$) junctions.— For a long transparent ($D = 1$) junction the spectrum of the Andreev-Kulik (AK) levels $|E_n| \ll \Delta$ takes the form [12]

$$E_n^{\pm} \simeq \pi \Delta_L \left(\pm \frac{\varphi}{2\pi} + \frac{1}{2} + n \right), \quad n = 0, \pm 1, \pm 2, \dots \quad (8)$$

Since this spectrum does not depend on the superconducting gap Δ , for temperatures $T \ll \Delta$ all the thermodynamic properties of a long ballistic junction are essentially material independent. Evaluation of the Josephson current in this case is equivalent to the calculation of the persistent current for chiral fermions on a ring with circumference $2L$ [13]. The corresponding phase-dependent part of the thermodynamic potential $\Omega_A(\varphi)$ can be evaluated for the AK spectrum, yielding

$$\tilde{\Omega}_A(\varphi) = 4T \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \frac{\cos k\varphi}{\sinh(2\pi T k/\Delta_L)}. \quad (9)$$

The force oscillations induced by the AK level structure in a single-channel long SNS junction are ($|\varphi| \leq \pi$)

$$\tilde{F}_A \simeq \begin{cases} \frac{\Delta_L}{2\pi L} (\varphi^2 - \frac{\pi^2}{3}), & T \ll \Delta_L, \\ -16\pi \frac{T^2}{L\Delta_L} \exp(-\frac{2\pi T}{\Delta_L}) \cos \varphi, & T \geq \Delta_L. \end{cases} \quad (10)$$

We focus here only on the phase-dependent part of the thermodynamic potential, $\tilde{\Omega}_A(\varphi)$, and the resulting Andreev (or, equivalently, Casimir [14]) force \tilde{F}_A , since, as aforementioned, the force in superconducting junctions [Eq. (10)] is added to a much larger phase-independent term ($\sim \varepsilon_F/\lambda_F$) that dominates the cohesive force in metallic NWs.

In a multichannel junction the thermodynamic potential is the sum over transverse channels (ln)

$$\tilde{\Omega} = \sum_{ln} \tilde{\Omega}_A^{(ln)}(\varphi), \quad (11)$$

where $\tilde{\Omega}_A^{(ln)}(\varphi)$ is given by Eq. (9) with $\Delta_L^{(ln)} = \hbar v_F^{(ln)}/L$ substituted for Δ_L . For a long junction the Fermi velocity enters explicitly the expression for a single-channel supercurrent, and the total current in a multichannel junction strongly depends on the junction geometry [15].

We model the normal part of a long SNS junction by a cylinder of length L and cross-section area $S = V/L$. Assuming hard-wall boundary conditions, the electron longitudinal velocity in the (ln)th channel is

$$v_F^{(ln)}(L) = \sqrt{2(\varepsilon_F - a\gamma_{ln}^2)/m}, \quad (12)$$

where γ_{ln} are the Bessel function zeroes: $J_l(\gamma_{ln}) = 0$, and $a = \hbar^2 \pi L/2mV$. The dependencies of the AF on the phase difference (φ) and on the length (L) of the NW are displayed, respectively, in Figs. 1 and 2, where we

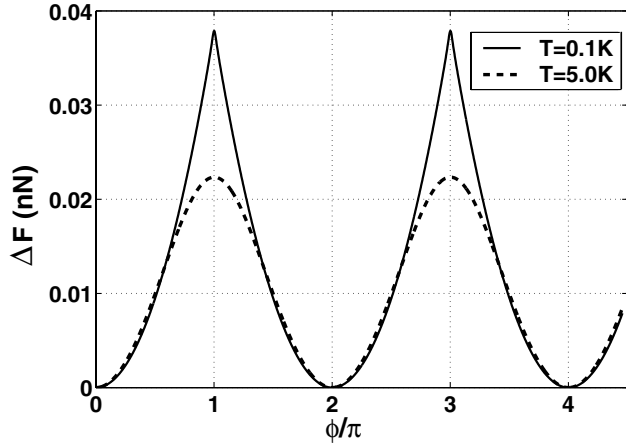


FIG. 1. $\Delta F(\varphi)$ vs the phase difference for a NW SNS junction. We use the physical parameters for Pb, i.e., $\varepsilon_F = 1.5 \times 10^{-11}$ erg and $v_F = 1.83 \times 10^8$ cm/s. The volume $V = 5 \times 10^{-15}$ cm³, and the length of the junction $L = 10^{-4}$ cm. Results are shown for two temperatures, both below $T_c(\text{Pb}) = 7.18$ K. The force was calculated as follows: first, using Eq. (9), the grand canonical potential was found for each transverse mode [each with a different $\Delta_L^n = \hbar v_F^n/L$; see Eq. (12)]. The total potential is the sum over all the transverse modes [Eq. (11)], and its derivative with respect to L was evaluated numerically.

show $\Delta F(\varphi) = F_A(\varphi) - F_A(0)$. From Fig. 1 we observe that the force is enhanced at special values of the phase difference $\varphi_r = \pi(2r + 1)$ ($r = 0, \pm 1, \pm 2, \dots$). At $\varphi = \varphi_r$ one of the AK bound states coincides with the Fermi energy, and, most significantly, this state is $4N_\perp$ -fold degenerate [16], thus amplifying its contribution. Direct observation of the SC-induced nanomechanical effect predicted here may be obtained through the following: (i) generation of a NW of length L via separation of an AFM tip-surface contact, using a superconducting material (e.g., Pb) at $T < T_c$, followed by (ii) measurement of the force required to maintain the NW length (L) as a function of variations of the phase difference across the SNS junction (as seen from Fig. 1, this force maximizes at $\varphi = \pi$).

The variation of the elongation force (for $\varphi = \pi$) with the NW length is shown in Fig. 2. We note first that even though the number of open channels is very large for the NW junction shown in Fig. 2, the magnitude of the forces is significantly smaller than in the case of short junctions

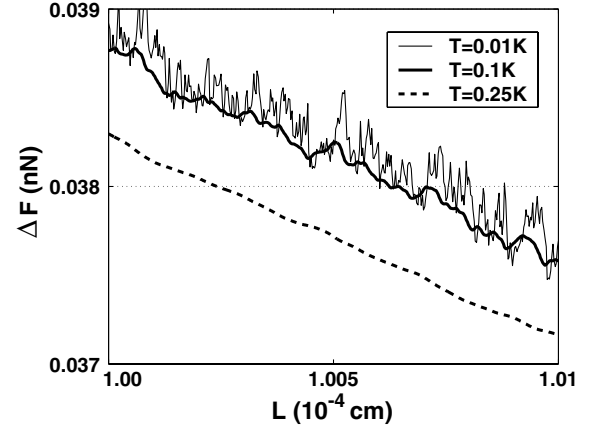


FIG. 2. $\Delta F(\varphi = \pi)$ vs the length of the junction, calculated for different temperatures. The parameters of the junction and the method of calculation are as in Fig. 1.

(see previous subsection) [17]. The aperiodic variations of the AF originating from the change in the number of open channels upon elongation, are particularly pronounced at lower temperatures. Note, however, that such aperiodic variations occur also for normal-metal NW [4,6] and, consequently, separation of the SC-induced contribution may be difficult.

Next we consider a multichannel superconducting long junction in a weak magnetic field $\mu_B B \ll \Delta$ (where μ_B is the Bohr magneton), applied locally (i.e., only to the normal-metal nanowire part of the SNS junction). Here, the only [18] influence on the AK levels is through the Zeeman coupling of the electron spin s to the magnetic field, $H_Z = g\mu_B \mathbf{s} \cdot \mathbf{B}$ (g is the g factor). The thermodynamic potential $\delta\Omega_A(\varphi, B) \equiv \Omega_A(\varphi, B) - \Omega_A(0, 0)$ takes the form [see Eq. (9)]

$$\delta\Omega_A(\varphi, B) = -4T \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \frac{(1 - \cos k\varphi \cos k\chi)}{\sinh(2\pi kT/\Delta_L)}, \quad (13)$$

where $\chi \equiv \Delta_Z/\Delta_L$ and $\Delta_Z = g\mu_B B$ is the Zeeman energy splitting. Note that the influence of the Zeeman interaction on the thermodynamics of the SNS junction is equivalent to the influence of a gate voltage on the thermodynamic properties of quantum rings [19].

The magnetization $M_A = -\partial\delta\Omega_A(\varphi, B)/\partial B$ at low ($T \ll \Delta_L$) and high ($T > \Delta_L$) temperatures is given for a single-channel junction as

$$M_A \approx \begin{cases} -g\mu_B \frac{\chi}{\pi}, & T = 0, |\chi| \leq \pi, \\ -8g\mu_B \frac{T}{\Delta_L} e^{-(2\pi T)/\Delta_L} \cos\varphi \sin\chi, & T \geq \Delta_L. \end{cases} \quad (14)$$

Note that the SC-induced magnetization M_A can be of the order of several μ_B (if $g \gg 1$) even for a single-channel junction, and it is insensitive to the superconducting phase difference at low temperatures. For a multichannel quantum junction at low temperatures the dependence of $M_A(\varphi)$ exhibits typical resonant behavior at the resonant phases φ_r , as shown in Fig. 3. This is a manifestation of

the effect of “giant oscillations,” known previously for conductance oscillations [20]. At these phases Andreev states of energies $E_A = \pm g\mu_B B$ become $2N_\perp$ -fold degenerate [20], leading to giant enhancement of thermodynamic and kinetic characteristics of ballistic junctions in magnetic fields.

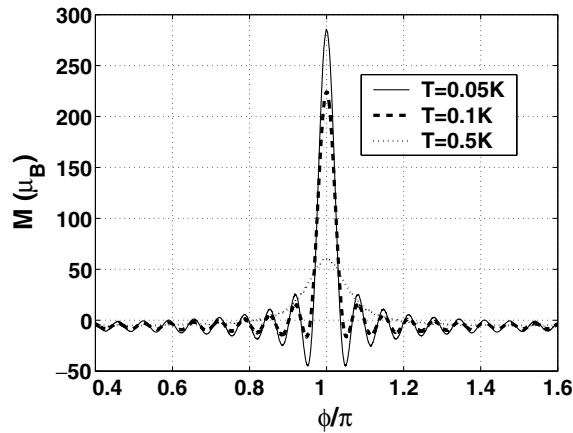


FIG. 3. Magnetization of the junction in a magnetic field of 5 Oe plotted vs the phase difference ϕ for several temperatures. The parameters of the junction are as in Fig. 1. The magnetization was evaluated as a numerical derivative of $\delta\Omega(\varphi, B)$ [see Eq. (13)] with respect to B .

Since at resonance the coherent contribution ($\propto N_{\perp}$) of all transverse modes dominates the magnetization, we predict at low temperatures ($T \ll \Delta_Z$): (i) a giant response ($\propto N_{\perp}$) of an SNS junction to a magnetic field, and (ii) a steplike behavior of the magnetization as a function of the wire diameter. At other values of the phase difference, different transverse channels contribute to $\delta\Omega_A$ with different periods (i.e., in general, incoherently), resulting in a complex structure of the magnetic oscillations.

In most cases a supercurrent is suppressed by the Zeeman interaction [21]. A magnetic field would also suppress the predicted Andreev force $\delta F_A(\varphi, B) = -\partial\delta\Omega_A(\varphi, B)/\partial L$. At low temperatures ($T \ll \Delta_L$) the force (which is periodic both in the phase φ and in the dimensionless Zeeman energy splitting $\chi = \Delta_Z/\Delta_L$) can be written for a single-channel junction as ($|\varphi|, |\chi| \leq \pi$): $\delta F_A \approx (\Delta_L/2\pi L)[(\varphi)^2 - (\chi)^2]$.

In summary, we predicted and illustrated that superconductivity induces in quantum wires phase-dependent forces correlated with the supercurrent. At resonance values of the superconducting phase difference, these Andreev forces become measurable (nN scale). Furthermore, we predict giant magnetization (of the order of $N_{\perp}\mu_B$) of ballistic SNS junctions in a weak magnetic field at low temperatures $T \ll \Delta_Z$. Since low-temperature STM with superconducting tips has already been demonstrated [22,23] and used to form Josephson junction [24,25], and in light of improved force-detection capabilities (extending to 10^{-1} – 10^{-3} nN) (8), the above predictions provide the impetus for future experiments.

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 - [17] For a long junction ($L \gg \xi_0$) with only a very small number of channels ($N_{\perp} \sim 1$), the force is negligibly small $\tilde{F}_A \sim \hbar v_F/L^2 \sim (\epsilon_F/\lambda_F)(\lambda_F/L)^2$. On the other hand, for a relatively short (i.e., $L \sim 0.1\xi_0 \sim 10$ nm) ballistic NW, the SC-induced contribution to the force can become quite large. For such a NW with a diameter $2R = L$ we estimate (using the semiclassical expression $(k_F R)^2/4$, where $k_F(Pb) \sim 1.6 \times 10^8$ cm $^{-1}$) that the number of channels is ~ 1500 . From Eqs. (4) and (5) we obtain an AF (evaluated at $\varphi = \pi$) of 10^{-1} nN for $T = 0.1$ K, and half this force for $T = 6$ K.
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