Radiation-Spin Interaction, Gilbert Damping, and Spin Torque

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Magnetization relaxation processes, which are represented by the Gilbert damping term and the spin torque term in the Landau-Lifshitz-Gilbert (LLG) equation, are described by the radiation-spin interaction (RSI), where the radiation field is produced by magnetization precessional motion itself. It is shown that the LLG equation including the Gilbert damping term and the spin torque term is derived from the spin Hamiltonian containing the RSI. The derivation of the LLG equation is given in a self-consistent method. It is also shown that, according to RSI, the magnitude of the magnetization vector deviates from the magnetization saturation with the order of $\mathcal{O}(\alpha^2)$, where α is the Gilbert damping parameter.

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With increasing interest in advanced magnetic information storage and data process elements, a detailed understanding of the micromagnetic structure in microsized and nanosized magnets becomes more crucial. In general, a physical description of micromagnetic phenomena is based on the use of the Landau-Lifshitz-Gilbert (LLG) equation [1], which is a phenomenological equation of motion for the magnetization, $\mathbf{M}$,

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{M} \times \frac{d\mathbf{M}}{dt}, \qquad (1)$$

where γ is the gyromagnetic ratio and α is the Gilbert damping parameter. The effective magnetic field $\mathbf{H}_{\text{eff}}$ is given by the energy variational with magnetization, $\mathbf{H}_{\text{eff}} = -\delta G/\delta \mathbf{M}$, where G is the free energy of the system.

The second term in the LLG equation, the Gilbert damping term [2], controls the magnetic relaxation process and determines how fast the magnetization is restored to its equilibrium position. Thus, a thorough understanding of the Gilbert damping term is essential for the research in magnetic memory devices employing fast magnetization reversal processes. The magnetization relaxation process has been described with various quantum processes such as spin-orbit coupling and two-magnon scattering [3].

Recently, Slonczewski [4] and Berger [5] have suggested an interesting magnetization relaxation process caused by a spin-polarized current applying in ferromagnetic multilayer systems, which typically consist of two ferromagnetic thin films spaced by a nonmagnetic normal metal. (For review, see [6].) One of the ferromagnetic films, i.e., "fixed" layer, polarizes electron spins of an applied current to be parallel to the direction of its magnetization, and the other, i.e., "soft" layer, is prepared to be easy to change its magnetization direction. Then, the magnetization of the soft film can be driven to oscillate and even to be switched by spin transfer from a spin-polarized current into the film. The spin torque effect has been experimentally verified in many cases [7].

The spin torque is included in the LLG equation in the form of [4]

$$\boldsymbol{\tau}_{\text{spin-torque}} = \overline{\gamma} \xi \mathbf{M} \times (\mathbf{m} \times \mathbf{M}), \qquad (2)$$

where ξ is the spin torque parameter and $\overline{\gamma}$ is a dimensionful constant containing the $s - d$ interaction coupling constant. $\mathbf{M}$ denotes the magnetization of soft ferromagnetic film, and $\mathbf{m}$ is the effective magnetization due to spin-polarized conduction electrons injected into the soft ferromagnetic film. At the normal metal and soft ferromagnetic film interface, the direction of $\mathbf{m}$ is the same as the direction of magnetization of the fixed ferromagnetic film. Then, it changes as conduction electrons cross the interface and propagate inside the soft ferromagnetic film. It must be noted that the spin state of conduction electrons satisfies its own Schrödinger equation, and one has to solve two coupled equations for $\mathbf{M}$, $\mathbf{m}$, and the conduction electron's Schrödinger equation. Following Slonczewski and Berger, some plausible physical interpretations for the spin torque have been suggested [8–11].

In this Letter, we discuss a microscopic description for magnetization relaxation processes, which are represented by the Gilbert damping term and the spin torque term in the LLG equation. It is shown that the Gilbert damping term and the spin torque term are derived from the spin Hamiltonian containing the interaction between the spin and the radiation field, which is induced by the precessing magnetization itself. The derivation of the LLG equation is given in a self-consistent method [12].

This work is partly motivated by the recent report by Heinrich *et al.* [13]. They demonstrated that the spin-pumping concept, which was used to describe the enhancement of the Gilbert damping parameter by spin-polarized currents at an interface between a normal metal and a ferromagnetic film [14], also gives a physical description for the spin torque effect. This indicates that there may exist a physical phenomenon describing the Gilbert damping and the spin torque *simultaneously.*

We begin with the definition of the magnetic moment operator (MMO) given by

$$\hat{\mathcal{M}} \equiv -\frac{\delta \mathcal{H}}{\delta \mathbf{H}_{\rm ext}}, \tag{3}$$

where $\mathcal{H}$ is the Hamiltonian operator and $\mathbf{H}_{\rm ext}$ is the external field. The definition of the MMO given by Eq. (3) uses the notion of the response of a magnetic system to an external field. According to the definition of MMO, the magnetization can be defined as an ensemble average of the response [12]

$$\mathbf{M} \equiv \frac{1}{V}\mathbf{Tr}\{\rho \hat{\mathcal{M}}\}, \tag{4}$$

where ρ is the density operator and V is the volume of the system.

Now we consider the spin Hamiltonian operator $\mathcal{H}_0$, which describes the spin dynamics in a ferromagnetic material, given by

$$\mathcal{H}_0 = -g\mu_B \sum_i \hat{S}_i \cdot \mathbf{H}_{\rm eff}, \tag{5}$$

where g is the Landé g factor, μ_B is the Bohr magneton, and $\hat{S}_i$ is the spin operator of the ith, e.g., atom. In the following, we use the subscript "0" to indicate that the corresponding term does not include the radiation-spin interaction (RSI) effect. The effective field $\mathbf{H}_{\rm eff}$ in Eq. (5) includes the exchange field, the external field, the anisotropy field, and the demagnetizing field. Then, the MMO and the magnetization become

$$\hat{\mathcal{M}}_0 = g\mu_B \sum_i \hat{S}_i, \tag{6}$$

$$\mathbf{M}_0 = \frac{g\mu_B}{V}\sum_i \mathbf{Tr}\{\rho \hat{S}_i\}, \tag{7}$$

respectively. The equation of motion of magnetization is obtained by differentiating both sides of Eq. (7) with respect to time [12],

$$\frac{d\mathbf{M}_0}{dt} = \frac{ig\mu_B}{V\hbar}\sum_i \mathbf{Tr}\{\rho[\hat{S}_i, \mathcal{H}_0]\} = -\gamma \mathbf{M}_0 \times \mathbf{H}_{\rm eff}, \tag{8}$$

where the commutation relation $[S_i^a, S_j^b] = i\hbar\epsilon_{abc}\delta_{ij}S_i^c$ and $d\rho/dt = 0$ have been used, and the validity of evaluation given in Eq. (8) is restricted to quasiadiabatic evolutions. Note that the equation given by (8) does not include the Gilbert damping term.

Our approach to understand the magnetization relaxation process is that the damping imposed on the precessing magnetization originates from the magnetization precessional motion itself. According to this basic concept, we introduce the RSI, where the radiation field is induced by the precessing magnetization itself, into the Hamiltonian. At a glance, the concept of RSI appears to be the same as that of radiation damping in nuclear magnetic resonance (NMR) spectroscopy [15]. However, it is rather similar to the concept of radiation damping of a charged particle in classical electrodynamics [16]. There is an important difference between the RSI and the radiation damping in NMR. We give a short comment on that at the end of this Letter.

The application of radiation damping in classical electrodynamics (not in NMR) to spin systems can be found in a recent paper [17]. Following [17], we put the dissipative torque due to RSI in correspondence with the radiation reaction force acting on the particle. According to the analogy, the dissipative torque is represented in terms of the dissipative part of radiation field $\mathbf{H}_r^d$ [18]

$$\chi \mathbf{M} \times \frac{d\mathbf{M}}{dt} = \mathbf{M} \times \mathbf{H}_r^d. \tag{9}$$

Replacing the term $d\mathbf{M}/dt$ in Eq. (9) with the LLG equation, Eq. (9) becomes

$$\mathbf{M} \times \mathbf{H}_r^d = -\frac{\gamma\chi}{1+\alpha^2 M^2}\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\rm eff} - \alpha M^2 \mathbf{H}_{\rm eff}), \tag{10}$$

where M is the magnitude of magnetization. Thus, the dissipative part of the radiation field is read as

$$\mathbf{H}_r^d = \lambda(\mathbf{M} \times \mathbf{H}_{\rm eff} - \alpha M^2 \mathbf{H}_{\rm eff}), \tag{11}$$

where $\lambda \equiv -\gamma\chi(1+\alpha^2M^2)^{-1}$ is a radiation parameter.

According to the expression for $\mathbf{H}_r^d$ given by Eq. (11), the spin Hamiltonian including the RSI is written as

$$\mathcal{H} = -g\mu_B \sum_i \hat{S}_i \times [(1 - \lambda J_\lambda \alpha M^2)\mathbf{H}_{\rm eff} + \lambda J_\lambda \mathbf{M} \times \mathbf{H}_{\rm eff}], \tag{12}$$

where J_λ is the coupling constant of spins and the dissipative radiation field. The MMO, then, becomes

$$\hat{\mathcal{M}} = g\mu_B \sum_i [(1 - \lambda J_\lambda \alpha M^2)\hat{S}_i + \lambda J_\lambda \hat{S}_i \times \mathbf{M}]. \tag{13}$$

The difference between $\mathbf{M} \equiv \mathbf{Tr}\{\rho\hat{\mathcal{M}}\}/V$ and $\mathbf{M}_0$ given by Eq. (7) can be considered as evaluating $\hat{\mathcal{M}} \times \mathbf{M}$ from Eq. (13). After some algebraic calculations, it is shown that $\mathbf{M}$ is parallel to $\mathbf{M}_0$, and its magnitude M is given by the relation of

$$M_0^s = \frac{M}{1 - \lambda J_\lambda \alpha M^2}. \tag{14}$$

It must be noted that the magnitude of magnetization vector M is not equivalent to the magnetization saturation M_0^s, but is modified by a correction of the order of $\mathcal{O}(\alpha^2)$ [because $\mathcal{O}(\lambda J_\lambda) = \mathcal{O}(\alpha)$]. Since, according to our proposal, magnetization damping is caused by the magnetization precessional motion itself, the magnitude of magnetization deviates from the magnetization saturation via its dynamical motion.

From the definition of magnetization, Eq. (4), and the new MMO (13), we obtain the LLG equation including the Gilbert damping term and an additional new torque:

$$\begin{aligned}\frac{d\mathbf{M}}{dt} &= \frac{ig\mu_B}{V\hbar}\sum_i \mathbf{Tr}\{\rho[(1-\lambda J_\lambda \alpha M^2)\hat{S}_i + \lambda J_\lambda \hat{S}_i \times \mathbf{M}, \mathcal{H}]\} + \frac{\lambda J_\lambda}{1-\lambda J_\lambda \alpha M^2}\mathbf{M}\times\frac{d\mathbf{M}}{dt} \\ &= -\gamma(1-\lambda J_\lambda \alpha M^2)\mathbf{M}\times\mathbf{H}_{\rm eff} + \frac{\lambda J_\lambda}{1-\lambda J_\lambda \alpha M^2}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma(\lambda J_\lambda)^2}{1-\lambda J_\lambda \alpha M^2}(\mathbf{M}\times(\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{\rm eff}))), \end{aligned} \tag{15}$$

where Eq. (14) is used. Note that the time derivative in the procedure has to be performed before the trace operation. Taking the relation of $\alpha = \lambda J_\lambda/(1-\lambda J_\lambda \alpha M^2)$ and using a simple vector algebra, we show that Eq. (15) becomes the LLG equation given by Eq. (1).

Now, we study the spin torque effect. First, let us turn on the $s-d$ interaction without introducing the RSI in the Hamiltonian. We use the mean field approximation for the $s-d$ interaction so that the Hamiltonian is written as

$$\mathcal{H}_0 = -g\mu_B\sum_i \hat{S}_i\cdot\mathbf{H}_{\rm eff} - \frac{j_{sd}}{g\mu_B}\sum_i \hat{S}_i\cdot\mathbf{m}, \tag{16}$$

where j_{sd} is determined by the interaction between the effective magnetization $\mathbf{m}$ and atomic spins of the soft ferromagnetic film. Using the definition of magnetization, we obtain the equation of motion of magnetization

$$\frac{d\mathbf{M}_0}{dt} = -\gamma\mathbf{M}_0\times\left(\mathbf{H}_{\rm eff} + \frac{j_{sd}}{g\mu_B}\mathbf{m}\right). \tag{17}$$

The second term on the right hand side represents a torque due to an effective field $(j_{sd}/g^2\mu_B^2)\mathbf{m}$ [11].

Next, let us introduce RSI in the Hamiltonian. Since there are two torques in Eq. (17) driving the magnetization to a precessional motion, we have to introduce RSI terms corresponding to each of them. That is, the Hamiltonian becomes

$$\begin{aligned}\mathcal{H} = &-g\mu_B\sum_i \hat{S}_i\cdot[(1-\lambda J_\lambda \alpha M^2)\mathbf{H}_{\rm eff} + \lambda J_\lambda \mathbf{M}\times\mathbf{H}_{\rm eff}] \\ &- \frac{j_{sd}}{g\mu_B}\sum_i \hat{S}_i\cdot[(1-\eta J_\eta \xi M^2)\mathbf{m} + \eta J_\eta \mathbf{M}\times\mathbf{m}],\end{aligned} \tag{18}$$

where η is a radiation parameter due to the effective magnetization $\mathbf{m}$ and J_η is the coupling constant between the radiation field $\eta(j_{sd}/g^2\mu_B^2)\mathbf{M}\times\mathbf{m}$ and the spin. Then, following the above procedure, we obtain the LLG equation

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times\left(\mathbf{H}_{\rm eff} + \frac{j_{sd}}{(g\mu_B)^2}[1-\eta J_\eta M^2(\xi-\alpha)]\mathbf{m}\right) + \alpha\mathbf{M}\times\frac{d\mathbf{M}}{dt} + [\eta J_\eta - \alpha(1-\eta J_\eta \xi M^2)]\frac{j_{sd}}{g\mu_B}\mathbf{M}\times(\mathbf{m}\times\mathbf{M}). \tag{19}$$

In Eq. (19), it is clear that the RSI leads to the spin torque as well as to the Gilbert damping in the LLG equation.

It appears that if $\alpha = \xi$ and $\lambda J_\lambda = \eta J_\eta$, the spin torque term vanishes in Eq. (19). However, rewriting the Gilbert damping term, we show that even in this case, the LLG equation still contains the spin torque term:

$$\frac{d\mathbf{M}}{dt} = -\frac{\gamma}{1+\alpha^2M^2}\mathbf{M}\times\left(\mathbf{H}_{\rm eff} + \frac{j_{sd}}{(g\mu_B)^2}\mathbf{m}\right) - \frac{\gamma\alpha}{1+\alpha^2M^2}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{\rm eff}) + \frac{\alpha j_{sd}}{g\mu_B(1+\alpha^2M^2)}\mathbf{M}\times(\mathbf{m}\times\mathbf{M}). \tag{20}$$

Thus, the Gilbert damping term in Eq. (19) already includes a part of the spin torque effect. This observation is directly related to the enhancement of the Gilbert damping constant by the spin-pumping effect discussed in [14]. In the equation of magnetization, Eq. (20), the last two terms on the right hand side compete with each other in the magnetization relaxation process [10].

The magnetization relaxation process determined by the RSI depends on the nature of coupling between the spin and the radiation field produced by precessing magnetization. As an example, there are no intermediate processes like the spin-coil coupling appearing in the radiation damping in NMR. Thus, the relaxation time due to the RSI should be longer than that of radiation damping [20].

On the other hand, the RSI is self-consistently controlled by magnetization dynamical motion itself. This means that dynamical effects of magnetic property, such as damping terms in the LLG equation and the $\mathcal{O}(\alpha^2)$-order deviation of the magnitude of the magnetization vector from the magnetization saturation given by (14), naturally vanish as taking a static limit.

In conclusion, the concept of RSI proposed in this Letter is not restricted to specific features of the systems considered, e.g., geometric structures. The details of effects are contained in the characteristic parameters appearing in the LLG equation without any significant modification of the procedure developed in this Letter.

That is, the concept of RSI might be extensively applied for various magnetization relaxation processes.

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[18] The radiation field, $\mathbf{H}_r$, contains the nondissipative part, $\mathbf{H}_r^n$, as well as the dissipative part, $\mathbf{H}_r^d$, i.e., $\mathbf{H}_r \equiv \mathbf{H}_r^d + \mathbf{H}_r^n$ [19]. Generally, $\mathbf{H}_r^n$, not like $\mathbf{H}_r^d$, is not proportional to $d\mathbf{M}/dt$ and cannot be neglected in many cases. We assume that $\mathbf{H}_r^n$ is parallel to $\mathbf{M}$ [17], and, then, $\mathbf{H}_r^n$ terms vanish on the procedure obtaining the equation of motion for the magnetization. The assumption necessarily accompanies the condition of slow precession of $\mathbf{M}$. For simplicity, we omit $\mathbf{H}_r^n$ terms in the following procedure.

[19] The radiation field $\mathbf{H}_r$, in principle, can be exactly evaluated from Maxwell's equations with the source of $d\mathbf{M}/dt$.

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