Vibration-Induced Granular Segregation: A Phenomenon Driven by Three Mechanisms

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The segregation of large spheres in a granular bed under vertical vibrations is studied. In our experiments, we systematically measure rise times as a function of density, diameter, and depth, for two different sinusoidal excitations. The measurements reveal that, at low frequencies, inertia and convection are the only mechanisms behind segregation. Inertia (convection) dominates when the relative density is greater (less) than one. At high frequencies, where convection is suppressed, fluidization of the granular bed causes either buoyancy or sinkage and segregation occurs.

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Many theoretical and experimental studies have been carried out in the past five decades aimed at revealing the physics of one of the most intriguing phenomena in granular matter: vibration-induced segregation [1–14]. However, when this intensively investigated phenomenon appeared to be well understood, new scientific puzzles came on the scene [15–22]. Air-driven segregation, inertia, and condensation are now added to the already vast list of concepts important in the subject. Thus, since this problem is an important concern to industries dealing with granulates, these recently disclosed effects should be further investigated. Granular segregation was first reported in 1939 by Brown [23] and studied ever since by the engineering community [1–5], until it was brought in 1987 to the physics realm with the suggestive name of the "Brazil nut problem" (BNP) [6]. The results related to this problem established themselves as benchmarks of granular segregation. But the question of why the Brazil nuts are on top seems to be yet an open matter of discussion. Both theoretical and experimental studies have focused on the influence of size, friction, density, and excitation parameters [7–10,12,13,16,22,24,25] and the results explain, or obscure, bit by bit the underlying mechanism behind the BNP. Some of these results support the idea that it is "void filling" beneath large ascending particles, the mechanism promoting the upward movement of an intruder in a shaken granular bed [6,14]. Other research claims that global convection is the driving force behind the BNP [8], and other that arching [7] or inertia [16,22] are crucial elements in explaining it. The dilemma is not yet settled with the advent of even more recent findings [15], the most relevant surprising result being that decreasing the density of the intruder does not necessarily mean a monotonic increasing of the rise time, as might have been previously suggested by studies in 3D [9] and 2D [16]. Furthermore, based on computer simulations, Hong *et al.* even dared to predict the reverse segregation effect in the BNP [17,19] (known now in the literature as the RBNP), which was immediately confronted by two groups [20,26], but nevertheless observed in the laboratory by *Breu et al.* [27]. Finally, *Yan et al.* [21] recently failed to confirm the experimental findings of Möbius *et al.* [15].

Based on this debate, a simple yet overwhelming conclusion arises: More research is needed if we want to uncover the physics of this elusive granular matter problem. This Letter aims to contribute to its final understanding.

Our experimental setup consists of a Plexiglas cylinder (closed at one end) of 10 cm inner diameter and 26 cm of length. The Plexiglas cylinder is fixed to a vibrating table fed with an amplified periodic voltage coming from a function generator (HP-33120A). In a typical experiment, the column is filled with small seeds or glass beads. Rise times are measured by a stopwatch. Excellent reproducibility of the data is obtained if the temperature and humidity do not change during the experiments.

In Fig. 1, we show the rise time as a function of the intruder relative density ($\rho_r = \rho/\rho_b$, where ρ_b is the density of the bed particles) for seven different starting depths: 5, 7, 9, 11, 13, 15, and 17 cm below the surface, at 5 Hz and $\Gamma = 3$. The bed column had 21 cm of height (hence, at 17 cm of depth our 4 cm diameter intruders touch the bottom of the container). The size and density of the bed particles we used (tapioca monodisperse spheres) are, respectively, 3.1 mm and 0.57 g/cc. The intruders are plastic spheres filled with different materials to change their densities.

Our first clear observation is that the ascension dynamics of the spheres, whose starting positions were at the bottom of the column, has a monotonic dependence on ρ_r, the curve diverging at $\rho_r \approx 1$. At any other depth, the spheres, regardless of their density, segregate to the surface following a nonmonotonic dynamics. This nonmonotonic ascension dynamics was previously observed by the group in Chicago [15], although the peak they observed was positioned at a relative density less than one and their measurements correspond to only one depth (around 5 cm). We can normalize the data with $\rho_r > 1$ in Fig. 1 by making the rise time of the heaviest sphere equal to one (for each depth). In this way, the rise time of every sphere will be measured with the time scale of the

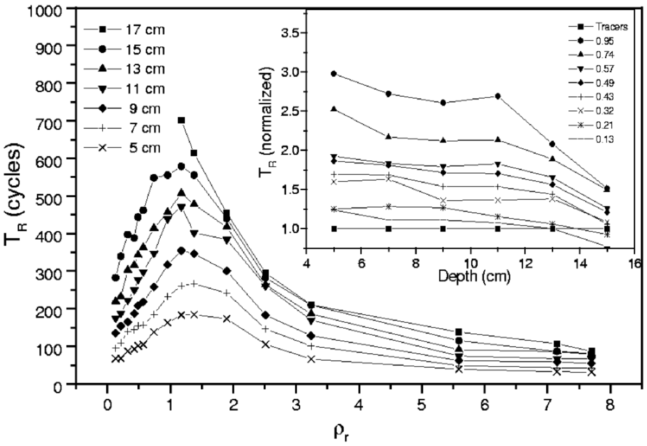

FIG. 1. Dimensionless rise time T_R, as a function of relative density for seven different depths. The height of the granular bed is 21 cm and the diameter of the sphere 4 cm. Inset: dimensionless intruder rise times as a function of depth for different relative densities less than one. Data are normalized with measured rise times for tracer particles ascending by convection. Intruders ascend slower than tracer particles, represented by the horizontal line.

fastest (the densest) one. These normalized results, for the spheres denser than the granulate, collapse into the same curve (see Fig. 2). Based on this result, we believe that these spheres segregate mostly by inertia. This concept has been evoked in the literature for some time already [9,15,16,18,22], but never used to quantify granular segregation in 3D. In a recent paper [22], we proposed a simple theoretical model to explain the rise dynamics of heavy spheres in a vibrated granulate. The model is based on energy considerations and states that, on each cycle, the kinetic energy of the intruder is lost by friction during its penetration into the granular bed: $1/2mv_{to}^2 = \beta(h)P_l$ [where v_{to} is the "take-off" velocity the bead has when the granulate reaches a negative acceleration $a = -g$, m is the mass of the bead, and $\beta(h)$ is the friction force exerted upon it by the granulate]. Therefore, since in our experiments the volume of the intruders and the vibration conditions are constants, the penetration length per vibration cycle should be directly proportional to the density. Thus, the number of cycles for a sphere to segregate to the surface is inversely proportional to it. The smooth line in Fig. 2 was obtained by using $a\rho_r^b$, where the best fit gives $b = -1.09$. What is very interesting is that all peaks in Fig. 1 are at $\rho_r \approx 1$ and, afterwards, lighter intruders start to ascend faster. In the inset of Fig. 2 we show normalized rise times for $\rho_r < 1$. For this case, normalization was done using also the fastest sphere (the lightest). We will come back to this plot later.

We carried out a second experiment. The rise time of light spheres was measured again as we change ρ_r, but now the spheres are always positioned at the bottom of a container filled by a granulate of 6 cm of height. The frequency we use is 50 Hz (with $\Gamma = 3$). We can see that the lighter the spheres the faster they ascend (see Fig. 3). This is contrary to what happens at low frequencies, where regardless of how shallow the granular bed on top of the spheres is, they cannot segregate if they are at the bottom. Indeed, on one hand, at low frequencies

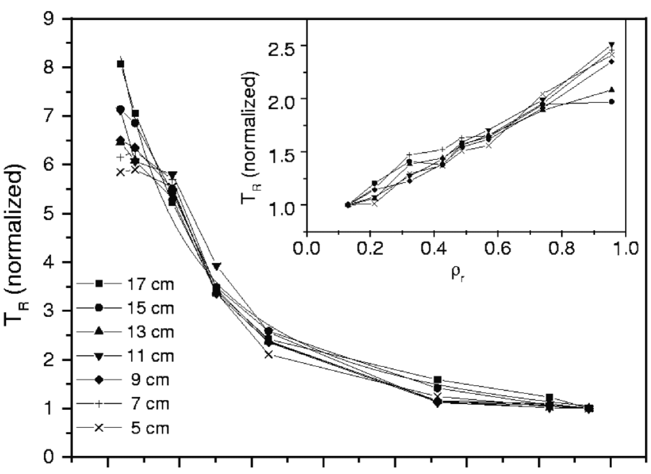

FIG. 2. Dimensionless rise times for intruders with $\rho_r > 1$ for all depths, normalized with the rise times of the fastest (densest) sphere, at each depth. The line is the best fit obtained with $a\rho_r^b$, giving $a = 9.84$ and $b = -1.09$. Inset: Dimensionless rise times for intruders with $\rho_r < 1$ for different depths, normalized with the rise time of the fastest (lightest) sphere, at each depth.

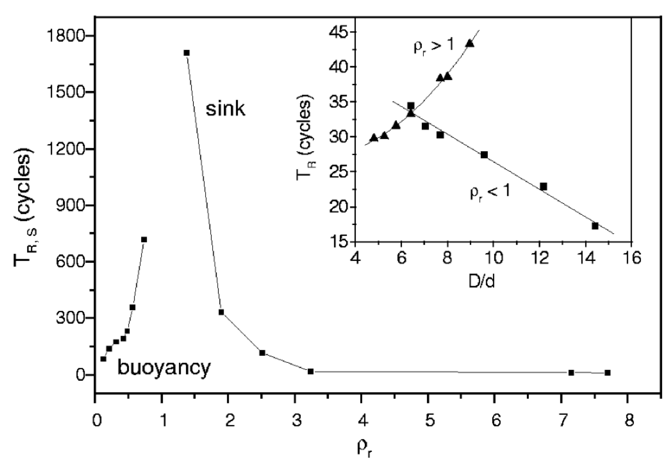

FIG. 3. Dimensionless rise (sink) times as a function of relative density for only one depth (5 cm), at 50 Hz. In the left part of the figure ($\rho_r < 1$), we note that intruders ascend faster the lighter they are, while in the right part ($\rho_r > 1$) we see that the denser sink faster. Inset: Rise times as a function of relative diameters for $\rho_r > 1$ (where the solid line represents a parabolic fit) and $\rho_r < 1$ (where the solid line is only to help the eye). Both cases are at constant mass (7.1 and 2.2 g, respectively), 5 Hz, $\Gamma = 3$, and 5 cm depth.

spheres with $\rho_r < 1$ cannot ascend from the bottom as seen in Fig. 1 (but spheres with $\rho_r > 1$ can). On the other hand, at high frequencies spheres with $\rho_r < 1$ can ascend from the bottom (but spheres with $\rho_r > 1$ cannot). During this high-frequency experiment, tracer particles were put at the base of the container, alone or together with the light spheres, and the latter emerge but the tracer particles do not (i.e., there is no convection). To explain this effect, instead of "void filling," we prefer to use a term used in fluids and mentioned already by some authors in the granular field [28]: buoyancy. Is this granular effect caused by fluidization the missing term for explaining the results reported by the group in Chicago [15] and ours in Fig. 1? The answer is no.

The condition for buoyancy to happen, when neither convection nor inertia are present, is pure fluidization. Here, we would like to point out that fluidization with no convection is achieved if along the granular column there is not a "temperature" gradient; were the particles of the granulate move only around their equilibrium positions, whether or not the bed crystallizes [12].

Hence, at high frequencies the granulate fluidizes and buoyancy takes place, segregating to the top any light intruder buried inside it. Furthermore, at these fluidized conditions, where light particles are buoyant, heavy intruders, as in common fluids, must sink. In Fig. 3, we also depict sink times as a function of ρ_r for $\rho_r > 1$. The sink curve shows that the heavier the intruder is, the faster it sinks (we plot the time it takes for each sphere to sink its own diameter). The above concepts explain the RBNP predicted by Hong *et al.* [17,19] and later observed by Breu *et al.* [27]. It also explains why Canul-Chay *et al.* [20] could not observe it in their own experiments [29].

To understand the still unexplained part of the curve of Möbius *et al.* and ours in Fig. 1, let us explore carefully the "convection connection," postulated by the group in Chicago [8]. We can plot the same data already plotted in Fig. 1 in a different fashion (see the inset of the figure). Normalized rise times are plotted as a function of depth for all relative densities less than one. These data were normalized using rise times of tracer particles, measured at the same excitation conditions for each depth, while no intruders were in the bed. The horizontal line corresponds precisely to the tracer particle rise times normalized to themselves. We note that most of the curves are above the tracer line, indicating that convection is faster than the ascension of these spheres. We suggest then that pure convection (neither inertia because particles are light nor buoyancy because the bed is not fluidized at these low frequencies) is the mechanism needed to explain the drop of rise time curves at $\rho_r \approx 1$. Looking again to the inset of Fig. 2, we see that the normalized rise times reasonably collapse into the same curve. The fact that they collapse does not mean that the spheres rise at the tracer times, as one can clearly see in the inset of Fig. 1. Obviously, although convection is behind the ascension, their inertia is still non-negligible. Therefore, the heavier they are, the harder to be dragged to the top by the streaming flux or convective cell. This can be understood if we look at the transfer of momentum, where both the "streaming" and "collisional" modes participate in the transport of intruders having non-negligible masses [28].

Our results plotted in Fig. 1 seem to be in contradiction with prior experimental studies conducted by Shinbrot *et al.* [9] and Yan *et al.* [21]. On one hand, Shinbrot *et al.* measure convection periods where divergence is seen for light intruders. On the other, Yan *et al.* observe that the lighter the intruders the faster they sink. In both cases, the differences with our experiments lie on the following two issues: (i) in our experiments we always positioned the intruders at specific depths and measure rising times, instead of convection periods [9] or sinking times [21]; (ii) the size of the bed particles. We are convinced that, if Shinbrot *et al.* had positioned their intruders at specific depths in a granular bed with larger particles, they would have observed the nonmonotonic rise dynamics we observe in Fig. 1. Furthermore, Yan *et al.* suggested that, in order to understand the nonmonotonic segregation dynamics shown in Fig. 1, we need to understand first the role of the air pressure gradient acting on the granular bed [21]. According to our results, we conclude that this suggestion is not correct. Air is indeed an important ingredient only in granulates with very small particles where reverse [9] or negative [21] buoyancy is observed.

Finally, let us remind the reader that all theoretical and experimental studies published thus far on granular segregation agree on the following: The larger the intruder the faster it rises through a vibrating bed. Moreover, some geometrical approaches conclude that there is a critical diameter below which intruders do not ascend [7,13]. The above conclusions are true only when the density remains constant as one changes the size of the intruders (see, for instance, the recent work reported by the group in Chicago [15]). Under these circumstances, the mass varies with the cube of the diameter and therefore, due to their greater value, larger intruders ascend faster. However, the opposite behavior is observed when the mass (not the density) remains constant as one changes the size of the intruders (see the inset of Fig. 3). In this case, the faster intruders are the smaller (in contradiction with the mentioned size paradigm). Our above model, which predicts the hyperbolic dynamics for intruders denser than the granulate (at constant intruder volume, see Fig. 2), predicts also the parabolic behavior found for size segregation (see the inset of Fig. 3) at constant mass with $\rho_r > 1$ [30]. In the inset of Fig. 3, we show rising times versus diameter for $\rho_r < 1$ (constant mass), verifying in this case what granular scientists have concluded in the past: larger intruders rise faster.

We report experimental results that shed definitive light to explain the fundamental aspects of the fascinating

BNP. We conclude that there are only three physical mechanisms behind the segregation of a large or a small, heavy or light, particle in a vibrated granulate: inertia, convection, and buoyancy (sinkage). The first two are always present at $\Gamma > 1$ and low frequencies, where a nonmonotonic ascension dynamics is observed as one changes the relative density of the intruders. Inertia dominates when $\rho_r > 1$, and convection does it when $\rho_r < 1$. Segregation, by buoyancy or sinkage, is present at excitation conditions where the granulate is fluidized with no convection ($\Gamma > 1$, small amplitudes and high frequencies). Finally, when intruders have the same mass but diameters are varied, keeping their relative densities greater (less) than one, the smaller (larger) they are, the faster they segregate to the free surface of the bed.

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[29] Breu *et al.* acknowledge in their article [27] that, if the small particle layer is not shallow, the RBNP is not observed. Canul-Chay *et al.* experiments [20] were carried out with thick granular layers. In their case, inertia always segregates the heavy and large particles to the top, but since convection produces reentrance the final phase is mixed. Moreover, Canul-Chay *et al.* did their experiments at low frequencies where convection is dominant.
[30] In the expression, $1/2mv_{to}^2 = \beta(h)P_l$, the left part is constant. Since $\beta(h)$ is proportional to the intruder cross section (D^2), T_R is also proportional to it.