

Universal Decoherence in Solids

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Symmetry implications for the decoherence of quantum oscillations of a two-state system in a solid are studied. When the oscillation frequency is small compared to the Debye frequency, the universal lower bound on the decoherence due to the atomic environment is derived in terms of the macroscopic parameters of the solid, with no unknown interaction constants.

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The problem of tunneling and coherence in the presence of dissipation has been extensively studied in the past; see, e.g., the review of Leggett *et al.* [1]. In this Letter we revise the case of quantum oscillations of a particle (an electron, an atom, or a defect) in a double-well potential in a solid in the presence of weak dissipation due to the nonconducting atomic environment. We demonstrate that all previous works on this subject have missed two important facts. The first fact is that the double-well potential formed by the local arrangement of atoms in a solid is defined in the coordinate frame of that local atomic environment, not in the laboratory frame. The second fact is that the interactions of the tunneling variable with phonons must be invariant with respect to global translations and rotations. When these facts are taken into account, a simple universal result for the decoherence rate can be obtained.

Let the degenerate minima of the double-well potential be located at the points X_0 and $-X_0$ inside the solid. In the case of weak dissipation, if, at the moment of time $t = 0$, the particle is prepared in the state $X = X_0$, then for any moment $t > 0$

$$\langle X(0)X(t) \rangle = X_0^2 e^{-\Gamma t} \cos(\omega_0 t), \quad (1)$$

where $\langle \dots \rangle$ means quantum average, $\hbar\omega_0 = \Delta$ is the tunneling splitting, and $\Gamma \ll \omega_0$ is the decoherence rate. When addressing the effect of dissipation one should answer two questions: (i) how is Δ renormalized by the atomic environment? and (ii) what is the value of Γ ? The answer to the first question requires the precise knowledge of the interactions with the environmental degrees of freedom. After the latter are integrated out from the total action [2], Δ can be computed via the instanton of the remaining effective action for the tunneling variable [3,4]. For the Ohmic interactions, Δ can be nicely ex-

pressed in terms of a single measurable dissipation constant [4]. In this Letter we show that a similar beautiful result follows for the decoherence due to the super-Ohmic interactions with phonons. In the limit when ω_0 is small compared to the Debye frequency, ω_D , the decoherence rate Γ can be expressed in terms of measurable constants of the solid, with no unknown interaction constants. Similar results for problems involving tunneling of the angular momentum have been obtained earlier [5,6]. Here we present a rigorous derivation of the universal expression for Γ due to phonons for an arbitrary double-well potential in a solid.

The Hamiltonian of the system is

$$\mathcal{H} = \frac{1}{2} m \dot{\mathbf{R}}^2 + U(\mathbf{R}) + \alpha_{iklm} R_i R_k u_{lm}(\mathbf{R}) + \dots + \int d^3 r \left(\frac{1}{2} \rho \dot{\mathbf{u}}^2 + \lambda_{iklm} u_{ik} u_{lm} \right), \quad (2)$$

where \mathbf{R}' and \mathbf{R} are the radius vectors of the particle of mass m in the laboratory coordinate frame and in the coordinate frame rigidly coupled with a solid, respectively, $U(\mathbf{R})$ is the double-well potential, $\mathbf{u}(\mathbf{r})$ is the phonon displacement field, $u_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$ is the strain tensor, ρ is the mass density of the solid, λ_{iklm} is the tensor of elastic coefficients, and α_{iklm}, \dots are coefficients of the expansion of the long-wave interaction between \mathbf{R} and \mathbf{u} into a series on u_{ij} . The invariance of the interaction with respect to the translation $[\mathbf{u}(\mathbf{r}) \rightarrow \mathbf{u}(\mathbf{r}) + \mathbf{a}]$ and rotation $[\mathbf{u}(\mathbf{r}) = \boldsymbol{\Phi} \times \mathbf{r}]$ of a solid as a whole rules out combinations of the forms $\mathbf{R} \cdot \mathbf{u}$ and $\mathbf{R} \cdot \nabla \times \mathbf{u}$. All interaction terms, therefore, must be even in the number of the spatial components of \mathbf{R} . Our result is based, in part, upon this observation. Another important observation is that the kinetic energy of the particle depends on $\dot{\mathbf{R}}'$, while the potential energy depends on \mathbf{R} . Substituting $\mathbf{R}' = \mathbf{R} + \mathbf{u}$ into Eq. (2), one obtains

$$\mathcal{H} = \frac{1}{2} m \dot{\mathbf{R}}^2 + U(\mathbf{R}) + m \dot{\mathbf{R}} \cdot \dot{\mathbf{u}}(\mathbf{R}) + \alpha_{iklm} R_i R_k u_{lm}(\mathbf{R}) + \dots + \int d^3 r \left(\frac{1}{2} [\rho + m \delta(\mathbf{r} - \mathbf{R})] \dot{\mathbf{u}}^2 + \lambda_{iklm} u_{ik} u_{lm} \right). \quad (3)$$

The renormalization of the mass density in Eq. (3) is totally insignificant in the long-wave limit (see below), but the presence of the effective dynamic interaction, $m \dot{\mathbf{R}} \cdot \dot{\mathbf{u}}$, which is linear in the tunneling variable, is absolutely crucial for our argument.

In this Letter we treat the renormalized value of Δ as a known parameter that can be obtained from experiment. We start with a symmetric double-well potential. Let the two degenerate minima of $U(\mathbf{R})$ be $\mathbf{R} = \pm \mathbf{R}_0$, so that the ground state and the first excited state of the particle are

$$\begin{aligned} |0\rangle &= \frac{1}{\sqrt{2}}(|\mathbf{R}_0\rangle + |-\mathbf{R}_0\rangle), \\ |1\rangle &= \frac{1}{\sqrt{2}}(|\mathbf{R}_0\rangle - |-\mathbf{R}_0\rangle). \end{aligned} \quad (4)$$

We assume that the energy gap, Δ , between these two states, renormalized by the environment, is small compared to the distance to other energy levels of the particle. Our goal is to compute the decoherence of the low-energy states, $|\psi\rangle = C_1|0\rangle + C_2|1\rangle$, due to the decay of $|1\rangle$ into $|0\rangle$, accompanied by the emission of a phonon of frequency $\omega_0 = \Delta/\hbar$. In the limit when $\omega_0 \ll \omega_D$, the phonon wavelength is large compared to the interatomic distance, which justifies the use of the long-wave limit of the elastic theory. To simplify calculations, we also assume that the tunneling distance, $2X_0$, is small compared to the wavelength of phonons of frequency ω_0 . Writing \mathbf{R}_0 as $(X_0, 0, 0)$, one has $\hat{X}|\pm\mathbf{R}_0\rangle = \pm X_0|\pm\mathbf{R}_0\rangle$, while $\hat{Y}|\pm\mathbf{R}_0\rangle = \hat{Z}|\pm\mathbf{R}_0\rangle = 0$. Consequently, $\hat{X}|0\rangle = X_0|1\rangle$ and $\hat{X}|1\rangle = X_0|0\rangle$, while \hat{Y} and \hat{Z} produce a zero result when acting on $|0\rangle$ and $|1\rangle$. It is now easy to see that any combination of the even number of operators \hat{X} , \hat{Y} , and \hat{Z} , including \hat{X}^2 , etc., has a zero matrix element between $|0\rangle$ and $|1\rangle$. Thus, the only decohering term in Eq. (3) is $m\hat{\mathbf{X}}\hat{\mathbf{u}}_x$, with

$$\langle 0|\hat{X}|1\rangle = \langle 0|\frac{i}{\hbar}(\hat{\mathcal{H}}\hat{X} - \hat{X}\hat{\mathcal{H}})|1\rangle = -i\omega_0 X_0, \quad (5)$$

where it is explicitly implied that $|0\rangle$ and $|1\rangle$ are the approximate eigenstates of the full Hamiltonian, so that $\Delta = \langle 1|\hat{\mathcal{H}}|1\rangle - \langle 0|\hat{\mathcal{H}}|0\rangle$ is the tunneling splitting renormalized by the interactions.

We first consider the case of zero temperature. The $T = 0$ decoherence rate is given by the Fermi golden rule:

$$\begin{aligned} \Gamma &= \frac{2\pi}{\hbar} \sum_{i \neq j} \langle i|\mathbf{m}\hat{\mathbf{R}} \cdot \hat{\mathbf{u}}|j\rangle \langle j|\mathbf{m}\hat{\mathbf{R}} \cdot \hat{\mathbf{u}}|i\rangle \delta(E_i - E_j) \\ &= \frac{2\pi}{\hbar} m^2 |\langle 0|\hat{X}|1\rangle|^2 \sum_{\mathbf{k}, i} |\langle \mathbf{k}, i|\hat{\mathbf{u}}_x|0_{\text{ph}}\rangle|^2 \delta(\hbar\omega_{\mathbf{k}i} - \Delta), \end{aligned} \quad (6)$$

where $|0_{\text{ph}}\rangle$ and $|\mathbf{k}, i\rangle$ are the states of the solid with zero phonons and one phonon of the wave vector \mathbf{k} and polarization i respectively, and $\omega_{\mathbf{k}i} = c_i k$ is the phonon frequency, with c_i being the speed of sound of the polarization i . The canonical quantization of the phonon field [7] yields

$$\hat{\mathbf{u}}_x = \frac{-i}{\sqrt{V}} \sum_{\mathbf{k}, i} \sqrt{\frac{\hbar\omega_{\mathbf{k}i}}{2\rho}} (a_{\mathbf{k}i} e^{i\mathbf{k}\mathbf{r}} - a_{\mathbf{k}i}^\dagger e^{-i\mathbf{k}\mathbf{r}}) e_i^x, \quad (7)$$

where V is the volume of the system, $a_{\mathbf{k}i}^\dagger$ and $a_{\mathbf{k}i}$ are operators of creation and annihilation of phonons, and \mathbf{e}_i are unit polarization vectors. Substitution of Eqs. (5) and (7) into Eq. (6), with account of $\langle (e_i^x)^2 \rangle = 1/3$, gives

$$\Gamma = \frac{\pi m^2 X_0^2 \omega_0^2}{3\hbar\rho V} \sum_{\mathbf{k}, i} \omega_{\mathbf{k}i} \delta(\omega_{\mathbf{k}i} - \omega_0). \quad (8)$$

Replacing the summation over \mathbf{k} by $V \int d^3k/(2\pi)^3$ one obtains

$$\Gamma = \frac{m^2 X_0^2 \omega_0^5}{6\pi\hbar\rho} \left(\frac{2}{c_t^3} + \frac{1}{c_l^3} \right), \quad (9)$$

where c_t and c_l are the speeds of the transversal and the longitudinal sound, respectively.

The following observation allows one to understand the above result in simple physical terms. Up to a numerical factor of order unity, Γ of Eq. (9) satisfies

$$\hbar\Gamma \sim p_0^2/2M, \quad (10)$$

with $p_0 = mX_0\omega_0$ being the rms value of the momentum of the oscillating particle and $M \sim \rho\lambda^3$ being the mass of the solid within the volume of dimensions λ , where λ is the wavelength of phonons of frequency ω_0 , averaged over polarizations. This volume of the solid adjacent to the particle must oscillate together with the particle in order to conserve the linear momentum. The latter is a consequence of the commonly neglected fact that the potential $U(\mathbf{R})$ is formed by atoms that are coupled to the rest of the solid, which makes the double well a part of the dissipative environment. The $p_0^2/2M$ width of the excited state is, therefore, a consequence of the conservation law that mandates the entanglement of the particle states with the states of the solid.

The super-Ohmic case has been studied in Ref. [1], based upon the spin-boson Hamiltonian,

$$\begin{aligned} \mathcal{H}_{\text{SB}} &= -\Delta_0 \hat{\mathbf{s}}_x + X_0 \hat{\mathbf{s}}_z \sum_{\alpha} C_{\alpha} x_{\alpha} \\ &+ \sum_{\alpha} \left(\frac{p_{\alpha}^2}{2m_{\alpha}} + \frac{1}{2} m_{\alpha} \omega_{\alpha}^2 x_{\alpha}^2 \right). \end{aligned} \quad (11)$$

Here $\hat{\mathbf{s}}$ is the spin-1/2 operator, Δ_0 is the bare splitting, α labels coordinates x_{α} , momenta p_{α} , frequencies ω_{α} , and masses m_{α} of harmonic oscillators, and C_{α} are constants of the linear coupling between the macroscopic variable X and the oscillators. The following result was obtained for the decoherence rate [1]: $\Gamma = (X_0^2/4\hbar)J(\Delta)$, where $J(\omega) = \sum_{\alpha} (\pi C_{\alpha}^2/2m_{\alpha}\omega_{\alpha}) \delta(\omega - \omega_{\alpha})$ and Δ is the renormalized splitting. This, of course, is a correct answer to the mathematical problem posed by Eq. (11). According to Ref. [1], for a nonlinear coupling of the form $F_{\alpha}(X)x_{\alpha}$, the quantity C_{α} in the above formulas should be replaced by $X_0^{-1}[F_{\alpha}(\frac{1}{2}X_0) - F_{\alpha}(-\frac{1}{2}X_0)]$. Then, when only even powers of X in $F_{\alpha}(X)$ are allowed by symmetry, the result for the decoherence rate is zero. As we have seen above, in a physical problem of quantum oscillations of a particle or

a defect in a double-well potential in a solid the linear coupling of the tunneling variable to the phonons is prohibited by symmetry, unless one identifies the coupling with the Galilean transformation term $m\dot{\mathbf{R}} \cdot \dot{\mathbf{u}}$ and chooses $C_\alpha \propto \omega_\alpha^2$. We, therefore, conclude that in relevant physical problems the only correct answer for Γ is given by Eq. (9). The beauty of this answer is that it does not depend on any unknown interaction constants but is expressed in terms of measurable parameters only.

We now generalize our answer to include the case of an asymmetric double well and finite temperature. We start with the first and assume that the energy minima of $U(\mathbf{R})$ are shifted with respect to one another by a small energy ϵ . The origin of the coordinate system can always be chosen such that these minima correspond to $\mathbf{R} = \pm \mathbf{R}_0$, with $\mathbf{R} = (X_0, 0, 0)$. The relevant two-state Hamiltonian of the particle is

$$h = -\Delta\hat{s}_x + \epsilon\hat{s}_z. \quad (12)$$

The solution of the corresponding Schrödinger equation yields

$$\begin{aligned} |0\rangle &= \frac{1}{\sqrt{2}}(C_-|\mathbf{R}_0\rangle + C_+|-\mathbf{R}_0\rangle), \\ |1\rangle &= \frac{1}{\sqrt{2}}(C_+|\mathbf{R}_0\rangle - C_-|-\mathbf{R}_0\rangle), \end{aligned} \quad (13)$$

where

$$C_\pm = \sqrt{1 \pm \epsilon/\sqrt{\Delta^2 + \epsilon^2}}. \quad (14)$$

The energy gap between these states is

$$\hbar\omega = \sqrt{\Delta^2 + \epsilon^2}. \quad (15)$$

It is easy to see that the states (13) are still the eigenstates of \hat{X}^2 , as for $\epsilon = 0$, so that $\langle 0|\hat{R}_i\hat{R}_j|1\rangle = 0$, as before. Among terms in Eq. (3) which are linear in $\dot{\mathbf{R}}$, the matrix element responsible for the decoherence continues to be

$$\langle 0|\hat{X}|1\rangle = -i\omega X_0 C_+ C_- = -i\omega_0 X_0, \quad (16)$$

which is independent of ϵ . The phonon part in Eq. (6), however, must be modified by replacing Δ with the full gap, $\hbar\omega = \sqrt{\Delta^2 + \epsilon^2}$, in the δ function. At a finite temperature one should sum up the transitions between the initial state, $|1\rangle$, with $n_{\mathbf{k}i}$ phonons and the final state, $|0\rangle$, with $n_{\mathbf{k}i} + 1$ phonons, or vice versa for the transition $|0\rangle \rightarrow |1\rangle$. This gives

$$\Gamma = \frac{\pi m^2 X_0^2 \omega_0^2}{3\hbar\rho V} \sum_{\mathbf{k},i} \omega_{\mathbf{k}i} (2n_{\mathbf{k}i} + 1) \delta(\omega_{\mathbf{k}i} - \omega) \quad (17)$$

instead of Eq. (8). The integration over the phonon modes must be accompanied by thermal averaging over the number of phonons, with $\langle n_{\mathbf{k}i} \rangle = [\exp(\hbar\omega_{\mathbf{k}i}/T) - 1]^{-1}$. The final answer for a biased double well at $T \neq 0$ reads

$$\Gamma = \frac{m^2 X_0^2 \omega_0^2 \omega^3}{6\pi\hbar\rho} \left(\frac{2}{c_l^3} + \frac{1}{c_t^3} \right) \coth\left(\frac{\hbar\omega}{2T}\right). \quad (18)$$

Once again we emphasize that it does not contain any unknown interaction constants. The effect of the interactions has been entirely absorbed into the renormalized splitting, $\hbar\omega_0$.

Among various tunneling problems, the problem studied above is relevant to the width of a low-energy optical mode that corresponds to quantum oscillations of an atom between two isomeric positions in the unit cell of a crystal. It describes a kind of an ammonia-molecule arrangement of atoms embedded in a solid. For, e.g., an atom of mass $m \sim 3 \times 10^{-23}$ g, oscillating at $\omega_0 \sim 10^{12}$ s $^{-1}$ in a symmetric double well with $X_0 \sim 2 \times 10^{-8}$ cm in a crystal with $\rho \sim 5$ g/cm 3 and $c_t \sim 10^5$ cm/s $< c_l$ at $T < \hbar\omega_0$, the width given by Eq. (18) is of the order of 10^{10} s $^{-1}$. For tunneling of electrons, the effect is generally small due to the smallness of the electron mass. One should remember, however, that Γ of Eq. (18) represents the ultimate lower limit of the decoherence rate which is mandated by the conservation law.

To have a complete picture we show how the above treatment of the particle problem can be transformed to include the problems of the decoherence of quantum oscillations of the angular variable, e.g., spin or an orbital moment. This problem is relevant to tunneling of the magnetic moment of a molecule between up and down directions in a crystal field [5,8]. Here we give its general solution in the presence of a bias field. When the tunneling variable is the angular momentum \mathbf{L} , the potential $U(\mathbf{R})$ in Eq. (3) must be replaced with

$$\mathcal{H}_0 = \beta_{ik} L_i L_k + \beta'_{iklm} L_i L_k L_l L_m + \dots, \quad (19)$$

where β_{ik} , β'_{iklm} , etc., are tensors determined by the symmetry of the crystal. The analogy with the particle problem is that \mathbf{L} in Eq. (19) is the angular momentum in the coordinate frame coupled to the local crystal axes.

Rotations of \mathbf{L} with respect to the crystal axes couple to the transversal phonon modes satisfying $\nabla \cdot \mathbf{u} = 0$. Consequently, $\dot{\mathbf{u}}(\mathbf{R})$ can be written as $\boldsymbol{\Omega} \times \mathbf{R}$, with $\boldsymbol{\Omega}$ given by $\boldsymbol{\Omega} = \frac{1}{2} \nabla \times \dot{\mathbf{u}}$. This allows one to write $m\dot{\mathbf{R}} \cdot \dot{\mathbf{u}}(\mathbf{R})$ in Eq. (3) in terms of the angular momentum,

$$m\dot{\mathbf{R}} \cdot \dot{\mathbf{u}}(\mathbf{R}) = (\mathbf{R} \times m\dot{\mathbf{R}}) \cdot \boldsymbol{\Omega} = \mathbf{L} \cdot \boldsymbol{\Omega} = \frac{1}{2} \mathbf{L} \cdot (\nabla \times \dot{\mathbf{u}}). \quad (20)$$

While this equation has been derived for \mathbf{L} of an orbital nature, it must equally apply to a spin. One comes to this conclusion by considering the effect of the rotation in a stationary coordinate frame which axes are determined by the local crystal field at the location of the spin. The rotation of that frame due to a transversal phonon is equivalent to the magnetic field, which results in the same effective interaction with the spin as it is for the orbital moment.

The other interactions of \mathbf{L} with the phonon field are of the magnetostriction form,

$$\mathcal{H}_{\text{int}} = \alpha_{iklm} L_i L_k u_{lm}(\mathbf{R}) + \dots \quad (21)$$

In the absence of the external field, the symmetry with respect to the time reversal requires that all terms in Eq. (21) contain an even number of L components. The external field, \mathbf{H} , adds the Zeeman term $-\gamma\mathbf{L} \cdot \mathbf{H}$ to the Hamiltonian, with γ being the gyromagnetic ratio. Thus, the total Hamiltonian becomes

$$\begin{aligned} \mathcal{H} = & \beta_{ik} L_i L_k + \beta'_{iklm} L_i L_k L_l L_m + \cdots - \gamma \mathbf{L} \cdot \mathbf{H} \\ & + \frac{1}{2} \mathbf{L} \cdot [\nabla \times \dot{\mathbf{u}}(\mathbf{R})] + \alpha_{iklm} L_i L_k u_{lm}(\mathbf{R}) + \cdots \\ & + \int d^3r \left(\frac{1}{2} \rho \dot{\mathbf{u}}^2 + \lambda_{iklm} u_{ik} u_{lm} \right). \end{aligned} \quad (22)$$

The main difference of the quantum problem for \mathbf{L} from the quantum problem for \mathbf{R} is that different components of the operator $\hat{\mathbf{L}}$ do not commute with each other. For certainty, we study the situation when the classical energy minima of the potential correspond to the classical vector \mathbf{L} looking in one of the two directions along the Z axis. This could be, e.g., the case of a biaxial crystal field and a weak magnetic field applied along the Z axis, $\mathcal{H}_0 = -\beta_z L_z^2 + \beta_x L_x^2 - \gamma L_z H_z$, with $\beta_z, \beta_x > 0$. At $\beta_x = 0$ the $|L\rangle$ and $|-L\rangle$ eigenstates of \hat{L}_z would coincide with the eigenstates of \mathcal{H}_0 . Since \hat{L}_z does not commute with \hat{L}_x , any small $\beta_x L_x^2$ term mixes these states:

$$\begin{aligned} |0\rangle &= \frac{1}{\sqrt{2}} (C_- |L\rangle + C_+ |-L\rangle), \\ |1\rangle &= \frac{1}{\sqrt{2}} (C_+ |L\rangle - C_- |-L\rangle), \end{aligned} \quad (23)$$

where C_{\pm} are given by Eq. (14) with $\epsilon = 2\gamma LH$.

As in the particle problem, the effect of the interactions in Eq. (22) on the states $|0\rangle$ and $|1\rangle$ of Eq. (23) is twofold. First, interactions renormalize Δ in the expression for the energy gap, Eq. (15). Second, they generate the finite width of $|1\rangle$ due to the finite probability of the decay $|1\rangle \rightarrow |0\rangle$. The latter, as in the particle problem, must be considered with an account of the symmetry of Eq. (22). In the quantum problem the products of the L components in Eq. (22) must be replaced by the symmetric combinations of the \hat{L}_i operators in order to preserve the Hermitian property of the Hamiltonian, e.g., $L_i L_k \rightarrow \frac{1}{2}(\hat{L}_i \hat{L}_k + \hat{L}_k \hat{L}_i)$, or, equivalently, the symmetry of the tensors β_{ik} , etc., with respect to the transposition of indices must be enforced. It is then easy to see that $\langle 0 | [\hat{L}_i \hat{L}_k + \hat{L}_k \hat{L}_i] | 1 \rangle = 0$. This is true for any combination of the even number of the operators \hat{L}_i . Thus, the only term in Eq. (22) responsible for the decoherence of quantum oscillations between L and $-L$ is $\frac{1}{2} \mathbf{L} \cdot [\nabla \times \dot{\mathbf{u}}(\mathbf{R})]$, which is independent of any interaction constants. Consequently, the relevant matrix element of the operator of the angular momentum is

$$\langle 0 | \hat{L}_z | 1 \rangle = L C_+ C_- = L \omega_0 / \omega, \quad (24)$$

and

$$\begin{aligned} \Gamma = & \frac{\pi}{2\hbar^2} |\langle 0 | \hat{L}_z | 1 \rangle|^2 \\ & \times \left\langle \sum_{\mathbf{k}, i} |\langle n_{\mathbf{k}i} + 1 | (\nabla \times \dot{\mathbf{u}})_z | n_{\mathbf{k}i} \rangle|^2 \delta(\omega_{\mathbf{k}i} - \omega) \right\rangle_T, \end{aligned} \quad (25)$$

where $\langle \cdots \rangle_T$ means thermal average. Substituting here the quantized phonon field of Eq. (7), one obtains

$$\Gamma = \frac{L^2 \omega_0^2 \omega^3}{12\pi\hbar\rho c_l^5} \coth\left(\frac{\hbar\omega}{2T}\right), \quad (26)$$

which is the angular equivalent of Eq. (18). This formula generalizes the result of Ref. [5] obtained for $H = 0$. It contains no unknown interaction constants, the same as Eq. (18).

As in the coordinate tunneling problem, it is interesting to notice that at $H = 0$ and $T = 0$, Eq. (26) can be written as $\hbar\Gamma \sim L^2/2I$, where $I \sim \rho\lambda^5$ is the moment of inertia of the part of the solid of dimensions $\lambda \sim 2\pi c_l/\omega_0$, adjacent to the particle (molecule) whose angular momentum tunnels between L and $-L$. Thus, the physical origin of the decoherence given by Eq. (26) is the entanglement of the angular states of the particle (molecule) with the angular states of the solid, required by the conservation of the angular momentum.

In conclusion, we have studied quantum oscillations in a double well coupled with a solid. When the oscillation frequency is small compared to the Debye frequency, the decoherence due to phonons is given by the universal formula which contains only directly measurable parameters. It provides the ultimate lower limit for the decoherence rate, mandated by the invariance with respect to global translations and rotations.

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