Critical Field for Complete Vortex Expulsion from Narrow Superconducting Strips

Gheorghe Stan and Stuart B. Field
Department of Physics, Colorado State University, Fort Collins, Colorado 80523, USA

John M. Martinis
National Institute of Standards and Technology, 325 Broadway, Boulder, Colorado 80305, USA
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We have measured the maximum field for which vortices are completely expelled from a thin-film superconducting strip. Niobium strips of width W were field cooled and imaged with a scanning Hall-probe microscope. Below a critical field $B_m \approx \Phi_0/W^2$ all flux was expelled; above this field vortices were observed with a density increasing approximately linearly with field. The small value of the critical field, which is orders of magnitude less than in the bulk, implies that superconducting devices should be designed with narrow wires to eliminate the generation of noise from vortex motion.

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A defining characteristic of superconductivity is the Meissner effect, the complete expulsion of an external magnetic field when a superconductor is cooled through its transition temperature T_c. In general, however, superconductors rarely exhibit a *complete* field expulsion; grain boundaries, normal inclusions, and other defects serve to trap flux within the superconductor. This is particularly true for type-II superconductors, where trapped flux exists in the form of quantized filaments of flux, or *vortices*. Flux expulsion can be further impeded by sample geometry, such as a flat plate in a perpendicular magnetic field. Thus, for actual experiments, the Meissner effect is typically incomplete, inhomogeneous, and strongly dependent on sample preparation and geometry.

The situation becomes remarkably simple, however, in the technologically important case of a thin-film type-II superconducting strip of width W, in the limit where the in-plane penetration depth is much greater than W. As we discuss below, this limit is the experimentally realized one when a strip is field cooled through T_c. In this Letter, we show that such a strip exhibits a *complete* Meissner expulsion of vortices below a critical field B_m, essentially *independent* of the details of pinning and material parameters such as the coherence length ξ and the penetration depth λ. Furthermore, this expulsion has a *universal* characteristic, with $B_m \sim \Phi_0/W^2$, where $\Phi_0 = h/2e$ is the superconducting flux quantum.

We report the experimental confirmation of this critical field using the technique of scanning Hall-probe microscopy (SHM) to image vortices in thin-film niobium strips. Our data can distinguish between two theories that give slightly different predictions [1–4]. Because the strip geometry is common in superconducting applications, this problem also is of current technological relevance. For instance, flux noise generated by the motion of vortices can limit the ultimate sensitivity of superconducting quantum interference devices (SQUIDs), and may produce decoherence in superconducting qubits. Our work shows that vortices can be eliminated by designing devices with leads narrow enough to expel the ambient field [1,2,5], a simpler solution than other proposed techniques [6–8].

Consider a strip field cooled through T_c. Near T_c, the in-plane penetration depth $\Lambda = 2\lambda^2/d$ is much greater than W, leading to weak screening and a field that penetrates nearly uniformly through the strip. Vortices begin to nucleate in the strip, and near T_c they are mobile and can seek out configurations that lower the free energy of the system. As the strip is cooled further pinning increases rapidly, causing the vortices to freeze out at a temperature T_f very close to T_c. As the temperature is lowered further, the vortices remain pinned. Thus, the vortex configurations we image at low temperature are properly described by a theory applicable to this high-temperature regime where $\Lambda > W$.

Several authors [1–4] have investigated the theory of vortex expulsion in this regime. The existence or expulsion of vortices is determined by two competing forces [9–12]. First, a vortex is attracted towards the edges of the strip by a force that can be thought of as arising from image antivortices outside the strip which enforce the proper boundary conditions on the current. This force tends to *expel* the vortex. Second, the vortex is attracted to the center of the strip from interactions with (weak) Meissner screening currents that flow parallel to the strip's edges. This second force is proportional to B, and at high enough field overcomes the outward image forces and creates a position of stable equilibrium along a line centered on the strip.

The effect of these forces can be described by a Gibbs free energy $G(x)$ for a single vortex [1–4], where x is the distance of the vortex to the center of the strip. In Fig. 1, we plot $G(x)$ for a 10-μm-wide niobium strip at $t_f \equiv 1 - T_f/T_c = 0.0015$, which, as we will discuss, corresponds to the experimentally determined value of the vortex freezing temperature. The free energy at low applied fields (e.g., $B = 10\ \mu\mathrm{T} = 0.1$ G) has a domelike shape that expels vortices. As the field is further increased, a flat maximum ($\partial^2 G/\partial x^2 = 0$) in G appears at $B_0 = 16\ \mu$T. For $B > B_0$, a local minimum appears at $x = 0$ in which vortices may be metastably trapped. As B is further increased, this minimum become absolutely stable at a field $B_s = 39\ \mu$T; above this field the free energy of the strip with one vortex is lower than the free energy with no vortex.

Thus, there are two fields, B_0 and B_s, which may be identified as possible critical fields for complete vortex expulsion. Clem [1] and Maksimova [4] argue that the relevant field is that for metastable equilibrium, B_0, which can be calculated as

$$B_0 = \frac{\pi\Phi_0}{4W^2}. \tag{1}$$

On the other hand, Likharev [3] claims that vortices can exist only in the strip when they are absolutely stable, that is, above the field B_s given by

$$B_s = \frac{2\Phi_0}{\pi W^2}\ln\left(\frac{\alpha W}{\xi}\right). \tag{2}$$

The constant α is related to calculating the vortex energy when it comes within $\sim\xi$ of the edge, and was found to be 1/4 by Likharev [3] and $2/\pi$ by Clem [2].

In our experiments, the strips are cooled to below T_c at a fixed field B; this is also the usual case for SQUID devices, where the SQUID might be cooled in the Earth's field. In this case, we argue on physical grounds that the experimentally relevant critical field will be B_s, not B_0. Consider the strip of Fig. 1 at $B_s = 27\ \mu$T, just above B_0. The energy barrier for escape from this shallow well is fairly large ($\approx 47\, kT_c$). However, during field cooling we will have passed through temperatures even nearer to T_c where the barrier is much smaller. For instance, when $t = 1 - T/T_c = 0.0005$ the escape barrier is only $\approx 18\, kT_c$, while the barrier for reentry is $\approx 42\, kT_c$. This situation makes it very likely that any such metastable vortex would escape and not be able to return. If, however, the sample were cooled in a field just above B_s, a vortex in the center would be absolutely stable at all temperatures.

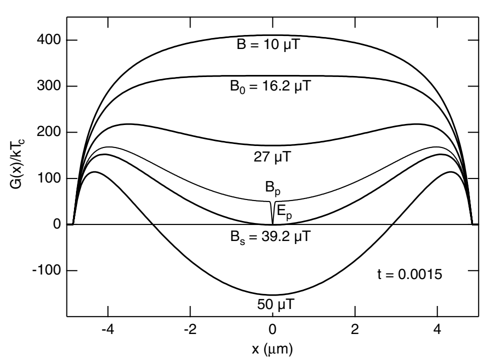

FIG. 1. The Gibbs free energy (Ref. [2]) of a single vortex located at position x at several values of the applied field B, at a reduced temperature $t = 1 - T/T_c$ of 0.0015. The curve at $B_p = 36\ \mu$T includes schematically a pinning well of depth $E_p \approx 50\, kT_c$.

To test these ideas, we fabricated strips from sputtered niobium films of thickness $d = 210$ nm using photolithography and subsequent reactive ion etching. The film studied had a transition temperature T_c of 8.848 K. The strip lengths were 4 mm, and had nominal widths of 1.6, 10, and 100 μm. The strips were imaged far from their ends using a low-temperature SHM with a wide-field scanning head [13], allowing us to image many vortices to get good counting statistics. The probe had an active region of about 1.1 μm on a side and was scanned at a height between 1.0 and 1.5 μm from the surface. A magnetic shield enclosed the entire cryostat and reduced the ambient field to less than 1 μT.

We determined the vortex density by applying a field B and then cooling the strips through T_c to 7 K where the SHM images were taken. The images were independent of the cooling rate near T_c down to 10 mK/s. Representative images are shown in Fig. 2. Figure 2(a) shows an image of 10 μm strips at $B = 85\ \mu$T. Although the vortices tend to lie along the center of the strip where their energy is the lowest, some vortices are pinned away from the center. In Fig. 2(b), we show a 1100 μm strip in an applied field of 5.3 μT. The vortices have begun to spread out over the strip, although at lower fields (not shown) they again tend to lie near the center.

We determined the critical field for each strip by taking images at many applied fields B and counting the number of vortices N in each image. As plotted in Fig. 3, for each width we find a field range centered about zero for which all vortices are expelled from the strip. The data at high field has a linear dependence of N given approximately by $N = (B - B_m)A/\Phi_0$, where A is the total area of the strips in the image. We have defined the maximum field B_m for complete vortex expulsion by extrapolating this linear

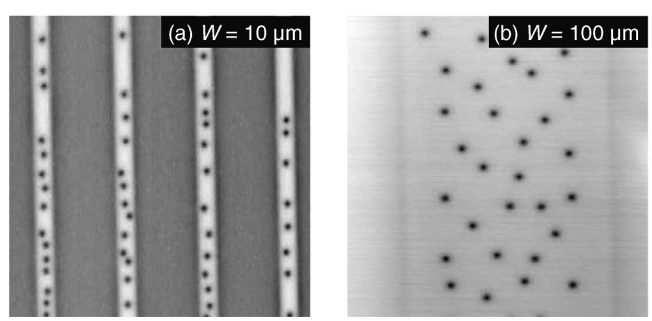

FIG. 2. (a) 10 μm strip after field cooling in 85 μT. The strips appear light because of the Meissner expulsion of the field, but many vortices (darker spots) are visible. (b) 100 μm strip after field cooling in 5.3 μT. Both images are 140 μT full scale, and about 145 μm wide.

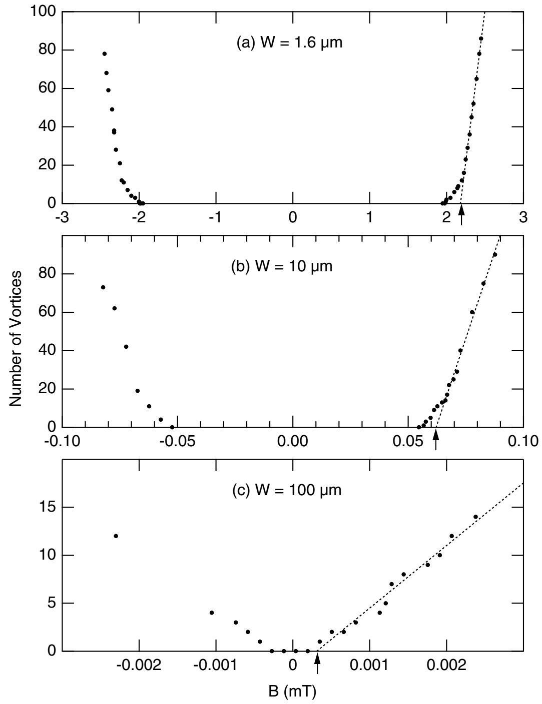

FIG. 3. Number of vortices N in an image as a function of applied field B for three different widths of strips. The arrows indicate the values of B_m determined by extrapolation of the linear regime (dotted lines), which have slopes of 0.85, 1.17, and 0.99 A/Φ_0, for (a), (b), and (c), respectively, where A is the total area of the strips visible in an image.

dependence of N down to $N = 0$. The data from Fig. 3 show that B_m changes rapidly with strip width. We will discuss later the region of reduced slopes at fields slightly less than B_m for the 1.6 and 10 μm strips.

In Fig. 4 we plot the experimentally determined values of B_m and compare them with the two theoretical predictions B_0 [Eq. (1)] and B_s [Eq. (2)]. We observe good agreement in the approximate magnitude of B_m as well

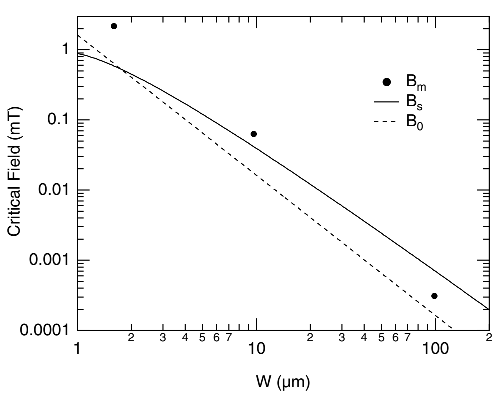

FIG. 4. Experimental and theoretical critical fields vs strip width W. B_m is the experimental data from this paper; B_s and B_0 are theoretical curves derived from criteria of absolute (B_s) and relative (B_0) stability.

as its scaling as $1/W^2$. To make a more quantitative comparison, we must estimate $\xi(T)$ in Eq. (2) at the temperature T_f at which the vortices become pinned.

We measured T_f as follows. First, vortices were nucleated in the 10 μm strip at a field 62 μT, slightly above B_m. The strip was then cooled to well below T_c, freezing in the vortices. After the field was lowered to slightly below B_m, the strip was continually imaged as the temperature was very slowly (0.1 mK/s) raised towards T_c. We found that vortices first moved when $t = 1 - T/T_c = 0.0015$, identifying the temperature T_f. This value is consistent with previous measurements [9,10]. At this temperature we estimate ξ to be 320 nm, taking [14] $\xi_0 = 38.9$ nm and a value of the Ginzburg-Landau parameter $\kappa = 5.0$, derived from the normal-state resistivity of our films [14,15]. We note that, since ξ appears only logarithmically in Eq. (2), B_s is not sensitive to our determination of either T_f or ξ.

The theoretical prediction for B_s is plotted in Fig. 4 for this value of ξ and $\alpha = 2/\pi$. The prediction of B_0 [which is independent of ξ, Eq. (1)] is consistently below our data, whereas the prediction for B_s matches the data better, especially for the strip of intermediate width. We note that the theoretical assumption $\xi \ll W \ll \Lambda$ is well satisfied only for this intermediate strip. For the narrowest strip, ξ/W is about 0.2, causing the London model used in the theoretical treatments to begin to break down, and altering the calculations of the vortex-image forces and free energy. For the widest strip at $t = 0.0015$, we calculate that $\Lambda = 24$ μm, which is less than W. When $\Lambda \ll W$, the critical field has been calculated [3,16] for the 100 μm strip to be *lower* than B_s by a factor of about 3, consistent with our data. Since the data for the 10 μm strip matches well the predictions for B_s but not B_0, we confirm that vortices are expelled unless the free energy G is *absolutely* stable.

We now turn to understand the deviation in Fig. 3 of the linear behavior of N vs B, for B slightly less than the critical field. This data can be explained by pinning, which is represented schematically in Fig. 1 as a narrow well of depth E_p in $G(x)$. Since such wells lower the free energy, they allow the first vortices to enter at a *lower* field than they would with no pinning. In Fig. 1, as the field is changed from B_p, where the first vortex enters the pinning well, to B_s, where vortices would have first entered without pinning, the free energy changes by $\Delta G = E_p$. Thus, we have

$$\Delta B/B_s \approx \frac{1}{B_s}\frac{\Delta G}{\partial G/\partial B} \approx \frac{1}{B_s}\frac{E_p}{\partial G/\partial B} = \frac{8\pi^2\Lambda}{\Phi_0^2 \ln(2W/\pi\xi)} E_p. \tag{3}$$

Here ΔB is the difference between the critical field (the extrapolation of the linear regime to $N = 0$ in Fig. 3) and B_p (where the first vortex enters), and we have used the result [2]

$$\partial G/\partial B|_{x=0} = \frac{\Phi_0 W^2}{16\pi\Lambda}.$$

Equation (3) predicts, within logarithmic accuracy, that the fractional width $\Delta B/B_s$ of the pinning region is independent of width. Experimentally, we find $\Delta B/B_m = 0.09$ for the 1.6 μm strip, and 0.10 for the 10 μm strip, confirming this prediction [17]. Taking $\Delta B/B_m = 0.1$ and using Eq. (3), we estimate a pinning energy $E_p \sim 51\, kT_c$. This energy can be compared with E_p estimated from the freeze-out temperature T_f. Since the vortex motion is thermally activated and has a rate described by a Boltzmann factor, the rate becomes small—and the vortices freeze out—when $E_p \sim 20\, kT_f$, a value that is reasonably close to E_p obtained above using Eq. (3).

Finally, we discuss the dependence of the vortex density on applied field, shown in Fig. 3. Ignoring the knee region, N is nearly linear in $B - B_m$. This is quite different from the situation in a bulk superconductor, where there is a very sharp increase in the vortex density just above H_{c1}, followed by a more gradual increase thereafter [14,18]. This rapid increase is due to interactions between vortices which are weak until the vortices come within $\sim\lambda$ of each other. In the strips, however, the interactions have a long range $(1/r)$ force, which implies a more gradual increase in the vortex density with applied field. Interestingly, the measured slope dN/dB is quite close to the simple value of $S_0 \equiv A/\Phi_0$, where A is the total area of the strips visible in the image. This slope corresponds to the number vortices expected if the total flux through the strips above T_c nucleated into vortices. It might have been expected, however, that because of the small effective width available to the vortices at fields just above B_m (e.g., Fig. 1, 50 μT), this slope would be rather less than S_0.

Maksimova [4] has calculated the free energy as a function of N near the metastable minimum B_0, from which it can be shown that $N \approx (B - B_0)^{3/2}$ for small N. We may extend her calculations to the region near B_s, and find

$$N = \frac{A}{2\Phi_0}(B - B_s).$$

While linear, in agreement with the data, this expression predicts a slope about one-half of the observed value. We note that in Maksimova's derivation it is assumed that the vortex density is high enough to be taken as a smooth continuum, and so her expressions should not be expected to agree quantitatively with our results where the vortices are quite discrete. Indeed, an analytic theory in this low-N regime is likely to be difficult to formulate. A more fruitful approach may be the use of computer simulations of vortices, using analytically derived forces and energies.

In conclusion, we have imaged vortices with a scanning Hall-probe microscope and shown a well-defined maximum field B_m for vortex expulsion. Our data verifies the theoretical prediction $B_m \approx \Phi_0/W^2$. The magnitude of B_m supports the criterion that vortices are observed in the film when the free energy of a vortex at the center of the strip is negative. The critical field, which is many orders of magnitude less than the bulk value, implies that superconducting devices should be designed with narrow wires to eliminate trapped vortices and the generation of noise from their motion.

[1] J. R. Clem, Bull. Am. Phys. Soc. **43**, 411 (1998).
[2] J. R. Clem (unpublished).
[3] K. K. Likharev, Sov. Radiophys. **14**, 722 (1972).
[4] G. M. Maksimova, Phys. Solid State **40**, 1610 (1998).
[5] E. Dantsker, S. Tanaka, and J. Clarke, Appl. Phys. Lett. **70**, 2037 (1997).
[6] R. Wördenweber, A. M. Castellanos, and P. Selders, Physica (Amsterdam) **332C**, 27 (2000).
[7] R. Wördenweber and P. Selders, Physica (Amsterdam) **366C**, 135 (2002).
[8] C.-S. Lee, B. Jankó, I. Derényi, and A.-L. Barabási, Nature (London) **400**, 337 (1999).
[9] J. Sok and D. K. Finnemore, Phys. Rev. B **50**, 12 770 (1994).
[10] D. Kouzoudis, M. Breitwisch, and D. K. Finnemore, Phys. Rev. B **60**, 10 508 (1999).
[11] J. Guimpel, L. Civale, F. de la Cruz, J. M. Murduck, and I. K. Schuller, Phys. Rev. B **38**, 2342 (1988).
[12] S. Brongersma, E. Verweij, N. J. Koeman, D. G. de Groot, R. Griessen, and B. I. Ivlev, Phys. Rev. Lett. **71**, 2319 (1993).
[13] S. Field and J. Barentine, Rev. Sci. Instrum. **71**, 2603 (2000).
[14] H. W. Weber, E. Seidl, C. Laa, E. Schachinger, M. Prohammer, A. Junod, and D. Eckert, Phys. Rev. B **44**, 7585 (1991).
[15] P. G. deGennes, *Superconductivity of Metals and Alloys* (Addison-Wesley, Reading, MA, 1966), Eq. 7-51.
[16] I. L. Maksimov and G. M. Maksimova, JETP Lett. **65**, 423 (1997).
[17] For the 100 μm strip, the scatter of the data would make a difference of 10% in B_m too small to observe.
[18] P. G. deGennes, *Superconductivity of Metals and Alloys* [Ref. [15], p. 68].