Gap Solitons in Waveguide Arrays

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Bright and dark spatial gap solitons are demonstrated in waveguide arrays. These gap solitons travel across the array at zero transverse velocity, in complete analogy with stationary (immobile) temporal gap solitons. Furthermore, the launching configuration for observing these stationary gap solitons is shown to be the analog of an "ideal experiment" for observing stationary temporal gap solitons, never observed so far. A clear distinction is established between the family of Floquet-Bloch solitons in general and discrete solitons in particular, and the limiting case of gap solitons.

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Light interaction with nonlinear periodic media yields a diversity of fascinating phenomena, among which two solitonic phenomena have been studied most intensively, namely, discrete (or lattice) solitons [1–3] and gap (or Bragg) solitons [4–9]. While discrete solitons are spatial phenomena in two- or three-dimensional arrays of coupled waveguides, gap solitons are usually considered as a temporal phenomenon in one-dimensional (1D) periodic media (e.g., an optical fiber in which the refractive index varies periodically, for reviews, see [10–12]). In such 1D periodic media the dispersion relation for the electromagnetic field (relating the frequency ω to the Bloch wave number K) is divided into bands, separated by gaps in which propagating modes are not allowed. At each band edge, that is, at the edge of each Brillouin zone, where the light resonates with the periodic medium, the group velocity $d\omega/dK$ reaches zero and the reflection of the medium reaches unity, leading to Bragg reflection of a beam incident onto the medium. If we allow for a positive Kerr nonlinearity, a high-power beam having a frequency located inside a gap may induce a nonlinear shift of the gaps so that *the beam tunes itself outside the gap*. The result is a *gap soliton,* propagating inside the periodic medium *slower* with respect to a continuous medium with the same (average) refractive index. Most peculiar is the stationary gap soliton, which is completely immobile.

A major experimental difficulty involved in the excitation of gap solitons stems from the high reflectivity of the periodic medium in and near the gaps. It is obviously difficult to couple enough light from a continuous region into the periodic medium, in order to excite a soliton. In fact, it takes a nonlinear instability to occur in the periodic medium, under the effect of a very powerful beam, for the excitation of a gap soliton [11]. Propagating gap solitons have been demonstrated in fiber gratings [13,14] and in ridge waveguide grating [15], however, *stationary* (or nearly stationary) gap solitons have not been observed yet. In an ideal experiment for observing stationary gap solitons, one would arrange inside the nonlinear medium a high-power pulse at a frequency which corresponds to the midgap, composed of two counterpropagating components, such that its shape matches that of the gap soliton, and only then "turn on" the periodic structure. This results in a localized, immobile, structure of light, preserving a constant width and shape. Great experimental simplification may be achieved if the 1D periodic medium (along x) is expanded by another homogenous dimension, z. The gap soliton may now travel along this extra spatial dimension, if supplied with momentum in that direction. This will change nothing in the physical mechanism, which underlies the creation of gap solitons, and the stationary soliton is now merely traveling freely along the z direction. However, in this configuration, the field constituting the gap soliton is constantly injected into the medium, eliminating time dependence while creating a *spatially localized* structure—a *spatial* gap soliton—propagating along the z direction. Obviously, the 2D structure now closely resembles the waveguide-array geometry of discrete solitons.

Indeed, a formal mapping between the 1D + time system (e.g., fiber grating) and the 2D system of the waveguide array may be established, in which the waveguide direction, z, in the array (and its conjugate variable β) replaces time (and its conjugate variable ω) in the 1D + time system. This results in the very same band-gap structure for the two systems [16]. We would like to point out that discrete solitons at the top of band No. 1 (see Fig. 1) also move at zero transverse velocity; however, this comes about not as a consequence of a Bragg reflection (as in the case of gap solitons) but from the obvious reason that these solitons possess zero transverse momentum ($k_x = 0$), just as continuum solitons at $k_x = 0$ do. Indeed, such discrete solitons have no analog in the 1D + time system. This is because these waves propagate with

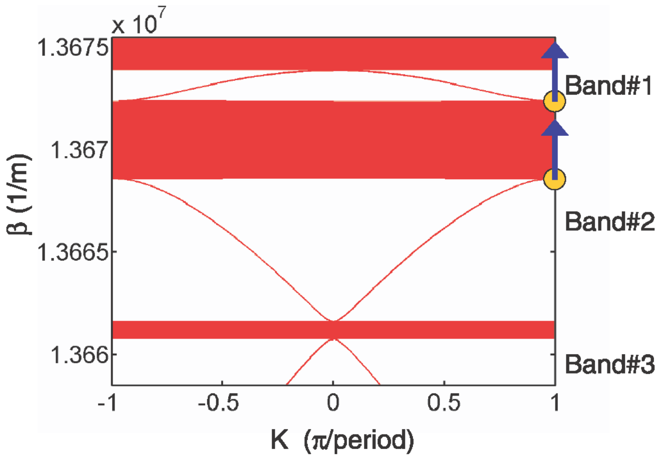

FIG. 1 (color online). The band-gap diagram of a typical waveguide array, relating the propagation constant (β) to the Bloch wave number (K). The three highest bands are shown, with the shaded regions representing the gaps. The excited gap-soliton modes (at band No. 1 and band No. 2) are marked by circles and their propagation direction by arrows.

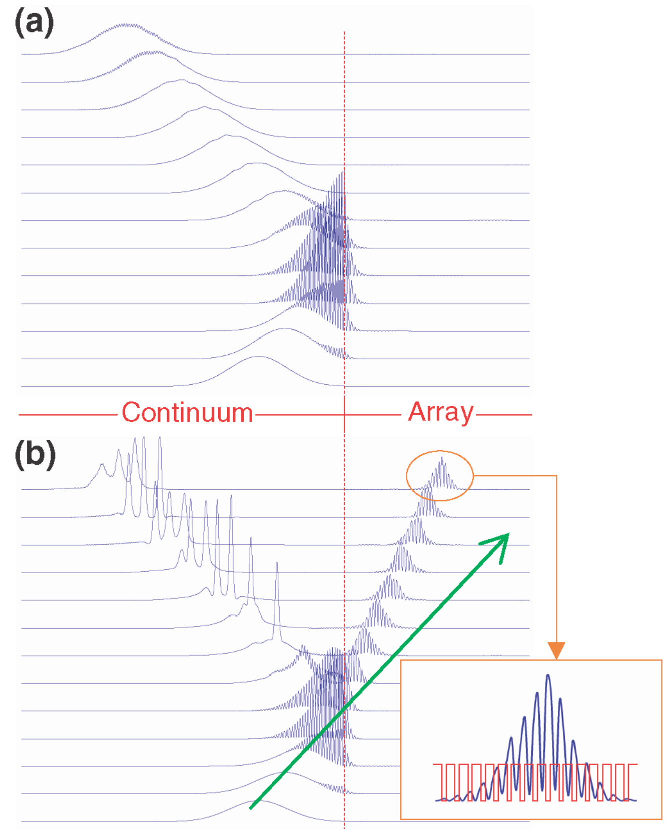

FIG. 2 (color online). Side-coupling geometry, for exciting a pure FB mode in a waveguide array. The beam is injected into the array from the continuum region along its edge. (a) At low power, the shown beam is Bragg reflected from the array into the continuum. (b) At high power, part of the beam penetrates the array, while exciting a gap soliton, traveling at a slow transverse velocity, as compared to that of the input beam (marked by an arrow). The inset shows an enlarged plot of the modal shape of the excited gap soliton.

evanescent fields in the low index regions, a mode not possible in 1D + time. This is true also for the rest of band No. 1 solitons in waveguide arrays (moving discrete solitons), and, in particular, to the immobile gap solitons at the bottom of band No. 1. As it turns out, the spatial analog of the usual temporal gap soliton is actually at the top of band No. 2, which is the mode with fields that peak in the low index regions of the structure.

In this Letter, we use waveguide arrays to excite spatial gap solitons, traveling at zero transverse velocity, using a configuration, which is completely analogous to the ideal experiment described above. Some aspects of spatial gap solitons in waveguide arrays have been discussed before for regular arrays [3,17–19] and for modulated waveguide arrays which possesses a "ghost gap" inside band No. 1 [20]. In addition, staggered discrete (band No. 1) gap solitons have been excited in optically induced nonlinear photonic lattices [21].

As discussed above and demonstrated experimentally recently [16], the diffraction relation (relating the propagation constant β to the Bloch wave number K) of waveguide arrays possesses bands, separated by gaps in which propagating modes are not allowed (see Fig. 1). It has also been shown that any particular mode in any band may be excited in the array when using a side-coupling geometry (see Fig. 2), by tilting the input beam to the required angle. Angles that correspond to a gap will lead to Bragg reflection from the array interface back into the continuum, as shown in Fig. 2(a). When the beam power is raised, part of the beam may penetrate the array [Fig. 2(b)] and form a spatial gap soliton, which propagates at a transverse velocity substantially smaller than the continuum velocity [marked by an arrow in Fig. 2(b)]. The gap soliton excited in the array is, in fact, a pure band No. 2 Floquet-Bloch (FB) soliton (discussed in detail in Ref. [16]), traveling at small velocity because it is excited close to the upper edge of band No. 2 (see Fig. 1), where $d\beta/dK$ vanishes. The inset of Fig. 2(b) shows the modal shape of the excited gap soliton, which is identical to that of the band No. 2 FB mode. This approach for exciting gap solitons in waveguide arrays is difficult to realize for the same reason as in the 1D system, namely, the high reflectivity of the array close to a band edge.

A different approach for exciting a pure FB soliton in the array is to inject the light in the head-on geometry (Fig. 3) using an input beam that is spatially shaped to match exactly the modal shape of the desired FB mode [16]. In this manner, excitation of *a pure FB mode that is right at the band edge* may be realized. Figure 3 shows an enveloped beam having the modal shape of a band No. 2 FB mode, at the upper band edge, that is for $K = \pi/p$ (p being the array's period). Such a narrow-enveloped beam will propagate in the array at zero transverse velocity, while diffracting sideways. When the power level of such a beam is increased, the band-gap structure in the high-power region of the beam shifts upwards (toward higher

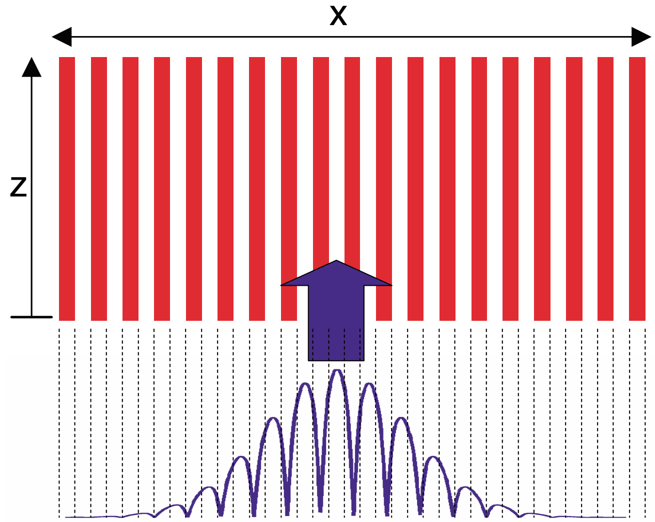

FIG. 3 (color online). Head-on geometry, for exciting a pure FB mode in a waveguide array, by shaping the input beam such that it matches the modal shape of the desired FB mode. Band No. 2 modal shape is shown, for the marked FB mode in Fig. 1 (at $K = \pi/\text{period}$).

β's) under the influence of the Kerr effect, such that the high-power region of the beam is located inside the gap of the *linear* diffraction relation, thereby creating a spatial gap soliton, which is the stationary limit of the gap soliton shown in Fig. 2(b). The soliton is propagating at zero velocity, although it is located around a nonzero wave number ($K = \pi/p$). Injecting such a beam into a continuum will cause it to split into two beams (as shown below), which will propagate at opposite directions *at finite transverse velocities.*

In order to shape an input beam into a (nearly) pure FB mode, we used two beams, which form an interference pattern [17]. When considering shallow etched arrays (i.e., index step of 0.001 to 0.007), this interference pattern turns out to be quite similar to the modal shape of the FB modes at the bottom edge of band No. 1, or at the top edge of band No. 2 (both at $K = \pi/p$), depending on whether the interference peaks are centered onto the array's waveguides or in between them, respectively. This interference pattern was injected head-on (as shown in Fig. 3), onto the input facet of our waveguide-array sample. [A different approach for understanding this excitation scheme is as follows. Each of the Gaussian (unshaped) beams excites both band No. 1 (bottom edge) and band No. 2 (top edge), traveling both at zero transverse velocity (Fig. 1), as discussed in Ref. [16]. However, by changing the relative phase of the two beams, a destructive interference of band No. 2 components of the two beams may be achieved, in which case band No. 1 components of the two beams excited in the array will interfere constructively, leading to a pure excitation of a band No. 1 FB mode (and vice versa for band No. 2).] It is clear that at the bottom edge of band No. 1, where the diffraction is anomalous [16], only *dark* gap solitons may be excited (in a media with positive Kerr nonlinearity), while *bright* gap solitons may be excited at the top edge of band No. 2, where the diffraction is normal (Fig. 1).

The experimental system is similar, in principle, to that used in previous experiments [22]. The sample consisted of an 8 μm period waveguide array, with a waveguide width of 4 μm. 130 fsec pulses emitted from a synchronously pumped optical parametric oscillator at 1.53 μm were split into two beams, interfering at an angle of 11° to form an 8 μm interference pattern on the input facet of the sample. These input beams therefore are counterpropagating with a velocity of $\pm c/35$ in the x direction. The light emerging from the array was imaged onto an IR camera. The input pattern was carefully adjusted to the band edge (i.e., exactly $K = \pi/p$), by verifying that both band No. 1 and band No. 2 excitations emerge from the very same location (identical with the input location), implying that *the excitation travels at zero transverse velocity.*

Figure 4 shows experimental results, in which a band No. 2 beam of six array periods width is launched into the array. The figure shows photographs of the beam at the array output, at low and high power. At low power, the input beam excites a band No. 2 FB mode, which diffracts to about twice its initial width. At high power, the beam focuses until reaching its initial width, implying that a bright gap soliton has been excited in the array, traveling at zero transverse velocity. It should be stressed that the FB solitons excited in Ref. [16] were *not* gap solitons, as they propagated at a "normal" velocity (i.e., unaffected by the presence of the gap), and may be regarded in some sense as continuum solitons, which are merely perturbed by the periodic potential of the waveguide array. In contrast to the general case of FB solitons, the (FB) gap solitons excited in this experiment travel at zero transverse velocity, under the effect of their resonant interaction with the array.

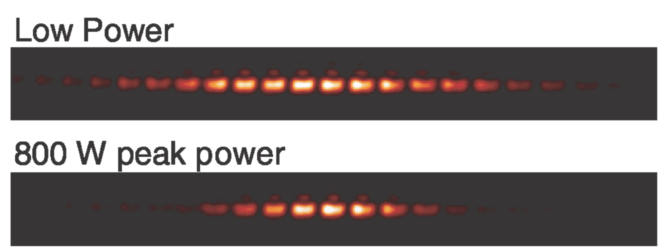

FIG. 4 (color online). Experimental results of a bright gap-soliton measurement, excited exactly at the top edge of band No. 2 (marked in Fig. 1), using a shaped input beam. Shown are photographs of the beam at the array output, for low and high powers. At low power, the input beam diffracts into about twice its initial width. At high power, a bright gap soliton is formed, traveling across the array at zero transverse velocity. We note that high index-step samples were used in this measurement, which possess a slightly double-humped modal shape of band No. 2, unlike the simulations shown in previous figures, where a smaller index step was assumed, which possess a single-humped modal shape of band No. 2.

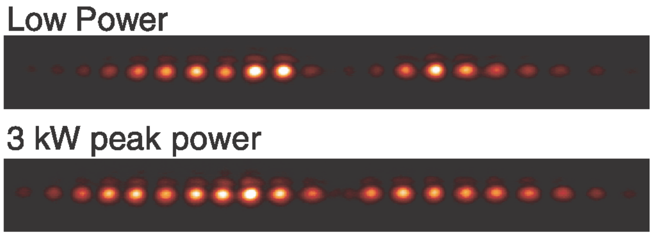

FIG. 5 (color online). Experimental results of a dark gap-soliton measurement, excited exactly at the bottom edge of band No. 1 (marked in Fig. 1), using a shaped input beam. Shown are photographs of the beam at the array output, for low and high powers. At low power, the narrow notch at the input beam diffracts into about a three array-period wide notch. At high power, the beam experiences nonlinear defocusing, while the notch narrows, until a dark gap soliton of 1.5 array-periods width is formed, traveling across the array at zero transverse velocity.

In order to excite a dark gap soliton, a π step was introduced at the center of the interference pattern, which resulted in a dark narrow notch in the shaped input beam. Figure 5 shows experimental results, in which a wide band No. 1 beam, with a π step in its center, is launched into the array. The figure shows photographs of the beam at the array output, at low and high power. At low power, the input beam excites a band No. 1 FB mode, in which the dark narrow notch at the array input diffracts into a three array period wide notch. At high power, the notch narrows and becomes more distinct, until reaching a width of about 1.5 array periods. Beam propagation simulations verify that dark gap solitons are excited under these conditions [23], traveling at zero transverse velocity. In should be noted that discrete (i.e., band No. 1) dark solitons were observed before [24], excited by tilting a *single* (unshaped) input beam such that two $K = \pi/p$ FB modes were excited, at bands No. 1 and No. 2, leading to a *nonpure* excitation of the dark discrete gap soliton.

In conclusion, bright and dark spatial gap solitons have been demonstrated in waveguide arrays, by using an experimental configuration which is analogous to an "idealized" configuration for forming stationary (immobile) temporal gap solitons. The resonant interaction of the beam with the array at the band edges leads to a pronounced effect induced by the array on the soliton's dynamics, leading to a vanishing transverse velocity of the soliton, in complete analogy with the stationary temporal gap solitons. Moreover, the identical physical mechanism, which underlies temporal and spatial gap solitons, leads us to conclude that they are not just analog to each other, but rather they constitute the very same phenomenon, expressed in different systems. In the 1D + time system these solitons are described as the high-power central region of a *pulse*, trapped between two low-power Bragg-reflecting regions. In the 2D system, this pulse is allowed to propagate along another spatial dimension such that the temporal pulse is mapped into a spatial beam. Now the soliton is described as the high-power central region of a *beam*, trapped between two low-power Bragg-reflecting regions.

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