Spinodal Decomposition during the Hadronization Stage at RHIC?

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The expansion of strongly interacting matter formed in high-energy nuclear collisions drives the system through the region of phase coexistence. The present study examines the associated spinodal instability and finds that the degree of amplification may be sufficient for the emergence of spinodal patterns that may be utilized as a diagnostic of the hadronization phase transition.

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A central goal of high-energy nuclear collisions is to explore the expected phase transition from the familiar hadronic world to a plasma of quarks and gluons. This phase change of strongly interacting matter has often been assumed to be of first order, with T_c in the range of 150–200 MeV, but its character (and dependence on baryon density) is yet to be determined experimentally.

When two heavy nuclei collide at sufficiently high energies, such as those provided by the BNL Relativistic Heavy-Ion Collider (RHIC), the matter in the central rapidity region is formed well above the critical temperature T_c in a state of rapid longitudinal expansion. The expansion then causes the energy density ε to continually drop, thus forcing the system through the critical phase region towards hadronization and subsequent chemical and kinetic freeze-out. During this evolution the system maintains local equilibrium and one may employ fluid dynamics which has the advantage that the equation of state, $p(\varepsilon)$, enters explicitly in the equations of motion.

The starting point for the present discussion is the observation that the occurrence of a first-order phase transition is intimately linked with phase coexistence and associated spinodal instability. Indeed, a first-order phase transition occurs when the appropriate thermodynamic potential exhibits a convex anomaly [1]. There is then an interval of energy density throughout which the pressure has a negative derivative, $\partial p/\partial\varepsilon < 0$. This anomalous behavior identifies the region of spinodal instability where small deviations from uniformity are amplified, as the system seeks to break its uniformity and separate into the two coexisting phases consistent with the given ε.

However, in the absence of reliable models for the transition region, fluid-dynamical studies of nuclear collisions have usually employed the (constant) pressure describing the phase-separated system, thus suppressing entirely the spinodal instability (see, for example, Refs. [2–4]). Employing a simple spline technique to obtain the approximate form of the anomalous pressure curve, the present study examines the significance of this inherent instability and seeks to ascertain the degree to which it may generate spatial patterns that might offer a diagnostic tool for probing the phase transition.

The occurrence of spinodal decomposition in heavy-ion collisions has already been studied in various contexts. At intermediate energies the mechanism appears to play a key role in nuclear multifragmentation [5], while studies of spinodal decomposition in high-energy collisions have primarily been carried out in connection with the chiral phase transition [6–11]. Our present study is closer in spirit to the recent work by Bower and Gavin [12] which focuses on the enhancement of baryon fluctuations within a fluid-dynamical framework.

To estimate the behavior of $p(\varepsilon)$ through the region of phase coexistence, we employ a cubic spline function to interpolate between the hadron and plasma phases. As is commonly done [2–4], we approximate the former phase by an ideal gas of hadrons, and we take a bag of noninteracting quarks and gluons for the latter.

In general, the contributions to p and ε from a nondegenerate particle of mass m are

$$p_\kappa(m) = \frac{T^4}{2\pi^2}\sum_{n=1}^{\infty}\frac{\kappa^{n+1}}{n^4}\nu^2 K_2(\nu), \quad (1)$$

$$\varepsilon_\kappa(m) = \frac{T^4}{2\pi^2}\sum_{n=1}^{\infty}\frac{\kappa^{n+1}}{n^4}[3\nu^2 K_2(\nu) + \nu^3 K_1(\nu)], \quad (2)$$

where $\kappa = +/-$ for bosons/fermions and $\nu \equiv nm/T$.

The hadronic phase is approximated as an ideal gas of 14 hadronic species $i = \pi, K, \eta, \rho, \ldots, N, \Lambda, \Sigma, \Delta$ with masses m_i and degeneracies g_i (including antiparticles where appropriate). Since we are focusing on the central rapidity region of collisions at RHIC, we assume that all the chemical potentials vanish, $\mu_B = \mu_Q = \mu_S = 0$,

$$p_{\rm had}(T) = \sum_{i=\pi}^{\Delta} g_i p_{\kappa_i}(m_i), \qquad \varepsilon_{\rm had}(T) = \sum_{i=\pi}^{\Delta} g_i \varepsilon_{\kappa_i}(m_i). \quad (3)$$

The hadronic equation of state is shown in Fig. 1. At the critical temperature, for which we use $T_c = 170$ MeV, the pressure is $p_c = 79$ MeV fm^{-3}, while the energy density is $\varepsilon_H = 418$ MeV fm^{-3}. The slope of $p(\varepsilon)$ gives the square of the sound speed at T_c, $(\partial p_{\rm had}/\partial\varepsilon)_H = v_H^2 = 0.19$.

For the plasma phase, we assume that the system can be approximated as an ideal gas of quarks and gluons

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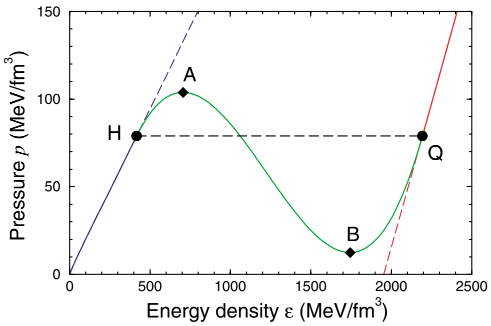

FIG. 1 (color online). The equation of state for *uniform* strongly interacting matter (solid lines) as obtained by interpolating with a cubic spline through the coexistence region between an ideal hadronic gas for $\varepsilon \leq \varepsilon_H$ and a bag with quarks and gluons for $\varepsilon \geq \varepsilon_Q$. The dashed line through the phase coexistence region represents the pressure of an equilibrium *mixture* of the two phases. The spinodal region extends between the two extrema of $p(\varepsilon)$ where uniform matter is mechanically unstable, $\partial p/\partial\varepsilon < 0$.

confined within a bag of suitable (negative) pressure $-B$,

$$p_{\rm qgp}(T) = \left[g_g + \frac{7}{8}g_q\right]\frac{\pi^2}{90}T^4 + g_s p_-(m_s) - B, \quad (4)$$

$$\varepsilon_{\rm qgp}(T) = \left[g_g + \frac{7}{8}g_q\right]\frac{\pi^2}{30}T^4 + g_s \varepsilon_-(m_s) + B. \quad (5)$$

The gluons as well as the u and d quarks are taken to be massless, m_g, $m_q = 0$, while the s quark is given a mass of $m_s = 150$ MeV (though the results would change very little if it were zero also). The corresponding degeneracies are $g_g = 16$, $g_q = 24$, and $g_s = 12$.

The plasma pressure starts at $-B$, while the energy density starts at B. These quantities increase more rapidly than their hadronic counterparts and the plasma pressure overtakes the hadronic pressure at the critical temperature. This condition determines the bag constant, $B^{1/4} = 246$ MeV. The resulting plasma equation of state $p_{\rm qgp}(\varepsilon)$ is shown in Fig. 1. The critical energy density of the plasma is $\varepsilon_Q \equiv \varepsilon_{\rm qgp}(T_c) = 2194\ {\rm MeV\,fm^{-3}}$, so that the latent heat of the phase transition is $\Delta\varepsilon \equiv \varepsilon_Q - \varepsilon_H = 1777\ {\rm MeV\,fm^{-3}}$. The function $p_{\rm qgp}(\varepsilon)$ is practically linear (its slight nonlinearity arises from m_s) and it has the slope $(dp_{\rm qgp}/d\varepsilon)_Q = v_Q^2 = 0.33$ at T_c.

Through the region of phase coexistence we employ a cubic spline function that matches both the pressure and its derivative (i.e., the speed of sound) at both ends, $p(\varepsilon) = p_c + \xi(\varepsilon)[(\varepsilon_Q - \varepsilon)v_H^2 - (\varepsilon - \varepsilon_H)v_Q^2]\bar{\xi}(\varepsilon)$, where $\xi \equiv (\varepsilon - \varepsilon_H)/\Delta\varepsilon$ and $\bar{\xi} \equiv (\varepsilon_Q - \varepsilon)/\Delta\varepsilon$. The resulting curve is shown in Fig. 1. This curve rises initially at ε_H with the slope v_H^2, but it gradually bends over and turns downwards. After crossing the mixed-phase line $p = p_c$ it starts bending upwards and finally joins the plasma curve at ε_Q with the slope v_Q^2. The crossing occurs at the energy density $\varepsilon_\times$ given by $(v_H^2 + v_Q^2)\varepsilon_\times = v_H^2\varepsilon_Q + v_Q^2\varepsilon_H$.

In the cubic spline approximation, the slope $\partial p/\partial\varepsilon$ is parabolic in the phase coexistence region,

$$\frac{\partial p}{\partial\varepsilon} = v_s^2 = \left[1 - 4\left(\frac{\varepsilon - \tilde{\varepsilon}}{\varepsilon_B - \varepsilon_A}\right)^2\right]\tilde{v}_s^2. \quad (6)$$

The inflection point in $p(\varepsilon)$ is at $\tilde{\varepsilon} = \frac{1}{3}(\varepsilon_H + \varepsilon_\times + \varepsilon_Q)$ and its extrema (located at ε_A and ε_B) are separated by

$$\varepsilon_B - \varepsilon_A = \frac{\sqrt{2}}{3}[(\Delta\varepsilon)^2 + (\varepsilon_\times - \varepsilon_H)^2 + (\varepsilon_Q - \varepsilon_\times)^2]^{1/2}. \quad (7)$$

The steepest slope of $p(\varepsilon)$ occurs at $\tilde{\varepsilon}$ and is given by

$$\left(\frac{\partial p}{\partial\varepsilon}\right)_{\tilde{\varepsilon}} = \tilde{v}_s^2 = -\frac{3}{4}[v_H^2 + v_Q^2]\left(\frac{\varepsilon_B - \varepsilon_A}{\Delta\varepsilon}\right)^2 \approx -0.132. \quad (8)$$

This value is rather robust against changes in T_c and m_s. The ε dependence of the sound speed is shown in Fig. 2.

The cubic spline is of course not the only functional form that joins the single-phase pressure curves smoothly, and it is adopted here merely as a convenient means for a rough estimate. The equation of state can be further constrained by the demand of thermodynamic consistency, which can be expressed as a Maxwell-type integral condition. As it turns out [13], this additional condition is, in fact, very well satisfied by the employed cubic spline, for the specific scenarios considered here.

The system is regarded as an ideal fluid with no conserved charges. The energy-momentum tensor $T^{\mu\nu}$ is then of ideal gas form [14], $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$, where $u^\mu = \gamma(1, \boldsymbol{v})$ is the four velocity of the energy flow, and local conservation of four momentum yields the basic equations of motion, $\partial_\mu T^{\mu\nu} = 0$, i.e.,

$$\partial_t[(\varepsilon + p)\gamma^2 - p] = \boldsymbol{\nabla}\cdot[(\varepsilon + p)\gamma^2\boldsymbol{v}], \quad (9)$$

$$\partial_t[(\varepsilon + p)\gamma^2\boldsymbol{v}] = \boldsymbol{\nabla}[(\varepsilon + p)\gamma^2 v^2 + p]. \quad (10)$$

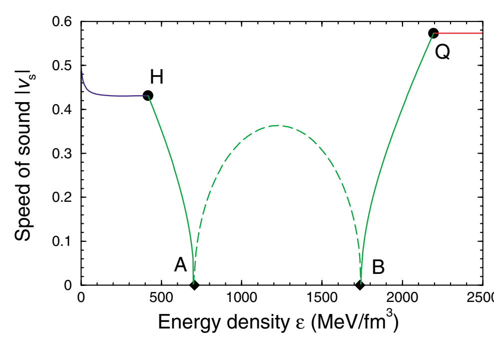

FIG. 2 (color online). The magnitude of the sound speed, $|v_s| = |\partial p/\partial\varepsilon|^{1/2}$, as a function of ε from zero to above the phase-coexistence region. While v_s is approximately constant in each of the single-phase regions, it drops rapidly as the metastable phase-coexistence region is entered and becomes zero at the spinodal boundaries. It is imaginary inside the spinodal region where the thermodynamical potential is convex and spontaneous phase separation occurs. In the mixed-phase scenario v_s is zero throughout the entire phase coexistence region.

These coupled equations are closed by the specification of the equation of state $p(\varepsilon)$.

We now consider a small perturbation on a uniform static system, $\varepsilon(t, \boldsymbol{r}) = \varepsilon_0 + \delta\varepsilon(t, \boldsymbol{r})$ with $\delta\varepsilon(t, \boldsymbol{r}) \ll \varepsilon_0$. Since the flow speed v is of the order of $\delta\varepsilon/\varepsilon_0$, it may be regarded as small, $v \ll 1$, and we may ignore terms of order v^2 and thus use $\gamma = 1$. Then Eqs. (9) and (10) simplify,

$$0 \approx \partial_t \varepsilon - \nabla \cdot [(\varepsilon + p)\boldsymbol{v}] \approx \partial_t \delta\varepsilon - (\varepsilon_0 + p_0)\nabla \cdot \boldsymbol{v}, \quad (11)$$

$$\boldsymbol{0} \approx \partial_t[(\varepsilon + p)\boldsymbol{v}] - \nabla p \approx (\varepsilon_0 + p_0)\partial_t \boldsymbol{v} - v_s^2 \nabla \delta\varepsilon, \quad (12)$$

where we have used $\nabla p(\varepsilon(\boldsymbol{r})) = (\partial p_0/\partial \varepsilon_0)\nabla\varepsilon = v_s^2 \nabla\varepsilon$, with $p_0 \equiv p(\varepsilon_0)$ being the pressure in the unperturbed uniform system. The speed of sound was displayed in Fig. 2. Taking the gradient of (12) and using the derived relation (11) between $\delta\varepsilon$ and $\boldsymbol{v}$ then yields the familiar equation for the propagation of sound waves, $\partial_t^2 \delta\varepsilon(t, \boldsymbol{r}) = (\partial p_0/\partial \varepsilon_0)\nabla^2 \delta\varepsilon(t, \boldsymbol{r})$. For harmonic perturbations, $\delta\varepsilon \sim \exp(i\boldsymbol{k} \cdot \boldsymbol{r} - i\omega t)$, it yields a linear relation between the wave number and the frequency, $\omega_k = v_s k$. Thus, if a physical system is brought into the spinodal region where $\partial p_0/\partial \varepsilon_0 < 0$ (such a quenching can be accomplished by rapid expansion and/or cooling), the frequency is imaginary and small deviations from uniformity evolve exponentially.

The linear dispersion relation becomes unrealistic once the wavelength of the perturbation, $\lambda_k = 2\pi/k$, has shrunk to the size of the range of the interaction between the constituents of the fluid. At such small scales, the physical justification for employing fluid dynamics is questionable. However, a physically meaningful treatment can still be made within that framework by employing a suitable smoothing, as we now explain.

The equation of state $p(\varepsilon)$ gives the pressure in a spatially uniform system as a function of its energy density. When the energy density varies sufficiently gently with position, one may obtain the corresponding local pressure as $p(\boldsymbol{r}) \approx p(\varepsilon(\boldsymbol{r}))$; i.e., one may calculate p as if the energy density had everywhere the same value as at $\boldsymbol{r}$. This local-density approximation becomes increasingly inaccurate as the scale of the spatial variations in $\varepsilon(\boldsymbol{r})$ is decreased, because the finite range of the forces reduces the response to fine ripples. A simple way to account approximately for the effect of the interaction range is to average the pressure over the interaction volume. While this procedure is, in fact, exact in certain cases (e.g., semiclassical one-body models of nuclear systems [15]), it is adopted here merely as a simple and transparent approximation which needs ultimately to be justified or improved on the basis of microscopic models of the system.

Thus we assume that the local pressure $p(\boldsymbol{r})$ can be obtained by a convolution with a (normalized) kernel $g(r)$, $p(\boldsymbol{r}_1) \approx \int d^3\boldsymbol{r}_2 g(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)p(\varepsilon(\boldsymbol{r}_2))$. Since the gradient of the pressure is proportional to the gradient of the energy density, a harmonic perturbation $\sim \exp(i\boldsymbol{k} \cdot \boldsymbol{r})$ yields $\nabla p(\boldsymbol{r}) \approx (\partial p_0/\partial \varepsilon_0) g_k \nabla\varepsilon(\boldsymbol{r}) = i\boldsymbol{k} g_k (\partial p_0/\partial \varepsilon_0)\delta\varepsilon(\boldsymbol{r})$, where g_k is the Fourier transform of $g(r)$. Hence the refinement amounts to simply modulating the local-density term $\partial p_0/\partial \varepsilon_0$ in the dispersion relation by g_k, yielding $\omega_k^2 = (\partial p_0/\partial \varepsilon_0) g_k k^2$. As a result of this convenient factorized form, the largest collective frequency occurs at the same wave number k_0 for any energy density, thus enhancing the formation of a distinctive spinodal pattern of a characteristic scale.

The spatial scale of the pattern is determined by the width of $g(r)$, which one would expect to be in the range of 0.5–1.0 fm, with the precise value depending on the specific microscopic treatment. For simplicity, we employ here a Gaussian form factor, $g(r) \sim \exp(-r^2/2a^2)$, so we have $g_k = \exp(-a^2k^2/2)$. The growth rates γ_k for the spinodal modes can then be written in a simple form,

$$\gamma_k(\varepsilon) = \left(-\frac{\partial p}{\partial \varepsilon}\right)_{\tilde{\varepsilon}}^{1/2} \left[1 - 4\left(\frac{\varepsilon - \tilde{\varepsilon}}{\varepsilon_B - \varepsilon_A}\right)^2\right]^{1/2} k e^{(-1/4)a^2k^2}. \quad (13)$$

This dispersion relation is illustrated in Fig. 3.

As the system expands and the energy density traverses the spinodal region, the growth rate $\gamma_k(\varepsilon(t))$ of a given mode increases from zero at ε_B, exhibits a maximum at $\tilde{\varepsilon}$, and then reverts to zero at ε_A. The total amplitude growth factor is then approximately equal to $G_k = \exp(\Gamma_k)$ where the amplification coefficient is given by $\Gamma_k \equiv \int_{t_B}^{t_A} \gamma_k(\varepsilon(t))dt \approx \gamma_k(\tilde{\varepsilon})\Delta t_{\rm eff}$. The effective time during which spinodal instability is encountered is $\Delta t_{\rm eff} \approx (\pi/4)(t_B - t_A)$ in the cubic spline approximation. For a quantitative estimate we use the value $t_B - t_A \approx 4$ fm/c gleaned from Ref. [4]. The resulting growth factors G_k are displayed in Fig. 4.

After rapidly reaching a maximum, $G(\lambda)$ subsides relatively slowly, thus allowing some degree of amplification of longer wavelengths. If $a = 0.5$ fm, the maximum growth factor of $G_{\max} \approx 8$ is obtained for $\lambda \approx 2$ fm, while $a = 0.7$ fm yields $G_{\max} \approx 4$ for $\lambda \approx 3$ fm. The corresponding component of the density-density

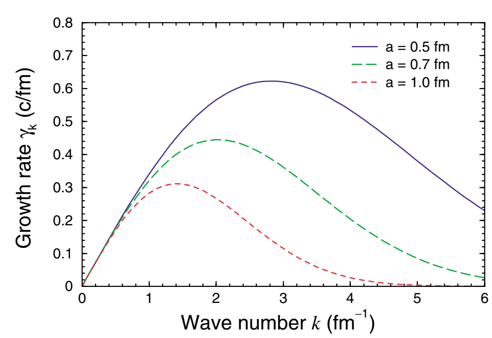

FIG. 3 (color online). The spinodal dispersion relation at the energy density $\tilde{\varepsilon}$ where the pressure decreases most rapidly. Results are shown for three values of the smearing range a.

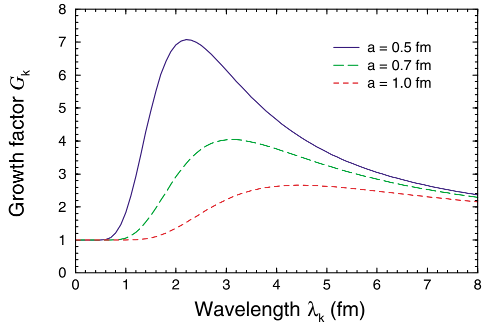

FIG. 4 (color online). The factor $G_k = \exp(\Gamma_k)$ by which the amplitude of undulations of wavelength $\lambda_k = 2\pi/k$ grows during the expansion through the spinodal region.

correlation function is proportional to the square of that, $\langle \delta\varepsilon(\boldsymbol{r}_1)\exp(i\boldsymbol{r}_{12}\cdot\boldsymbol{k})\delta\varepsilon(\boldsymbol{r}_2)\rangle \sim G_k^2$, and can thus reach quite appreciable magnitudes. (Without the finite-range modulation factor g_k there would be even larger amplification, but no characteristic scale would emerge.)

While the linear-response analysis provides the growth rates and also, when the expansion is invoked, the accumulated growth factors, the net result depends on the irregularities present as the system enters the spinodal region and their character and magnitude depend on the specific model employed. Generally, due to the high temperature in the plasma phase, such irregularities are expected to be considerable, even though they have usually been ignored in fluid-dynamical treatments.

Spinodal decomposition is intimately linked to the occurrence of a first-order phase transition and is important in many areas of physics as a mechanism for pattern formation. In nuclear physics it plays a key role for nuclear multifragmentation where it gives rise to highly nonstatistical fragment size distributions with a preference for equal masses [5]. Focusing here on high-energy nuclear collisions, we have investigated the importance of spinodal decomposition while the expanding matter hadronizes. Working within the framework of relativistic fluid dynamics and basing our analysis on commonly employed assumptions about the equation of state, we have found that a significant degree of amplification may occur as the matter passes through the mechanically unstable region of phase coexistence.

Such an onset of spinodal decomposition may have interesting dynamical consequences, most notably the development of characteristic patterns in the energy density, as the most rapidly amplified wavelengths grow predominant. The spatial scale of any such emerging spinodal pattern could, in principle, be probed by suitable correlation analyses. It is also conceivable that the azimuthal multipolarity of the favored modes would be reflected in the flow coefficients v_m (of which the best explored is v_2 quantifying the elliptic flow [16]). Investigation of these prospects would clearly be interesting.

If the appearance of spinodal patterns could, in fact, be examined experimentally, one could obtain fairly direct information on the equation of state in the critical region, since the relative prominence of various scales would reflect the degree of spinodal instability encountered by the corresponding wavelengths. Conversely, if the absence of such patterns can be established it may be concluded that no spinodal region has been encountered and, consequently, the phase transition cannot be of first order.

In view of the potential utility of spinodal decomposition as a diagnostic tool, it would seem worthwhile to pursue this issue with more realistic dynamical treatments that take better account of the spatiotemporal features of the system (including both the rapid longitudinal expansion and the finite transverse extent which may affect the present simple estimates). In particular, the quantitative importance of spinodal decomposition could be elucidated with existing fluid-dynamical models that have been modified to admit an equation of state containing the convexity anomaly which is a characteristic feature of first-order phase transitions.

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