Measurement of the Solar Gravitational Deflection of Radio Waves using Geodetic Very-Long-Baseline Interferometry Data, 1979–1999

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We used very-long-baseline interferometry (VLBI) to measure the deflection by the Sun of radio waves emanating from distant compact radio sources. This bending is characterized in the parametrized post-Newtonian formalism by γ, which is unity in general relativity. Using a large geodetic VLBI data set, we obtained $\gamma = 0.9998_3 \pm 0.0004_5$ (estimated standard error). We found no systematic biases from our analysis of subgroups of data.

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Introduction.—Measuring the bending of starlight by the Sun is a classic test of Einstein's theory of general relativity (GR). As expressed in the parametrized post-Newtonian (PPN) formalism (see, for example, Refs. [1,2]), the predicted angle θ through which an electromagnetic ray from a distant source is deflected by the Sun is given approximately by

$$\theta \cong \frac{(1+\gamma)GM_\odot}{c^2 b}(1+\cos\phi), \tag{1}$$

where G is the universal gravitational constant, c is the speed of light in vacuum, $M_\odot$ is the mass of the Sun, b is the impact parameter (distance of closest approach of the ray's path to the center of mass of the Sun), ϕ is the solar elongation angle (angle between the Sun and the source as viewed from Earth), and γ is the PPN parameter that characterizes the contribution of space curvature to gravitational deflection. For GR, $\gamma \equiv 1$, and for a ray that grazes the Sun's limb, $\theta \approx 1.75$ arcsec.

Since other metric theories of gravitation may predict different values of γ, high accuracy measurements of γ are of fundamental importance. The earliest measurements of γ were done with optical telescopes, but this method is limited by distortions of the optics and the photographic plates and have generally yielded results with uncertainties in γ of over 20%, i.e., standard errors greater than 0.2 (see, for example, Refs. [3,4]). In 1967, Shapiro [5] first proposed that γ could be estimated via interferometric observations of radio waves from celestial radio sources that are nearly occulted by the Sun. (In these observations, one would measure the differences in the times of arrival of the radio waves at antennas located around the globe. The expression appropriate for these differences in signal arrival times can be found in, for example, Refs. [6,7].) Since 1967, several such very-long-baseline interferometry (VLBI) experiments have been carried out, yielding estimates of γ with generally increasing accuracy [7–11].

The geodetic VLBI database, in particular, which is intended primarily to monitor various motions of Earth (e.g., precession, nutation, wobble of its crust, variations in its rotation, and tectonics), is a very useful data set for this purpose. These data are useful because of their inherent statistical accuracy, long record, and inclusion of a large number of radio sources well distributed on the sky, each with little internal structure. Robertson *et al.* [11] used the then-available geodetic VLBI data set and estimated γ to be 1.000 ± 0.002. The reported accuracy of this deflection experiment was then surpassed by the occultation-VLBI experiment of Lebach *et al.* [7], in which γ was found to be 0.9996 ± 0.0017. This estimate is the most accurate one previously obtained from a deflection experiment.

In this Letter, we report the results of our estimate of γ from a large geodetic VLBI data set. Our analysis of these data emphasizes the investigation of possible systematic errors, some of which may be a result of using a large data set with so many observations not very sensitive to γ.

Data and results.—Our data set consists of measurements from observations in about 2500 ~24-hour sessions ("experiments") spanning the years 1979–1999. The distribution of the solar elongation angles of the sources observed is shown in Fig. 1.

These data were acquired with the support of a myriad of U.S. and foreign agencies, and are maintained and made available by the NASA Goddard Space Flight Center (GSFC) VLBI group via the NASA Crustal Dynamics Data Information System [12]. (Other VLBI data were not used, for example because the experiments used mobile antennas or were conducted as equipment tests.)

The experiments that we used in this study involved 87 VLBI sites and 541 radio sources (Fig. 2), yielding a total of more than 1.7×10^6 ionosphere-corrected group-delay measurements [13]. These data were obtained from the raw observations using standard correlation and delay and rate estimation (i.e., "fringe-fitting") procedures [13].

To identify and delete significant outliers, we used the SOLVK software [14] to make a preliminary solution for each individual experiment. In these solutions, we

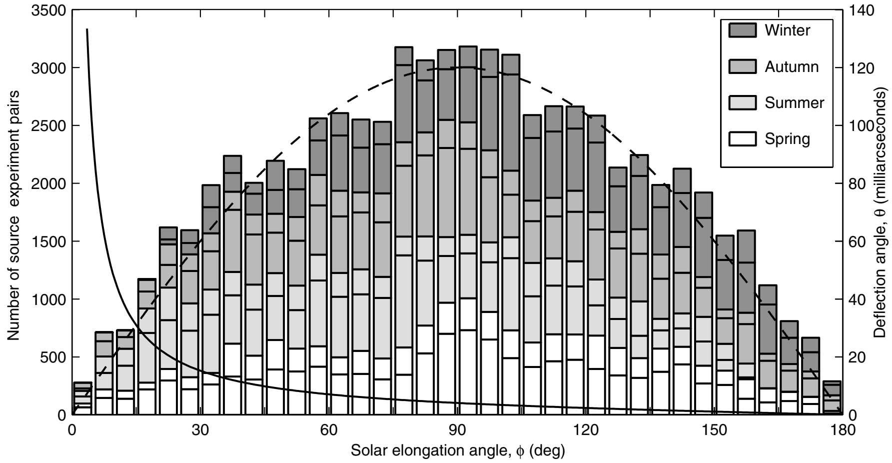

FIG. 1. Histogram of solar elongation angles associated with each source used in each experiment (see text). A source is counted once for each experiment in which it is observed. This histogram is divided into four large subgroups based on the season (see text) in which the observations were made. The lower portion of each seasonal bar represents sources with positive declination angles and the upper portion of each seasonal bar represents sources with negative declination angles. Overlaid on this plot is a dashed curve indicating the solar elongation angles expected if the sources were distributed uniformly on the sky and a solid line showing the relationship between solar elongation angle and predicted deflection angle (right-hand scale).

estimated a relatively small number of parameters specific to individual experiments such as atmosphere and clock parameters. Other parameters such as those representing source positions were fixed at values determined from the NASA GSFC solution 1102g [15]. Model values and partial derivatives for the group delays, also contained in the data files from GSFC, were calculated using version 9.10 of the computer program CALC [16]. This editing resulted in the deletion of 0.6% of the data, based on the criterion of the postfit residual of a measurement exceeding its standard error by at least a factor of 4.

Following this editing step, we used the SOLVK and GLOBK Kalman filter analysis software [14] to estimate values for a full set of parameters, consisting of radio source positions (right ascension and declination), three-dimensional antenna site positions, Earth orientation [precession, nutation, polar motion (“wobble”), UT1, and UT1 rate], variations of zenith atmospheric propagation delay separately for each site, clock variations at each site [14], and the PPN parameter γ. (The contributions of the atmosphere and clocks at each site to the measured group delays for each experiment were modeled

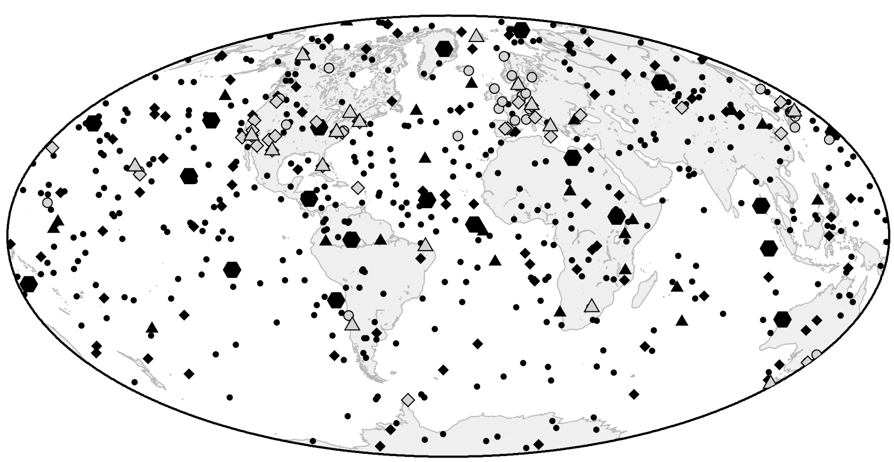

FIG. 2. Distribution of VLBI sites and radio sources. Sites: Gray shaded circles represent the 36 sites used in 1–10 experiments, gray shaded diamonds represent the 32 sites used in 11–100 experiments, and gray shaded triangles represent the 19 sites used in more than 100 experiments. Sources: Black shaded circles represent the 413 sources used in 1–100 experiments, black shaded diamonds represent the 84 sources used in 101–500 experiments, black shaded triangles represent the 24 sources used in 501–1000 experiments, and black shaded hexagons represent the 20 sources used in more than 1000 experiments. Note that the VLBI sites are not uniformly distributed around Earth. During the 1980s, very few negative declination sources were observed and these sources were used in only a few experiments.

as Gauss-Markov random walk stochastic processes [14].) We refer to the results from such an analysis for all of the data as a "global" solution and those from one experiment as a "daily" solution. In global solutions, we assumed radio source positions to be constant during the 1979–1999 period, although we allowed site positions, Earth wobble, rotation, and nutation to vary stochastically, in each case with appropriately loose *a priori* constraints. In some of our sensitivity tests, we also obtained estimates of γ from "partial" global solutions, in which we did not estimate radio source positions but instead used the positions obtained from our reference (see below) global solution. In obtaining our daily estimates of γ, we similarly did not estimate source positions but rather used the positions obtained from our reference global solution. We used the "NMF" atmospheric-delay mapping functions [17] although our sensitivity tests indicated that our results were insensitive to whether we used the NMF or the "CfA-2.2" [18] mapping functions. In addition, we tested the sensitivity of our results to the inclusion of horizontal refractivity gradients in our parametrization of the atmosphere and found no significant difference in our estimate of γ. We obtained a global estimate for γ of 0.9998_3 with a statistical standard error of 0.0002_6 where this standard error is based on the propagation through the Kalman filter of the SNR-derived standard errors in the group-delay measurements after each was added in quadrature with a 15 ps standard error representing the known noise added in the correlation process [13]. Typical group-delay standard errors range from about 65 ps in the early 1980s down to about 35 ps in the late 1990s.

Error analysis.—Our error analysis has two goals: (1) test the reliability of our estimate of the random contribution to the uncertainty in our estimate of γ, and (2) investigate sources of systematic error that could have a significant effect on our estimate of γ.

As a test of the reliability of the above standard error of 0.0002_6, we calculated the random component of our uncertainty in γ from our daily estimates of γ. We obtained 0.0004_5 as the sample standard error of the mean of these estimates. To be more conservative, we use this value as our estimate of our standard error. We did a spectral analysis of the time series of our daily estimates to search for periodic signals. Because the daily estimates are not evenly spaced in time, we used a technique based on the method developed by Lomb [19], programmed by Press *et al.* [20], and modified by us to handle data points with unequal standard errors. We found no significant periodic signal present in this time series.

In a further search for systematic errors, we generated additional global solutions using subsets of the VLBI data grouped by (a) solar elongation angle, (b) source elevation angle, ε, (c) source declination, and both the (d) season and (e) time period within our two decade span of data during which the observations were made. Results from these (global) tests were probed to try to uncover systematic errors, in particular, errors in the atmospheric propagation delay models (see, for example, Ref. [21]), that might affect estimates of γ.

The gravitational deflection of the radio waves by the Sun is in effect a change in the apparent position of the radio source, dependent on its solar elongation angle. We therefore performed "elongation angle cutoff" tests in which the elongation angle cutoff, $\phi_{\min}$, is the minimum allowable solar elongation angle of an observation used in a particular solution. In our global solution $\phi_{\min} \approx 0.7°$ (or roughly three solar radii); we refer to this solution as our "reference solution." We also made estimates of γ with elongation angle cutoff values of 10°, 20°, ..., 90° for suites of runs where we included separately group delays acquired from radio sources from (a) all of the sky, (b) only the northern celestial hemisphere (i.e., sources with positive declinations), (c) only the southern celestial hemisphere (i.e., sources with negative declinations), (d) only those experiments conducted before 1990, (e) only those experiments conducted between 1990 and 1999, (f) only those experiments conducted between 1990 and 1994, and (g) only those experiments conducted between 1995 and 1999. Because the Sun is higher in the sky during the summer than in the winter and atmospheric conditions can differ greatly during different seasons, we also separated our data into four groups based on season and compared, as functions of $\phi_{\min}$, estimates of γ obtained separately from each of these groups. The results from none of these (global) solar elongation angle tests suggest a systematic relationship between the choice of $\phi_{\min}$ and our estimate of γ; results from subsets *b* and *c* are plotted, as representative examples, in Fig. 3. As with the other subsets, there is no noticeable deviation from GR until $\phi_{\min} \approx 50°$, above which there is increased scatter about GR's prediction but still no indication of a significant systematic bias. Note also that, because successive points in each plot are based on data sets that differ only by the exclusion of data with solar elongation angles within a 10° range, the values of these points are highly correlated.

In addition, using the 0° cutoff solution as the reference, we separately varied the minimum source elevation angle, $\varepsilon_{\min}$, from 5° (same as our reference solution because we have no source elevation angles below this value) to 65° (above which there is insufficient sensitivity to γ to warrant study) in 10° increments. The results of these (global) elevation-angle tests (also see Fig. 3) similarly suggest the absence of a systematic relationship between the choice of $\varepsilon_{\min}$ and our estimate of γ.

Conclusion.—Our estimate of γ, $0.9998_3 \pm 0.0004_5$, is within one standard deviation of the value predicted by GR. We have found no systematic errors affecting our data set to require an increase in the value of our conservatively estimated standard error. Our estimated standard error is nearly 4 times smaller than that of any previously published measurement of gravitational deflection. Since

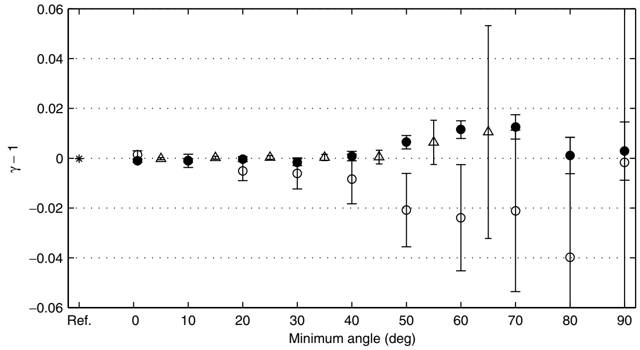

FIG. 3. Differences between global estimates of γ and GR's prediction as a function of minimum source elevation angle, $\varepsilon_{\min}$ (open triangles), and minimum solar elongation angle, $\phi_{\min}$, for positive declination sources (black shaded circles) and negative declination sources (open circles). These deviations from GR are plotted as a function of the smallest source elevation or solar elongation angle contained in each subset of data. For example, the open circle plotted with an abscissa of 50° represents the deviation in γ from GR's prediction determined from negative declination sources with $\phi_{\min} = 50°$, i.e., the difference from zero in the value of this plotted point is determined solely by data with solar elongation angles between 50° and 180°. "Ref." refers to $\gamma - 1$ for our reference solution ($\phi_{\min} \approx 0.7°$ and $\varepsilon_{\min} \approx 5°$), for which there are no cutoffs. Error bars represent the statistical standard error scaled by 1.7, the ratio of the standard error that we estimated for the mean of our daily estimates of γ, and the statistical standard error from our (complete) global solution (see text).

this paper was completed, Bertotti *et al.* [22] reported an estimate of γ from a spacecraft tracking time-delay experiment consistent with GR and with a stated standard error of $\approx 2 \times 10^{-5}$, about 20 times smaller than ours.

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