QCD Hard Scattering and the Sign of the Spin Asymmetry A_{LL}^{π}

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Recent preliminary PHENIX data are consistent with a negative and sizable longitudinal double-spin asymmetry A_{LL}^{π} for π^0 production at moderate transverse momentum $p_\perp \simeq 1$–4 GeV and central rapidity. By means of a systematic investigation of the relevant degrees of freedom, we show that the perturbative QCD framework at leading power in $p_\perp$ produces at best a very small negative asymmetry in this kinematic range.

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Introduction.—The determination of the nucleon's polarized gluon density is a major goal of current experiments with longitudinally polarized protons at the Relativistic Heavy Ion Collider (RHIC) [1]. It can be accessed through measurement of the spin asymmetries,

$$A_{\mathrm{LL}} = \frac{d\Delta\sigma}{d\sigma} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}, \tag{1}$$

for high transverse-momentum ($p_\perp$) reactions such as $pp \to \pi X$, jet X, γX. In Eq. (1), σ^{++} (σ^{+-}) denotes the cross section for scattering of two protons with same (opposite) helicities. High $p_\perp$ implies large momentum transfer, and the cross sections for such reactions may be factorized into long-distance pieces that contain the desired information on the (spin) structure of the nucleon, and short-distance parts that describe the hard interactions of the partons and are amenable to QCD perturbation theory (pQCD). All of the reactions listed above have partonic Born cross sections involving gluons in the initial state and may therefore serve to examine the gluon content of the scattering longitudinally polarized protons.

In this Letter, we will consider the spin asymmetry A_{LL}^{π} for high-$p_\perp$ π^0 production, for which very recently the PHENIX collaboration has presented first preliminary data [2] at center-of-mass (c.m.) energy $\sqrt{S} = 200$ GeV and central rapidity. The data are consistent with a sizable (up to a few per cent) negative A_{LL}^{π} in the region $p_\perp \sim$ 1–4 GeV. Even though the experimental uncertainties are still large and leave room for a different behavior of A_{LL}^{π}, the new data motivate us to entertain the unexpected possibility of A_{LL}^{π} being negative. To our knowledge, there have been no predictions of a substantially negative A_{LL}^{π} in the literature. This is in itself interesting, also because at moderate $p_\perp$ the spin asymmetry A_{LL}^{π} is expected to be particularly sensitive to gluon polarization.

Hard-scattering calculation.—We may write the polarized high-$p_\perp$ cross section as

$$\frac{d\Delta\sigma^{\pi}}{dp_\perp\, d\eta} = \sum_{a,b,c} \int dx_a \int dx_b \int dz_c\, \Delta a(x_a, \mu)\Delta b(x_b, \mu) \times \frac{d\Delta\hat{\sigma}_{ab}^{c}(p_\perp, \eta, x_a, x_b, z_c, \mu)}{dp_\perp\, d\eta} D_c^{\pi}(z_c, \mu), \tag{2}$$

where η is the pion's pseudorapidity. The Δa, Δb ($a, b = q, \bar{q}, g$) are the polarized parton densities; for instance,

$$\Delta g(x, \mu) \equiv g_+(x, \mu) - g_-(x, \mu) \tag{3}$$

(the sign referring to the gluon helicity in a proton of positive helicity) is the polarized gluon distribution. The sum in Eq. (2) is over all partonic channels $a + b \to c + X$, with their associated polarized cross sections $d\Delta\hat{\sigma}_{ab}^{c}$. These start at $\mathcal{O}(\alpha_s^2)$ in the strong coupling with the QCD tree-level scatterings: (i) $gg \to gg$, (ii) $gg \to q\bar{q}$, (iii) $gq(\bar{q}) \to gq(\bar{q})$, (iv) $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $qq \to qq$, $qq' \to qq'$, $q\bar{q} \to q'\bar{q}'$. The transition of parton c into the observed π^0 is described by the (spin-independent) fragmentation function D_c^{π}. The functions in Eq. (2) are tied together by their dependence on the factorization/renormalization scale μ which is of the order of the hard scale $p_\perp$, but not further specified. All next-to-leading order [NLO, $\mathcal{O}(\alpha_s^3)$] QCD contributions to polarized parton scattering are known [3]. Corrections to Eq. (2) itself are down by inverse powers of $p_\perp$ and are thus expected to become relevant if $p_\perp$ is not much bigger than typical hadronic mass scales. We neglect such contributions for now, but will briefly return to them later.

To set the stage for our further considerations, Fig. 1 shows NLO predictions for A_{LL}^{π}, for various gluon polarizations Δg, all proposed within the framework of the analysis of data from polarized deeply inelastic scattering (DIS) in [4]. Despite the fact that the Δg used in Fig. 1 are all very different from one another, none of the resulting A_{LL}^{π} are negative in the $p_\perp$ region we display.

Basic observations.—As a first step in our more detailed analysis of A_{LL}^{π}, we discuss the various ingredients

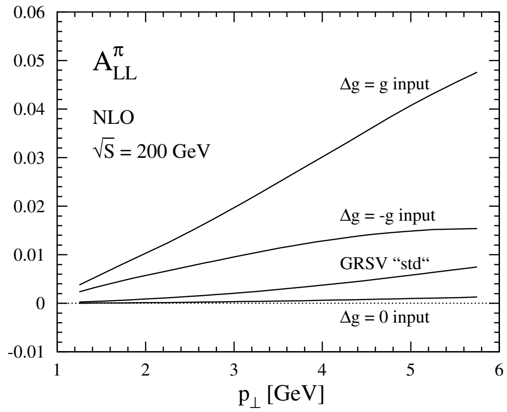

FIG. 1. NLO predictions for A_{LL}^{π} based on different assumptions about Δg at the input scale for the evolution in [4].

to Eq. (2), other than the unknown Δg that one hopes to probe: the Δq and $\Delta \bar{q}$ distributions, the calculated partonic cross sections, and the fragmentation functions D_c^{π}. They all jointly determine the analyzing power for Δg provided by A_{LL}^{π}.

To a first approximation, the Δq, $\Delta \bar{q}$ may be treated as "known." $\Delta q + \Delta \bar{q}$ has been probed extensively in polarized DIS. There certainly remains much room for improvement in our knowledge about them, but this does not affect A_{LL}^{π} too strongly. To give one example: For π^0 production the qg channel depends on the sum $\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d}$, so that uncertainties relating to the SU(2) flavor structure of the polarized sea are not relevant.

We now turn to the partonic cross sections and fragmentation functions. We focus on the lowest order (LO) which is expected to capture the qualitatively important points. Among the reactions (i)–(iii) listed above that have gluons in the initial state, process (ii) has a negative *partonic* spin asymmetry $\hat{a}_{\mathrm{LL}} \equiv -1$, while (i) and (iii) both have $\hat{a}_{\mathrm{LL}} > 0$ [1]. Also, for the $p_\perp$ region of interest, one has an average $\langle z_c \rangle \sim 0.3$–$0.4$, where quarks are more favored than gluons to fragment into pions. A first guess is, then, to attribute a negative A_{LL}^{π} to the negative $gg \to q\bar{q}$ cross section. However, this expectation is refuted by the numerical hierarchy in the partonic cross sections: at $\hat{\eta} = 0$ in the partonic c.m., which is most relevant for the PHENIX data, channel (i) is (in absolute magnitude) larger than (ii) by a factor of about 160. Therefore, if one wanted to suppress the (positive) $gg \to gg$ contribution, one would effectively have to switch off the gluon-to-pion fragmentation function D_g^{π}. Even though our knowledge of D_g^{π} is incomplete, this does not appear to be a sensible solution, for two reasons. First, data from e^+e^- collisions, in particular, analyses of hadron production in $b\bar{b}$ plus (mostly gluon) jet final states [5], do constrain D_g^{π} significantly, even at fairly large z_c. For example, the D_g^{π} in the sets of [6,7] are in reasonable agreement with these data, with the one of [7] arguably setting a lower bound on D_g^{π}. Second, elimination of the $gg \to gg$ channel would also strongly affect the unpolarized cross section and reduce it by up to an order of magnitude at RHIC energies. However, previous PHENIX measurements [8] of the unpolarized cross section for $pp \to \pi^0 X$ at $\sqrt{S} = 200$ GeV were found to be in excellent agreement with NLO calculations employing the D_c^{π} of [6].

We therefore exclude that the $gg \to q\bar{q}$ channel is instrumental in making A_{LL}^{π} negative, and we thus have to investigate possibilities within Δg itself, and its involvement in $gg \to gg$ and $qg \to qg$ scattering. Given that the polarized scattering cross sections for these reactions are both positive, and that the first process comes roughly with the square of Δg, it is immediately clear that a sizable negative asymmetry will not be easily obtained. In the next section, we will demonstrate this for the particularly instructive case of rapidity integrated cross sections.

A lower bound on $\mathbf{A}_{\mathrm{LL}}^{\boldsymbol{\pi}}$.—We consider the LO cross section integrated over all rapidities η. This is not immediately relevant for comparison to experiment, but it does capture the main point we want to make. From Eq. (2), one obtains

$$\frac{p_\perp^3 d\Delta\sigma^{\pi}}{dp_\perp} = \sum_{a,b,c} \int_{x_T^2}^1 dx_a\, \Delta a(x_a, \mu) \int_{\frac{x_T^2}{x_a}}^1 dx_b\, \Delta b(x_b, \mu) \times \int_{x_T/\sqrt{x_a x_b}}^1 dz_c\, D_c^{\pi}(z_c, \mu) \frac{p_\perp^3 d\Delta\hat{\sigma}_{ab}^c}{dp_\perp}(\hat{x}_T^2, \mu), \tag{4}$$

where $\hat{x}_T^2 \equiv x_T^2/z_c^2 x_a x_b$, $x_T \equiv 2p_\perp/\sqrt{S}$. It is then convenient to take Mellin moments in x_T^2 of the cross section,

$$\Delta\sigma^{\pi}(N) \equiv \int_0^1 dx_T^2 (x_T^2)^{N-1} \frac{p_\perp^3 d\Delta\sigma^{\pi}}{dp_\perp}. \tag{5}$$

One obtains (we suppress the scale μ from now on)

$$\Delta\sigma^{\pi}(N) = \sum_{a,b,c} \Delta a^{N+1} \Delta b^{N+1} \Delta\hat{\sigma}_{ab}^{c,N} D_c^{\pi,2N+3}, \tag{6}$$

where the $\Delta\hat{\sigma}_{ab}^{c,N}$ are the $\hat{x}_T^2$ moments of the partonic cross sections and, as usual, $f^N \equiv \int_0^1 dx\, x^{N-1} f(x)$ for the parton distribution and fragmentation functions. We now rewrite Eq. (6) in a form that makes the dependence on the moments Δg^N explicit:

$$\Delta\sigma^{\pi}(N) = (\Delta g^{N+1})^2 \mathcal{A}^N + 2\Delta g^{N+1} \mathcal{B}^N + C^N. \tag{7}$$

Here, $\mathcal{A}^N$ represents the contributions from $gg \to gg$ and $gg \to q\bar{q}$, $\mathcal{B}^N$ the ones from $qg \to qg$, and C^N those from the (anti)quark scatterings (iv) above; in each case, the appropriate combinations of Δq, $\Delta \bar{q}$ distributions and fragmentation functions are included.

Being a quadratic form in Δg^{N+1}, $\Delta\sigma^{\pi}(N)$ possesses an extremum, given by the condition

$$\mathcal{A}^N \Delta g^{N+1} = -\mathcal{B}^N. \tag{8}$$

We note in passing that the same equation may also be derived directly from Eq. (4) by regarding the cross section as a functional of Δg, using a variational approach, and taking Mellin moments of the resulting stationarity condition. In the following, we neglect the contribution from the $gg \to q\bar{q}$ channel which, as we discussed above, is much smaller than that from $gg \to gg$ for the $p_\perp$ in which we are interested. The coefficient $\mathcal{A}^N$ is then positive, and Eq. (8) describes a minimum of $\Delta\sigma^\pi(N)$, with value

$$\Delta\sigma^\pi(N)|_{\min} = -(\mathcal{B}^N)^2/\mathcal{A}^N + C^N. \tag{9}$$

It is then straightforward to perform a numerical Mellin inversion of this minimal cross section:

$$\frac{p_\perp^3 d\Delta\sigma^\pi}{dp_\perp}\bigg|_{\min} = \frac{1}{2\pi i}\int_\Gamma dN (x_T^2)^{-N} \Delta\sigma^\pi(N)|_{\min}, \tag{10}$$

where Γ denotes a suitable contour in complex-N space.

For the numerical evaluation, we use the LO Δq, $\Delta\bar{q}$ of GRSV (Glück, Reya, Stratmann, and Vogelsang) [4], the D_c^π of [6], and a fixed scale $\mu = 2.5$ GeV. We find that the minimal asymmetry resulting from this exercise is negative indeed, but very small: in the range $p_\perp \sim 1$–4 GeV its absolute value does not exceed 10^{-3}. The Δg in Eq. (8) that minimizes the asymmetry is shown in Fig. 2, compared to Δg of the GRSV LO "standard" set [4]. One can see that it has a node and is generally much smaller than the GRSV one, except at large x. The node makes it possible to probe the two gluon densities in the gg term at values of x_a, x_b where they have different sign, which helps in decreasing $A_{\rm LL}^\pi$.

Even though we have made some approximations in deriving the bound in Eq. (10), we do believe that it exhibits the basic difficulty with a sizable negative $A_{\rm LL}^\pi$ at moderate $p_\perp$: The fact that the cross section is a quadratic form in Δg effectively means that it is bounded from below. Note that this bound does not *always* imply that a negative $A_{\rm LL}^\pi$ is small: At higher $p_\perp$ it does allow a fairly large $A_{\rm LL}^\pi$.

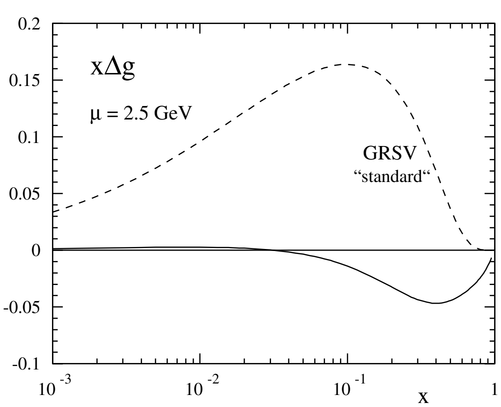

FIG. 2. $\Delta g(x, \mu = 2.5$ GeV) resulting from Eq. (8) (solid). The dashed line shows the GRSV LO "standard" Δg [4].

In its details, the qualitative picture drawn by our study is of course subject to a number of corrections. First of all, we have integrated the cross section over all rapidities, whereas for the PHENIX data $|\eta| \leq 0.38$. It is instructive to investigate the qualitative differences associated with this. Figure 3 shows the polarized LO cross section at $p_\perp = 2.5$ GeV versus the "distance" $|x_a - x_b|$ in parton momentum fractions in Eq. (2), for integration over all η and for $|\eta| \leq 0.38$. Here we have used the GRSV polarized parton densities. The larger the rapidity range probed, the more likely become collisions of partons with rather different momentum fractions. Indeed, the distribution for $|\eta| \leq 0.38$ is steeper, implying that a node in $\Delta g(x, \mu)$ will now be somewhat less efficient in promoting negative values for the asymmetry.

In a more realistic calculation, one would also prefer $\mu \sim p_\perp$ to a fixed μ. Furthermore, since the Δq and $\Delta\bar{q}$ are coupled to Δg via evolution, any change of Δg will require a retuning of the Δq, $\Delta\bar{q}$ densities, so that the agreement with the polarized DIS data remains intact. Inclusion of the NLO corrections is important as well.

All these points can be thoroughly addressed only in a "global" NLO analysis of the data, taking into account the results from polarized DIS as well. We will now report on such an analysis. Given that the data are still preliminary, this may seem premature. Our primary goal, however, is to investigate whether the findings of our somewhat idealized case, as summarized by Eqs. (9) and (10), hold true in general.

"Global" NLO analysis.—The main technical difficulty in a full global NLO analysis of polarized DIS and $pp \to \pi^0 X$ data is the numerical complexity of the evaluation of the NLO corrections for the latter cross section. A convenient way to alleviate this problem was presented in [9]. Starting from Eq. (2), one expresses the Δa, Δb by their Mellin inverses, e.g.,

$$\Delta a(x, \mu) = \frac{1}{2\pi i}\int_{\Gamma_N} dN x^{-N} \Delta a^N(\mu). \tag{11}$$

After interchange of integrations, one obtains

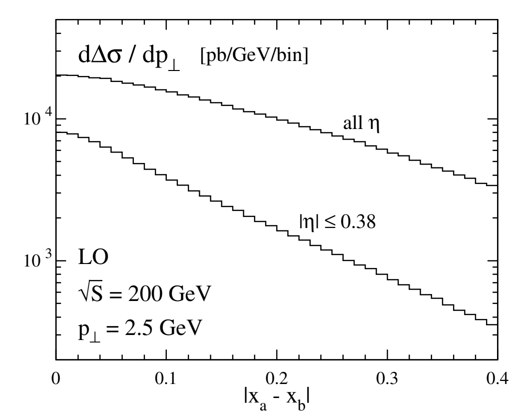

FIG. 3. $d\Delta\sigma^\pi/dp_\perp$ in bins of $|x_a - x_b|$.

$$\frac{d\Delta\sigma^{\pi}}{dp_{\perp}\,d\eta} = \sum_{a,b}\int_{\Gamma_N} dN \int_{\Gamma_M} dM\, \Delta a^N(\mu)\Delta b^M(\mu) \times \rho_{ab}^{\pi}(N, M, p_{\perp}, \eta, \mu), \tag{12}$$

where the ρ_{ab}^{π} contain the partonic cross sections, the fragmentation functions, and all integrations over momentum fractions, with the factors x_a^{-N} and x_b^{-M} as complex "dummy" parton distributions according to Eq. (11). The strength of this approach is that there is no dependence of ρ_{ab}^{π} on the moments Δa^N, Δb^M of the true parton densities. This means that the ρ_{ab}^{π} can be precalculated *prior* to the analysis on a specific array of the two Mellin variables N and M. One chooses a convenient functional form for the parton distributions, depending on a set of free parameters. The latter are then determined from a χ^2 minimization procedure. The double inverse Mellin transformation which finally links the parton distributions with the precalculated ρ_{ab}^{π} of course still needs to be performed in each step of the fitting procedure, but becomes extremely fast by choosing the values for N, M on the contours Γ_N, Γ_M simply as the supports for a Gaussian integration.

Following these lines, we have performed a simultaneous analysis of all data from polarized DIS and of the preliminary PHENIX data for $pp \to \pi^0 X$. We have used several different functional forms for the polarized gluon density, in particular, allowing it to have a node. The quark densities were allowed to vary as well. We have artificially decreased the error bars of the data points for A_{LL}^{π} in order to see whether the fit can be forced to reproduce a negative A_{LL}^{π} of about -2% in the region $p_{\perp} \simeq 1$–4 GeV. We have also slightly shifted individual data points to study the response of the fit. In no case have we been able to find a fit that yielded a negative A_{LL}^{π} with absolute value larger than a few times 10^{-3}. Even those fits, however, gave a negative A_{LL}^{π} only at the higher end of the $p_{\perp}$ interval, and invariably they led to a polarized gluon density that had a node and tended to violate positivity $|\Delta g| \leq g$ in certain ranges of x. The global analysis thus confirms our qualitative finding above that any negative A_{LL}^{π} is also very small.

Conclusions.—Our analysis demonstrates that pQCD at *leading power* in $p_{\perp}$ predicts that A_{LL}^{π} is bounded from below by $A_{\mathrm{LL}}^{\pi} \gtrsim \mathcal{O}(-10^{-3})$ in the region $p_{\perp} \simeq 1$–4 GeV. The observation relies on collinear factorization and on exploring the physically acceptable ranges of parton distribution and fragmentation functions.

For now, the data [2] do not allow a compelling conclusion on whether the bound is violated or not. What should one conclude if future, more precise, data will indeed confirm a sizable negative A_{LL}^{π}? As indicated earlier, corrections to Eq. (2) are associated with power-suppressed contributions to the cross section. Since $p_{\perp}$ is not too large, such contributions might well be significant. On the other hand, comparisons of unpolarized π^0 spectra measured at colliders with NLO QCD calculations do not exhibit any compelling trace of nonleading power effects even down to fairly low $p_{\perp} \gtrsim 1$ GeV, within the uncertainties of the calculation. It is conceivable that the spin-dependent cross section with its fairly tedious cancellations has larger power-suppressed contributions than the unpolarized one. One may attempt to model the effects by implementing an "intrinsic" transverse-momentum ($k_{\perp}$) smearing for the initial partons which generically leads to corrections by powers of $\langle k_{\perp}\rangle/p_{\perp}$, with $\langle k_{\perp}\rangle$ an average $k_{\perp}$. Such effects were shown to have indeed some potential impact on A_{LL}^{π} at $p_{\perp} \leq 5$ GeV [10]. A negative A_{LL}^{π} would open up a quite unexpected window on aspects of nucleon structure and limitations of pQCD thus far little explored.

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