Theory of Mach Cones in Magnetized Dusty Plasmas with Strongly Correlated Charged Dust Grains

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A theory for the formation of Mach cones in a magnetized dusty plasma with strongly correlated charged dust grains has been presented. We use the electron and ion susceptibilities for weakly correlated magnetized electrons and ions as well as the strongly correlated unmagnetized dust grain susceptibilities which are obtained by means of the quasilocalized charge approximation and a generalized hydrodynamic model. The plasma dielectric response of the present system reveals the parametric regimes for which Mach cones in a strongly coupled laboratory dusty magnetoplasma can be formed. We suggest conducting experiments in radiofrequency dusty plasma discharges with superconducting magnets for verifying the theoretical prediction of Mach cones that is made herein.

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Recently, the formation of Mach cones (V-shaped disturbances formed by an object moving with a superacoustic speed in a dispersive medium) in dusty plasmas has become an important issue from theoretical [1–7], experimental [8–11], and simulation [12] points of view. The experimentally observed Mach cones [8,9] in dusty plasma crystals have been theoretically interpreted by Dubin [2] in terms of constructive interference between dispersive dust lattice/acousticlike waves. On a view of the possibility for the formation of Mach cones in Saturn's dusty rings, theories for short wavelength dust-acoustic and dust-magnetoacoustic [4–6] Mach cones in magnetized plasmas with weakly coupled charged dust grains have been worked out.

However, in laboratory dusty magnetoplasmas with strongly correlated dust grains, because of the low dust temperature, short intergrain distance, and high dust grain charge, the intergrain (screened) Coulomb coupling parameter $\Gamma \simeq Z_d^2 e^2 \exp(-a_d/\lambda_D)/a_d T_d$ can be much larger than 1 and the average intergrain spacing could be of the order of the ion gyroradius ρ_i, where Z_d is the number of electrons residing on the dust grain surface, $a_d = (3/4\pi n_d)^{1/3}$ is the average intergrain spacing, λ_D is the characteristic Debye radius in dusty plasmas [6], T_d is the dust temperature, and n_d is the dust number density. As an example, for typical laboratory dusty plasma parameters [13], $T_e \simeq 2$ eV, $T_i \simeq 0.5T_e$, $T_d \simeq 0.07$ eV, $n_i \simeq 3 \times 10^9$ cm^{-3}, $Z_d \simeq 10^3$, $n_d \simeq 2 \times 10^5$ cm^{-3}, $r_d \simeq 0.3$ μm, and $B_0 = 10^4$ G, we have $a_d/\lambda_D \simeq 0.94$, $a_d/\rho_i \sim 1$, $\Gamma \simeq 193$, and $\Gamma_d \simeq 75$, respectively. Thus, in the present generation laboratory dusty plasma discharges, charged dust grains will remain strongly coupled, but weakly coupled ions and electrons would be magnetized. Since strong dust correlations significantly modify the dust grain susceptibility [14–17], the theory of Mach cones [1,4–6] (developed for weakly coupled dusty plasmas of Saturn's rings) is no longer valid for laboratory magnetoplasmas with strongly correlated charged dust particles.

In this Letter, we present a theory for the Mach cones in a dusty magnetoplasma which is composed of weakly coupled magnetized electrons and ions, and strongly coupled unmagnetized charged dust grains. We employ the susceptibilities for magnetized electrons and ions as well as newly found dust susceptibilities [14,15] that use the quasilocalized charge approximation (QLCA) and the (GHM), and derive the dispersion relations for the low-frequency (in comparison with the ion gyrofrequency) dusty plasma waves that are responsible for the formation of Mach cones in strongly coupled dusty magnetoplasmas.

We consider the propagation of low-frequency ($\omega \ll \omega_{ci}$, $\omega_{ci} = eB_0/m_i c$ is the ion gyrofrequency, B_0 is the magnitude of the external magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, m_i is the ion mass, e is the magnitude of the electron charge, c is the speed of light in vacuum, and $\hat{\mathbf{z}}$ is the unit vector along the z direction) electrostatic perturbations in our strongly coupled dusty magnetoplasma. Thus, at equilibrium we have $n_{i0} = n_{e0} + Z_d n_{d0}$, where n_{e0}, n_{i0}, and n_{d0} are the equilibrium electron, ion, and dust number densities, respectively. When the wavelength of the perturbations is comparable or shorter than the ion gyroradius $\rho_i (= v_{ti}/\omega_{ci})$, where $v_{ti} = (T_i/m_i)^{1/2}$ is the ion thermal speed and T_i is the ion temperature, we cannot use the fluid theory to determine the ion density perturbation n_{i1} in the electrostatic wave potential ϕ. Rather, we have to resort to the ion Vlasov equation to calculate the ion density perturbation. For $\omega \ll \omega_{ci}$ and arbitrary $b_i = k^2\rho_i^2$ values, where k is the propagation constant of the waves propagating in a plane perpendicular to the external magnetic field, we have $n_{i1} = -(k^2/4\pi e)\chi_i \phi$, where the ion susceptibility involving two-dimensional (in a

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plane perpendicular to the external magnetic field direction) weakly coupled ion motions is [5,6,18]

$$\chi_i \simeq \frac{1}{k^2\lambda_{Di}^2}\left[1 - \Lambda_0(b_i) + 2\Lambda_1(b_i)\frac{\omega^2}{\omega_{ci}^2}\right], \qquad (1)$$

where $\lambda_{Di} = (T_i/4\pi n_{i0}e^2)^{1/2}$, $\Lambda_{0,1} = I_{0,1}\exp(-b_i)$, and $I_0(I_1)$ is the zero (first) order modified Bessel function. On the other hand, for $b_e = k^2 v_{te}^2/\omega_{ce}^2 \ll 1$, the electron density perturbation is given by $n_{e1} = (k^2/4\pi e)\chi_e\phi$, where the electron susceptibility is

$$\chi_e \simeq \frac{\omega_{pe}^2}{\omega_{ce}^2}, \qquad (2)$$

where $v_{te} = (T_e/m_e)^{1/2}$, $\omega_{ce} = eB_0/m_e c$, $\omega_{pe} = (4\pi n_{e0}e^2/m_e)^{1/2}$, and m_e is the electron mass. We note that Eqs. (1) and (2) neglect the ion and electron motions parallel to the external magnetic field direction. This is justified because we are focusing on low-frequency perturbations with $k_z v_{ti} \ll \omega$ and $k_z/k \ll \omega/\omega_{ce} \ll 1$, where k_z is the component of the wave vector parallel to $\hat{\mathbf{z}}$.

For $\omega_{cd} \ll \omega$, where $\omega_{cd} = eZ_dB_0/m_dc$ is the dust gyrofrequency, strongly coupled dust grains can be considered unmagnetized. Using the QLCA and the GHM, the dust number density perturbation is $n_{d1} = (k^2/4\pi Z_d e)\chi_d\phi$, where the dust susceptibility in the collisionless limit is of the form [14–16]

$$\chi_d \simeq -\frac{\omega_{pd}^2}{\omega^2 - \omega_{pd}^2\mathcal{D}(k)}. \qquad (3)$$

Here $\omega_{pd} = (4\pi n_{d0}Z_d^2e^2/m_d)^{1/2}$ is the dust plasma frequency, and the $\mathcal{D}(k)$ term arises due to the strong correlation between dust particles. We now approximate $\mathcal{D}(k)$ by the QLCA in the limits ($\Gamma_d \gg 1$ and $d \leq \lambda_d$) considered by Rosenberg and Kalman [14], and the GHM in the limit ($1 \leq \Gamma \leq 200$) considered by Kaw and Sen [15]. That is, using the QLCA and long wavelength (in comparison with λ_D) limit, we can approximate $\mathcal{D}(k)$ as

$$\mathcal{D}(k) \equiv \mathcal{D}_{\rm RK}(k) \simeq fk^2a_d^2, \qquad (4)$$

where $f = -(4/45)(0.9 + 0.05a_d^2/\lambda_D^2)$ for $\Gamma_d \gg 1$ and $d \leq \lambda_D$. On the other hand, using the GHM we can approximate $\mathcal{D}(k)$ as

$$\mathcal{D}(k) \equiv \mathcal{D}_{\rm KS}(k) \simeq \gamma_d\mu_d k^2\lambda_{Dd}^2, \qquad (5)$$

where γ_d is the adiabatic index, $\lambda_{Dd} = (T_d/4\pi n_{d0}Z_d^2e^2)^{1/2}$, and $\mu_d = 1 + (1/3)u(\Gamma) + (\Gamma/9) \times [\partial u(\Gamma)/\partial\Gamma]$ is the compressibility [19], and $u(\Gamma)$ is the excess thermal energy of the system, which is often obtained by fitting data from Monte Carlo and molecular dynamic simulations or experiments, and can be approximated [16,20] as $u(\Gamma) \simeq -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81$ for $1 < \Gamma < 200$.

Now, using Eqs. (1)–(3) in $1 + \chi_e + \chi_i + \chi_d = 0$, the dispersion relation involving $\Psi = \omega/kV_d$ (a ratio of the wave phase speed $V_p = \omega/k$ to the dust particle speed V_d) can be expressed as

$$\alpha + \frac{2V_d^2\Lambda_1}{\lambda_{Di}^2\omega_{ci}^2}\Psi^2 - \frac{\omega_{pd}^2}{k^2V_d^2\Psi - \omega_{pd}^2\mathcal{D}(k)} = 0, \qquad (6)$$

where $\alpha = 1 + \omega_{pe}^2/\omega_{ce}^2 + (1 - \Lambda_0)/k^2\lambda_{Di}^2$. We note that for the QLCA we replace $\mathcal{D}(k)$ by $\mathcal{D}_{\rm RK}(k)$ defined by (4), and for the GHM we replace $\mathcal{D}(k)$ by $\mathcal{D}_{\rm KS}(k)$ defined by (5). The dust particle speed V_d in (6) is considered as a free parameter.

The Mach cones can be formed by any perturbation waves [e.g., waves defined by (6) in our case] if the object (viz. a dust particle in our case) speed V_d is larger than the wave phase speed V_p; i.e., $\Psi = V_p/V_d < 1$. If this condition is satisfied, the Mach cone opening angle θ is given by $\theta = \sin^{-1}\Psi$, where Ψ is defined by (6). To analyze the possibility for the formation of the Mach cones associated with the waves defined by (6), we numerically analyze (6), and find $\Psi = 1$ curves in (V_d, $k\rho_i$) space for typical laboratory dusty plasma parameters [13]: $T_e \simeq 2$ eV, $T_i \simeq 0.5T_e$, $T_d \simeq 0.07$ eV, $n_i \simeq 3 \times 10^9$ cm^{-3}, $Z_d \simeq 10^3$, $n_d \simeq 2 \times 10^5$ cm^{-3}, $r_d \simeq 0.3$ μm, $B_0 = 10^3$ G, $B_0 = 5 \times 10^3$ G, and $B_0 = 10^4$ G. The numerical results are displayed in Fig. 1. The $\Psi = 1$ curves in (V_d, $k\rho_i$) space will, obviously, determine the critical values of the dust particle speed V_d and the corresponding wavelength $\lambda = 2\pi/k$ for which the Mach cones are formed. The regions above the $\Psi = 1$ curves correspond to $\Psi < 1$, i.e., correspond to a regime for which the Mach cones in strongly coupled dusty magnetoplasmas are formed. The upper plot of Fig. 1 shows the critical values of the dust particle speed V_d and the corresponding wavelength $\lambda = 2\pi/k$ for which the Mach cones are formed for two different approaches, namely the QLCA and the GHM, and clearly indicates that for laboratory dusty magnetoplasma conditions [13] the QLCA and the GHM do not have any significant discrepancies, and for $k\rho_i \leq 1$ both approaches give exactly the same results.

The lower plot of Fig. 1, where $\mathcal{D} = \mathcal{D}_{\rm RK}$ is used, shows the critical values of the dust particle speed V_d and the corresponding wavelength $\lambda = 2\pi/k$ for which the Mach cones are formed for different values of the external magnetic field strength. It implies that, as we increase the magnitude of the external magnetic field, for the wave of fixed wavelength we need a dust particle of higher speed in order for the formation of the Mach cones.

In summary, we have theoretically investigated the possibility for the formation of the Mach cones in a dusty magnetoplasma whose constituents are weakly correlated magnetized electrons and ions, and strongly coupled unmagnetized charged dust grains. We have graphically shown the parametric regime (the region above the $\Psi = 1$ curves in Fig. 1) for which the Mach cones can be formed in laboratory dusty magnetoplasmas. In our analysis, we have considered the QLCA as well as the GHM, and have found that for laboratory dusty

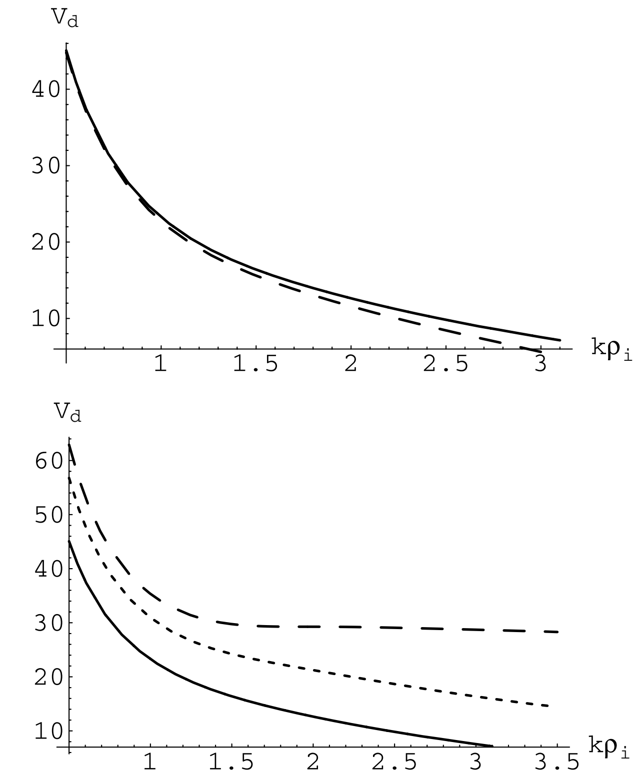

FIG. 1. $\Psi \equiv V_p/V_d = 1$ curves in $(V_d, k\rho_i)$ space for typical laboratory dusty plasma parameters (given in the text). The upper plot, where the solid (dashed) curve represents to the numerical results based on the QLCA (GHM), shows a comparison between the QLCA and the GHM. The lower plot, where $\mathcal{D} = \mathcal{D}_{\mathrm{RK}}$, $B_0 = 10^3$ G (solid curve), $B_0 = 5 \times 10^3$ G (dotted curve), and $B_0 = 10^4$ G (dashed curve), shows the effect of B_0 on $\Psi = 1$ curves in $(V_d, k\rho_i)$ space.

magnetoplasma conditions the QLCA and the GHM do not have any significant discrepancies, and for $k\rho_i \leq 1$ both approaches give exactly the same results. We have also examined the effect of the external magnetic field on the critical values of the dust particle speed V_d and the wavelength of the waves for which the Mach cones may be formed in a strongly coupled dusty magnetoplasma. We found that, as we increase the magnitude of the external magnetic field, for waves of fixed wavelength we need a dust particle of higher speed in order for the creation of Mach cones. From the opening angle of the Mach cone, one would be able to infer the plasma and dust parameters. It should be noted that the Mach cone result for an unmagnetized plasma cannot be obtained from the present theory in view of the approximations (viz. $\omega_{ce}k_z/k \ll \omega \ll \omega_{ci}$) used for calculating the electron and ion susceptibilities. Dust-acoustic Mach cones in a magnetized plasma could be formed only if electrons and ions have the Boltzmann distribution [5,6]; the latter appears when $\omega \ll k_z v_{te}, k_z\omega_{ce}/k$ and $b_i \gg 1$. We are hoping that the Garching group [21] will confirm our theoretical prediction of the Mach cone formation in her laboratory experiments in which strongly coupled dust grains, forming robust dust Coulomb lattices with intergrain spacing of the order of the ion gyroradius, will be held in a strong external magnetic field ($B_0 \simeq 4$ T).

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