Nature of Light Scalar Mesons from their Large-N_c Behavior

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We show how to obtain information about the states of an effective field theory in terms of the underlying fundamental theory. In particular, we analyze the spectroscopic nature of meson resonances from the meson-meson scattering amplitudes of the QCD low energy effective theory, combined with the expansion in the large number of colors. The vectors follow a $\bar{q}q$ behavior, whereas the σ, κ, and $f_0(980)$ scalars disappear for large N_c, in support of a $\bar{q}\bar{q}qq$-like nature. The a_0 shows a similar pattern, but the uncertainties are large enough to accommodate both interpretations.

Effective quantum field theories are very useful to deal systematically with the degrees of freedom of systems when more fundamental theories are not available or intractable. The paradigmatic example is QCD, which is not able to describe hadron dynamics at low energies, where it becomes nonperturbative. In particular, the existence and nature of the lightest scalar mesons is a long-standing controversial issue that has recently received relevant experimental and theoretical contributions. Concerning their existence, the implementation of the QCD spontaneously broken chiral symmetry leads to poles in the pion and kaon scattering amplitudes, associated with the most controversial states: the σ and the κ [1]. Such poles have been found in the most recent charm meson decay experiments [2]. About their nature, most chiral descriptions of meson dynamics do not include quarks and gluons and are hard to relate to QCD, and the spectroscopic nature is thus imposed from the start. In contrast, models with quarks and gluons, even those inspired in QCD, have problems with chiral symmetry, small meson masses, etc. Furthermore, both kinds of models are usually incompatible with the chiral expansion imposed by the low energy effective theory of QCD, known as chiral perturbation theory (ChPT).

ChPT [3] is the most general derivative expansion of a Lagrangian, respecting the QCD symmetries, containing only π, K, and η mesons. These particles are the Goldstone bosons of the spontaneous chiral symmetry breaking of massless QCD and are the QCD low energy degrees of freedom. For two-meson scattering ChPT is an expansion in even powers of momenta, generically denoted as $O(p^2), O(p^4)\dots$, over a scale $\Lambda_\chi \sim 4\pi f_0 \simeq 1$ GeV. Since u, d, and s quark masses are small compared with Λ_χ, they are introduced as perturbations, giving rise to π, K, and η masses, counted as $O(p^2)$. At each order in p^2 ChPT is the sum of all terms compatible with the symmetries, each multiplied by a "chiral" parameter, *thus avoiding any bias* in setting up a chiral model of mesons. Thus, ChPT allows for finite quantum field theory calculations, by absorbing loop divergences order by order in the chiral parameters. Once the set of

parameters up to a given order is determined from experiment, it describes, to that order, any other process involving mesons. At leading order there is only one parameter, the pion decay constant in the chiral limit, f_0, that fixes Λ_χ, so that all underlying theories breaking chiral symmetry at the same scale have the same leading term. Different underlying dynamics manifest through different chiral parameters at higher orders. We show in Table I the L_i parameters that determine meson-meson scattering up to $O(p^4)$. As usual after renormalization, they depend on an arbitrary regularization scale μ:

$$L_i(\mu_2) = L_i(\mu_1) + \frac{\Gamma_i}{16\pi^2}\log\frac{\mu_1}{\mu_2}, \tag{1}$$

where Γ_i are constants given in [3]. Of course, in physical observables the μ dependence is canceled through the regularization of the loop integrals.

The large N_c expansion [5] is the only analytic approximation to QCD in the whole energy region. Remarkably, it provides a clear definition of $\bar{q}q$ states that become bound states when $N_c \to \infty$. ChPT being the low energy QCD effective theory, the N_c scaling of its L_i parameters, listed in Table I, has been obtained in [3,6]. In addition, the π, K, η masses scale as $O(1)$ and f_0 as $O(\sqrt{N_c})$. There is still the question of what is the renormalization scale at which the N_c scaling should be

TABLE I. Chiral parameters from ChPT and the unitarized amplitudes (IAM) and their leading N_c scaling from QCD.

Parameter $\times 10^{-3}$	ChPT [3,4] $\mu = 770$ MeV	IAM $\mu = 770$ MeV	Large N_c behavior
$2L_1 - L_2$	-0.55 ± 0.7	0.0 ± 0.1	$O(1)$
L_2	1.35 ± 0.3	1.18 ± 0.10	$O(N_c)$
L_3	-3.5 ± 1.1	-2.93 ± 0.14	$O(N_c)$
L_4	-0.3 ± 0.5	0.2 ± 0.004	$O(1)$
L_5	1.4 ± 0.5	1.8 ± 0.08	$O(N_c)$
L_6	-0.2 ± 0.3	0.0 ± 0.5	$O(1)$
L_7	-0.4 ± 0.2	-0.12 ± 0.16	$O(1)$
L_8	0.9 ± 0.3	0.78 ± 0.7	$O(N_c)$

applied to the $L_i(\mu)$. The scale dependence is certainly suppressed by $1/N_c$ for $L_i = L_2, L_3, L_5, L_8$, but not for $2L_1 - L_2$, L_4, L_6, and L_7. Even though the subleading pieces will become proportionally less important at large N_c, the logarithmic terms can be rather large for $N_c = 3$ [7]. The separation between the large N_c leading and subleading parts of the *measured* L_i is not possible, but the leading N_c estimates work well around $\mu \simeq \Lambda_\chi \simeq$ 1 GeV (as we check below with the vector mesons). Indeed, the μ where the N_c scaling applies has been estimated between 0.5 and 1 GeV [3].

Since ChPT is an expansion in momenta and masses, it is limited to low energies. As the energy grows, the ChPT truncated series will violate unitarity. Nevertheless, in recent years ChPT has been extended to higher energies by means of unitarization [8–12]. The main idea is that when projected into partial waves of definite angular momentum J and isospin I, physical amplitudes t should satisfy an elastic unitarity condition:

$$\mathrm{Im}\, t = \sigma |t|^2 \Rightarrow \mathrm{Im}\frac{1}{t} = -\sigma \Rightarrow t = \frac{1}{\mathrm{Re}\, t^{-1} - i\sigma}, \quad (2)$$

where σ is the phase space of the two mesons, a well-known function. A ChPT calculation up to a given order does not satisfy this constraint, since the powers of momenta will not match in the left-hand equality. However, from the right-hand side we note that to have a unitary amplitude we need only $\mathrm{Re}\, t^{-1}$, and for that we can use the ChPT expansion; this is the inverse amplitude method (IAM) [8]. The results of this simple resummation are remarkable, since it generates resonances not initially present in ChPT such as the ρ, K^*, the σ, and the κ, ensuring unitarity in the elastic region and respecting the low energy ChPT expansion. When inelastic two-meson processes are present all partial waves t between all physically accessible states can be gathered in a symmetric T matrix. Then, the IAM generalizes to $T \simeq (\mathrm{Re}\, T^{-1} - i\Sigma)^{-1}$, where Σ is a diagonal matrix containing the phase spaces of all accessible two-meson states, again well-known [9–12]. With this generalization it was recently shown [11] that, using the one-loop ChPT calculations, it is possible to generate the four resonances mentioned above together with the $a_0(980)$, the $f_0(980)$, as well as the octet ϕ, extending the ChPT description of two body π, K, or η scattering up to 1.2 GeV, but keeping simultaneously the correct low energy expansion and with chiral parameters compatible with standard ChPT. We show in Table I the L_i obtained from a recent update of an IAM fit to the scattering data [11].

One may wonder how robust are these results. Similar unitarization methods [12,13] lead to similar results. In particular, the σ and κ are obtained as soon as one requires chiral symmetry and unitarity. The use of ChPT ensures that we are not forgetting any contribution up to $O(p^4)$, and that we could extend it to higher orders if we wished. Indeed, the IAM has been applied to $\pi\pi$ up to $O(p^6)$ finding basically the same results [14]. Also, with

an order of magnitude estimate for the leading $O(p^8)$ contribution it is even possible to go up to 1400 MeV in the $J = 2$ channel, generating the $f_2(1250)$ [15]. One could worry about crossing symmetry, but it has been shown that the amount of crossing violation is smaller than the present experimental uncertainties [14]. Furthermore, using the Roy equation formalism for $\pi\pi$, which respects also crossing symmetry, a similar pole for the σ has been recently found [16]. Unlike the IAM, all these improvements in $\pi\pi$ have not been applied to other processes because they become much more complicated.

Since we are interested in the specific underlying QCD dynamics, we have to consider at least $O(p^4)$ terms. It is possible to describe the scalar channels with the leading order plus a cutoff (or another regularization parameter) playing the role of some combination of higher order parameters. However, for the vector resonances we need at least the $O(p^4)$ parameters, and we want to generate the vectors to test that our approach is able to identify first of all the well established $\bar{q}q$ states, and the scale μ where the N_c scaling applies. For those reasons, we use the one-loop $O(p^4)$ meson-meson scattering amplitudes unitarized with the IAM [11]. Different IAM fits are due to different ChPT truncation schemes equivalent up to $O(p^4)$ and to the estimates of the large systematic uncertainties in the data; we have chosen a representative fit in Table I, but the results are similar for other sets. Note that these ChPT amplitudes are fully renormalized in the $\overline{MS} - 1$ scheme and are, therefore, scale independent. Hence all the QCD N_c dependence appears correctly through the L_i and cannot hide in any spurious parameter. If we had kept just the leading order and a regularization scale or a cutoff, we would not know if that cutoff is playing the role of, for instance, L_2 or L_8, or any other $O(N_c)$ combination of L_i.

Let us then scale $f_0 \rightarrow f_0\sqrt{N_c/3}$ for $m = \pi, K, \eta$, and $L_i(\mu) \rightarrow L_i(\mu)(N_c/3)$ for $i = 2, 3, 5, 8$, keeping the masses and $2L_1 - L_2$, L_4, L_6, and L_7 constant. Figure 1 shows, for increasing N_c, the modulus of the $(I, J) = (1, 1)$ and $(1/2, 1)$ amplitudes. We see the Breit-Wigner
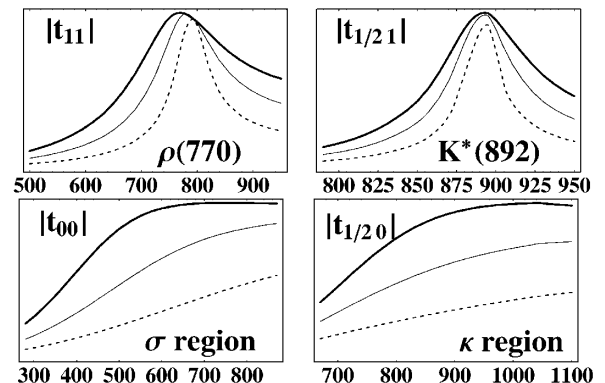

FIG. 1. Modulus of amplitudes in different meson-meson channels for $N_c = 3$ (thick line), $N_c = 5$ (thin continuous line), and $N_c = 10$ (thin dotted line), scaled at $\mu = 770$ MeV.
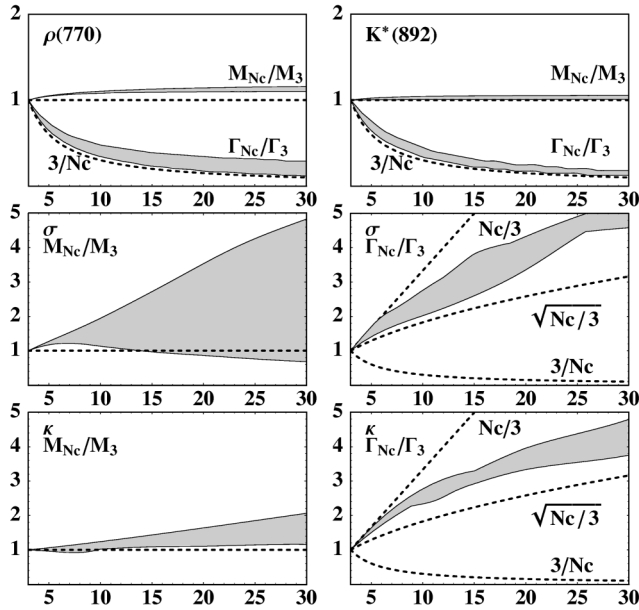
FIG. 2. N_c dependence of the ρ, K^*, σ, and κ pole positions, defined as $\sqrt{s_{\text{pole}}} \simeq M - i\Gamma/2$ normalized to their $N_c = 3$ value. The dashed lines show different N_c scaling laws, and the gray areas cover the uncertainty in $\mu \simeq 0.5$–1 GeV.

shape of the ρ and $K^*(892)$ vector resonances, respectively, becoming narrower as N_c increases, but with a peak at an almost constant position. In contrast all over both the σ and κ regions the amplitudes decrease with larger N_c.

In Fig. 2 we show the evolution of the ρ and K^* pole positions, related to the mass and the width as $\sqrt{s_{\text{pole}}} \simeq M - i\Gamma/2$ (as for Breit-Wigner resonances, but abusing the notation for the rest). We have normalized both M and Γ to their value at $N_c = 3$ in order to compare with the $\bar{q}q$ expected behavior: M_{N_c}/M_3 constant and $\Gamma_{N_c}/\Gamma_3 \sim 3/N_c$. The agreement is remarkable, not only qualitatively, but *also quantitatively* within the gray band that covers the uncertainty on the scale $\mu = 0.5$–1 GeV where to apply the large N_c scaling. We have checked that outside that band, the behavior starts deviating from that of $\bar{q}q$ states, which confirms that the expected scale range where the large N_c scaling applies is correct. In contrast, the σ and κ poles show a totally different behavior, since *their width grows with N_c*, in conflict with a $\bar{q}q$ interpretation. This was also suggested using the ChPT leading order unitarized amplitudes with a regularization scale [12,17].

In order to determine what these states could be, we have checked that *in the whole σ and κ regions*, the corresponding $\text{Im}\,t \sim O(1/N_c^2)$ and $\text{Re}\,t \sim O(1/N_c)$. Diagrammatically, imaginary parts can be generated only from graphs like those in Figs. 3(a) and 3(c), when the intermediate state (represented by the dotted line) is physically accessible. But Fig. 3(a) has an intermediate $\bar{q}q$ meson, with mass $M \sim O(1)$ and $\Gamma \sim 1/N_c$, so that at $\sqrt{s} \simeq M$ we expect $\text{Im}\,t \sim O(1)$ and a peak, as it is indeed the case of the ρ and K^*. Therefore, the σ and κ do not get
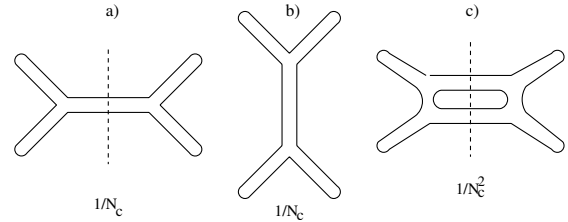

FIG. 3. Representative diagrams contributing to meson-meson scattering and their N_c scaling.

imaginary parts from graphs like that of Fig. 3(a), although they get a $1/N_c$ contribution to the real part from Fig. 3(b), usually interpreted as ρ or K^* t-channel exchange, respectively. The leading s-channel contribution in terms of quarks and gluons comes from the graph in Fig. 3(c). For the κ, which is a strange particle, this means a leading $\bar{q}\bar{q}qq$ (or two-meson) contribution. This kind of state is predicted to unbound and become the meson-meson continuum in the $N_c \to \infty$ limit [18]. The same interpretation holds for the sigma, but Fig. 3(c) also corresponds to a glueball exchange, which we cannot exclude with these N_c arguments alone. However, the lightest glueball is expected with a mass higher than 1 GeV and SU(3) symmetry would suggest that the κ and the σ should be rather similar. Thus, a dominant $\bar{q}\bar{q}qq$ component for the σ seems the most natural interpretation, although it can certainly have some glueball mixing.

Finally, Fig. 4 shows the large N_c behavior in the $f_0(980)$ and $a_0(980)$ regions, which are more complicated due to the distortions caused by the nearby $\bar{K}K$ threshold. The $f_0(980)$ is characterized by a sharp dip in the amplitude that vanishes at large N_c, contrary to the expectations for a $\bar{q}q$ state. Note that for smaller N_c, the position of the disappearing dip changes, but for $N_c > 5$ it follows again the $1/N_c^2$ scaling compatible with $\bar{q}\bar{q}qq$ states or
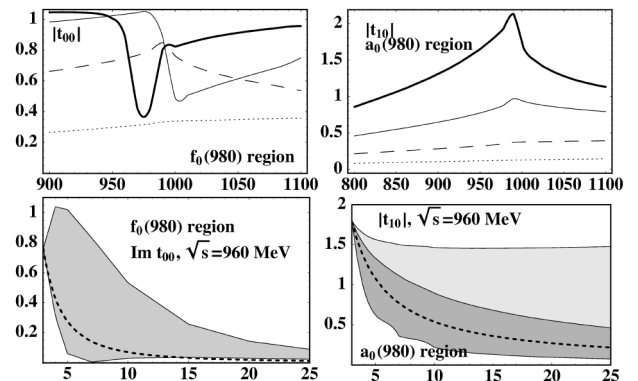

FIG. 4. Top: modulus of $(I, J) = (0, 0), (1, 0)$ amplitudes for $N_c = 3$ (thick line), $N_c = 5$ (thin continuous line), $N_c = 10$ (dashed line), and $N_c = 25$ (thin dotted line), scaled at $\mu = 770$ MeV. Bottom: imaginary part and modulus of amplitudes versus N_c in the resonant regions. Dark gray areas cover $\mu = 0.55$–1 GeV; the light gray area covers the uncertainty down to 0.5 GeV.
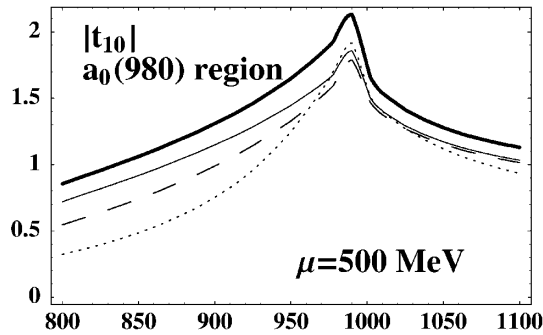
FIG. 5. Modulus of $(I, J) = (1, 0)$ amplitude for $N_c = 3$ (thick line), $N_c = 5$ (thin continuous line), $N_c = 10$ (dashed line), and $N_c = 25$ (thin dotted line), scaled at $\mu = 500$ MeV.

glueballs. The $a_0(980)$ behavior is more complicated. When we apply the $L_i(\mu)$ large N_c scaling at $\mu = 0.55$–1 GeV, its peak disappears, suggesting that this is not a $\bar{q}q$ state, and the imaginary part of the amplitude follows roughly the $1/N_c^2$ behavior in the whole region. However, as shown in Fig. 5, the peak does not vanish at large N_c if we take $\mu = 0.5$ GeV. Thus we cannot rule out a possible $\bar{q}q$ nature, or a sizable mixing, although it shows up in an extreme corner of our uncertainty band. For other recent large N_c arguments in a chiral context, see [19].

In conclusion, we have shown how by changing effective Lagrangian parameters according to some specific rules dictated by the underlying dynamics, we can learn about the structure of the states at the fundamental level. In particular, we have shown that the QCD large N_c scaling of the unitarized meson-meson amplitudes of chiral perturbation theory is in conflict with a $\bar{q}q$ nature for the lightest scalars [not so conclusively for the $a_0(980)$], and strongly suggests a $\bar{q}\bar{q}qq$ or two-meson main component, maybe with some mixing with glueballs, when possible.

The techniques here presented could be easily applied in other frameworks where unitarized effective Lagrangian amplitudes already exist, as heavy baryon chiral perturbation theory [20] or the strongly interacting symmetry breaking sector of the standard model [21]. With somewhat more effort they could also be applied when the fundamental theory is intractable but has a simpler description in terms of effective Lagrangians.

Note added.—The idea of this work and the pole movements were presented by the author in two workshops [22]. While completing the calculations and the manuscript the results without the scale uncertainties have been confirmed [23] for all resonances, using the approximated IAM [9].

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