Localization of Electromagnetic Waves in Three-Dimensional Fractal Cavities

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Three-dimensional fractals called the Menger sponge, with a fractal dimension $D = \log 20/\log 3$, were fabricated from epoxy resin by stereolithography. Clear attenuation of both reflection and transmission intensity was observed at 12.8 GHz for a cubic specimen with an edge size of 27 mm that was constructed up to the third stage of the self-similar patterns. The electromagnetic field was found to be confined in the central part of the specimen at this frequency. The localization is not caused by the presence of a photonic band gap as in photonic crystals but should be attributed to a singular photon density of states realized in the fractal structure. This is the first report on such localization of electromagnetic waves in three-dimensional fractal cavities.

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Propagation properties of electromagnetic waves in periodic and quasiperiodic structures have been of theoretical and practical interest for the past two decades. Photonic (electromagnetic) crystals, which consist of a periodic array of dielectric materials with different dielectric constants, have been studied intensively in terms of the formation of photonic band gaps, localization of light, control of spontaneous emission, etc. [1–4]. Light localization due to structural defects introduced to the regular structure of the photonic crystals is especially of interest because of the high Q (quality) factor of the localized modes that is expected to bring about a variety of optical device applications such as narrow-band filters and low-threshold lasers [4].

Light localization in photonic crystals is essentially based on the small (or ideally, zero) photon density of states accompanied by photonic band gaps. Because a complete band gap over the whole phase space is necessary to realize three-dimensional (3D) localization of electromagnetic waves, fabrication of 3D specimens has been a major concern in the research of photonic crystals. Another kind of light localization is also known that is characterized by the discrete and singular photon density of states. They are quasiperiodic and fractal (self-similar) structures [5,6]. One- and two-dimensional systems such as the Fibonacci lattice [7,8] and planer fractals [9] have been studied thus far. Regarding experimental studies, there are only a few reports on light localization in the Fibonacci multilayers [10]. On the other hand, enhancement of local fields due to resonant polarization in nanostructured materials has been known for a long time. A typical and most prominent example is the extraordinary large Raman signal observed for molecules adsorbed on thin silver films of island shapes, which is referred to as surface-enhanced Raman scattering (SERS) [11,12]. In this case, surface plasmon polaritons (SPP) induced by an incident electromagnetic field is accompanied by a local field, whose intensity is larger than that of the incident field by several orders of magnitude. Recently, Shalaev *et al.* discussed SERS in connection with the fractal nature of the metal films and clarified its characteristics in spatial and frequency dimensions [13–15]. Light localization in the self-similar structures that we discuss in this paper is distinguished from the local field enhancement with SPP by the facts that resonant polarization of matter is not relevant and that it is intrinsic to the singular photon density of states realized by a purely geometrical factor. Because the photon density of states is expected to depend strongly on the dimensionality of the specimens, fabrication and evaluation of 3D fractals are quite important for understanding and application of light localization in such optical self-similar structures.

We thus fabricated 3D fractals called the Menger sponge [6] in the frequency range of microwaves and investigated the wave propagation. The Menger sponge is the 3D version of the Cantor bar fractal [16]. The Cantor bar fractal is formed by extracting the center segment from three equivalent segments obtained by dividing an initial bar, and by repeating this process to the two remaining side segments. Similarly, the Menger sponge may be made from a dielectric cube. The initial cube should be divided into 27 ($=3^3$) identical cubic pieces, and seven pieces at the body and face centers should be extracted. By repeating the same extraction process for the 20 remaining pieces, we obtain the Menger sponge. The fractal dimension D is defined by the following relation: $N = S^D$, where N is the number of the self-similar units newly created when the size of the initial unit decreases to $1/S$. In the present Menger sponge, $N = 20$ and $S = 3$, so that $D = \log 20/\log 3$.

We designed the Menger sponges by using a CAD program (Toyota Keram Co. Ltd., Think Design Version 8.0) and then fabricated them from epoxy resin (D-Mec Ltd., SCR-730) using a rapid prototyping method called stereolithography (D-Mec Ltd., SCS-300P) [17]. This method enables us to form three-dimensional complex objects via a layer-by-layer process using CAD/CAM data. Each layer is solidified by scanning the surface of the photosensitive liquid resin with an ultraviolet laser beam. The 3D specimens are built up by forming such a layer on preceding layers.

In order to investigate how the results depend on the structure, we prepared the Menger sponges in four different stages (0, 1, 2, and 3) as illustrated in Figs. 1(a)–1(d). We also prepared simple-cubic photonic crystals [18] composed of air rods with each side of 3 or 1 mm in length to compare the wave propagation to that of the Menger sponges. The outer dimension of all samples was $27 \times 27 \times 27$ mm^3. The real and imaginary parts of the dielectric constant of the epoxy resin at the measured frequency range are 2.8 and 0.2, respectively. The spatially averaged dielectric constant of the Menger sponge of stage 3, for example, is 1.74.

Reflection and transmission spectra were measured with a network analyzer (Agilent Technology, HP-8720B) and two monopole antennas with characteristic impedance of 50 Ω as shown in Fig. 2 [19]. A monopole antenna for microwave emission and detection of reflection was placed parallel to the surface of the cubic sample. The other monopole antenna was placed parallel to the opposite surface of the sample to receive the microwave. The distance of the antennas away from the surfaces of the sample can be changed in the region between 10 and 30 mm. The thick layers of a low-density microwave absorber were placed on the interior surfaces of the cavity made of aluminum to suppress disturbance by superfluous reflection and diffraction.

Spectra observed by the antennas placed at 10 mm away from the surfaces of the sample are shown in Figs. 3(ii) and 3(iii). These spectra are normalized to those of free space. In the transmission spectrum of the Menger sponge of stage 3, a sharp dip of transmission intensity as large as -31 dB was observed at 12.8 GHz and a relatively broad attenuation of reflection as large as -7 dB was observed around 13.0 GHz. If we simply assume that the dip in the transmission spectrum is caused by a single mode, its Q factor estimated from the line shape is as high as 610 in spite of the relatively large dielectric loss of 0.2. In addition, for the Menger sponge of stage 2, a sharp attenuation as large as -15 dB and a corresponding broad attenuation of reflection as large as -5 dB were both observed around 12.8 GHz. It was confirmed that these features of attenuation in the transmission and reflection intensities were almost independent of the position of the antennas. On the other hand, such anomalous attenuations were not observed either for the simple-cubic photonic crystals or for the Menger sponges of stages 0 or 1. In the air-rod simple-cubic lattices with the low contrast (2.8:1) in dielectric constant and with the small volume fraction (7/27) of air

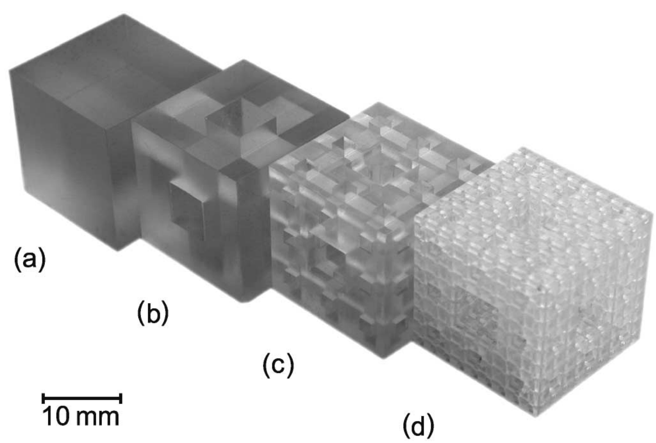

FIG. 1. Photographic images of the Menger sponges made by stereolithography: (a) stage 0, (b) stage 1, (c) stage 2, and (d) stage 3. Three-dimensional models were drawn with a CAD software and real fractal objects were formed from a photosensitive liquid epoxy resin using the CAD data. The outer dimensions of the specimens are all $27 \times 27 \times 27$ mm^3. The length of the side of the smallest hollow squares on the surface of the specimens is 9 mm for stage 1, 3 mm for stage 2, and 1 mm for stage 3, respectively.

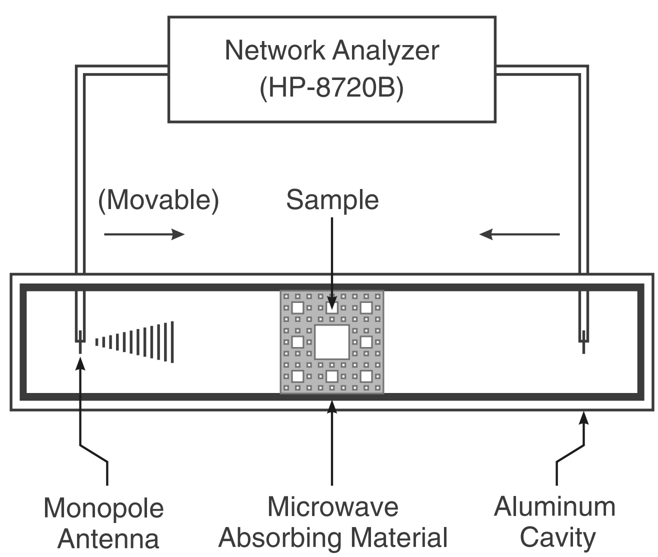

FIG. 2. Schematic illustration of experimental setup for the measurement of microwave transmission and reflection properties of a photonic fractal.

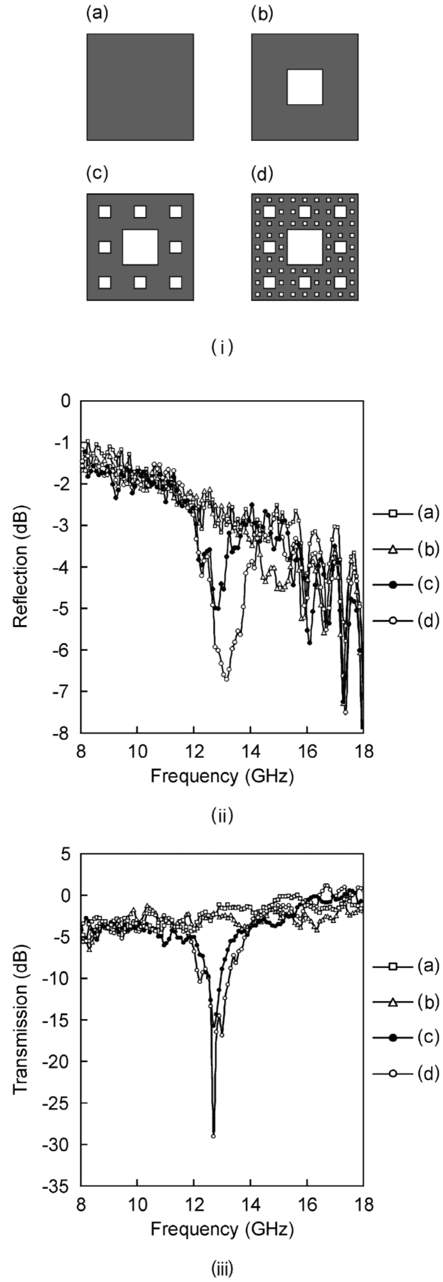

FIG. 3. (i) Illustration of the Menger sponges of stages 0, 1, 2, and 3. (ii) Reflection and (iii) transmission spectra of the Menger sponges of (a) stage 0, (b) stage 1, (c) stage 2, and (d) stage 3. A sharp attenuation maximum at around 12.8 GHz is observed in both the reflection and transmission spectra of the Menger sponge of stages 2 and 3. The Q factor of the 12.8 GHz mode is as large as 610 for the stage 3 sample.

rods, the photonic crystals do not present photonic band gaps in the frequency range considered in this study [20,21]. The specimens of the photonic crystals and the Menger sponges of stages 0 or 1 do not have the self-similar structure. Therefore, the origin of the anomalous attenuation observed both in the reflection and transmission spectra of the Menger sponges of stages 2 and 3 must be intrinsic to the self-similar fractal structure.

Let us compare these propagation characteristics of the Menger sponges to those of photonic crystals. First, consider the photonic band gap. Electromagnetic waves in the gap frequency region are totally reflected. The transmittance is quite low and the reflectance reaches almost 100% for the band gap frequency range [2]. Second, consider the localized mode due to structural defects that disturb the periodicity of the regular dielectric structure of the photonic crystal [3,4]. In this case, we must observe a sharp transmission peak in a nontransparent region due to the photonic band gap and a sharp dip of reflectance in the total reflection region. Third, consider the singular Bloch modes that appear in the photonic crystals with multiperiods [22]. In the multiperiodic photonic crystals, sharp transmission peaks in nontransparent regions and sharp dips in total reflection regions should be observed. The characteristics observed in Fig. 3 do not meet any of these features. The characteristics in Fig. 3 can rather be explained well if we assume that the mode at 12.8 GHz has a much shorter localization length and a much higher Q factor than other modes around this frequency. In this case, the effective dielectric loss for the mode at 12.8 GHz can be very large, which results in the large attenuation of transmittance and reflectance.

To confirm the localization, spatial distribution of the electric field in the Menger sponge was measured by a probe method using two monopole antennas. A monopole antenna for emission was placed at the same position as the transmission measurement and a receiver antenna was inserted into air rods in the specimen. The spectral intensity profile of electromagnetic waves was measured for every air rod and at all possible positions in the central air rod as shown in Fig. 4(i). Figure 4(ii) shows the spectral profile measured at several positions in the central air rod denoted in Fig. 4(i). At middle points (l), the intensity of the electric field at 12.8 GHz shows maximum values. The normalized intensity of the electric field along the central air rod ($\langle 100 \rangle$ direction) is shown in Fig. 4(iii). The electric field intensity shows a double maximum curve that is symmetric for the center point. Apparently, this mode is strongly localized almost within the central largest cubic air-cage region and the electric field decreases abruptly outside the air cube. The localized mode has a cubic-shell shape whose edges and corners are rounded.

The localization length estimated from Fig. 4(iii) is quite short. It is less than 10 mm and is even shorter than the wavelength of the relevant electromagnetic wave in free space, i.e., 23.4 mm. Because the amplitude of the field on the surface of the specimen is very small compared with the maximum value at point (l), coupling between the internal localized mode and the external field is also very small. This is the reason why we observed a large Q factor. The optical length of the Menger sponge of stage 3, which is calculated as a product of its actual thickness and spatially averaged refractive index,

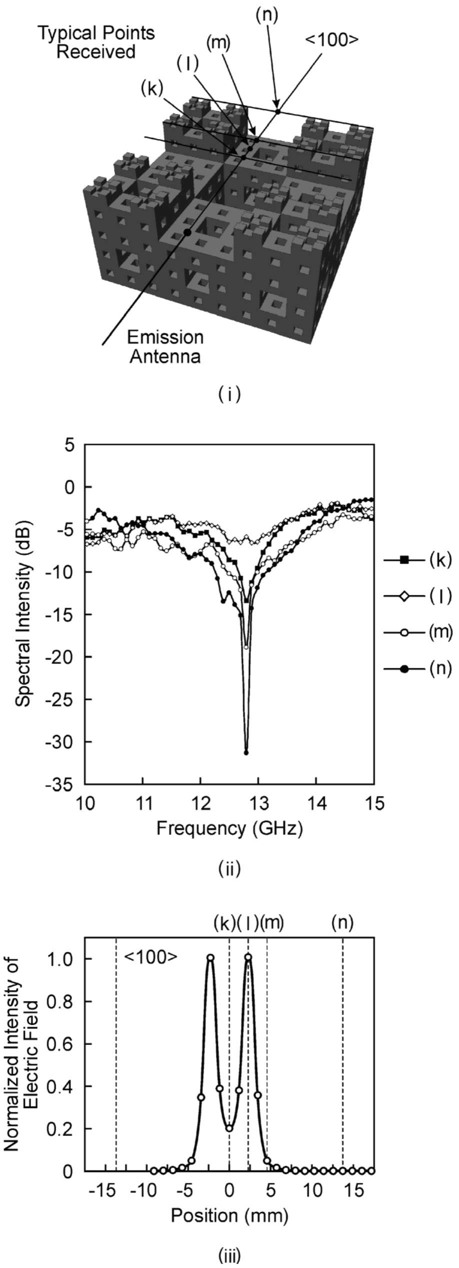

FIG. 4. (i) Typical observation points inside the Menger sponge of stage 3. (ii) Intensity spectra observed at the typical points inside the Menger sponge of stage 3 as denoted in (i). (k) and (l) denote the center and a half position of the central cubic air region along the $\langle 100 \rangle$ direction. (m) denotes the $1/3$ position of the central air rod. (n) denotes the boundary between the Menger sponge and free space. (iii) Spatial intensity profile of the electric field at 12.8 GHz along the central air rod axis ($\langle 100 \rangle$). The electromagnetic wave at 12.8 GHz is strongly localized almost within or close to the central cubic air region.

is 35.6 mm. This value is larger than the relevant wavelength by only 50%; that is, the Menger sponge confines the electromagnetic wave with a wavelength corresponding to $2/3$ of its optical length. On the other hand, in the case of photonic crystals, the localization length is several times the lattice constant of the crystal. So, the optical length necessary to confine a localized mode is several times the relevant wavelength in free space. We must, thus, conclude that the confinement of the electromagnetic energy by the Menger sponge is quite strong.

In conclusion, we observed localization of microwaves in the Menger sponges that is intrinsic to their fractal structure, and this is the first report on the light localization in 3D fractal cavities. This localization is not caused by the presence of a photonic band gap as in photonic crystals but should be attributed to a singular photon density of states realized in the fractal structure. Confinement of electromagnetic energy in the fractals is much stronger than in photonic crystals and may be useful for some device applications. We further design fractals by changing the size, shape, dielectric constant, stage number, and fractal dimension. Detailed studies are now in progress.

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