Universal Thermodynamics of Degenerate Quantum Gases in the Unitarity Limit

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We perform a systematic study of the thermodynamics of quantum gases in the unitarity limit. Our study is based on a "universality hypothesis" for the relevant energy scales which is supported by experiments and can be proven in the Boltzmann regime. It implies a universal form for the grand potential, which is specified by only a few universal numbers in the degenerate limit. This hypothesis provides a simple way to determine the density profile of a trapped fermion superfluid. It implies a superfluid bump in the density and gives the general expression of the second sound velocity of a homogeneous superfluid at unitarity.

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Feshbach resonance has introduced a whole new dimension in the research of degenerate quantum gases. Through this resonance, effective interactions between atoms are dramatically increased. Such resonance arises when the energy of a pair of scattering atoms is tuned close to that of a molecular bound state by an external magnetic field, thus causing substantial resonance scattering. At resonance, scattering reaches the *unitarity limit*: with a diverging scattering length a_{sc}, and a cross section reaching the maximum value $4\pi/k^2$, where k is the relative momentum of the scattering atoms. These properties are *universal* because they are independent of any feature of atomic potentials.

This universality, simple as it is, poses a challenging many-body problem, as there are no small parameters. On the other hand, it can lead to great simplification if one makes a reasonable assumption, referred to as universality hypothesis (UH) : that the only dominant length scale at unitarity in the ground state is interparticle spacing $n^{-1/3}$, where n is the density. The idea is that if the only relevant length scales in the effective theory are a_{sc} and $n^{-1/3}$, then a_{sc} must drop out from all physical quantities at resonance because it is infinity, leaving $n^{-1/3}$ the only relevant length scale. The word "hypothesis" is to indicate that although universality has emerged in approximate calculations [1,2], it has not been proven rigorously except in Boltzmann regime [3]. This hypothesis also implies that for both bosons and fermions, the only relevant energy scale is the "Fermi" energy $\mathcal{E}_F^o(n) \equiv (\hbar^2/2M)(3\pi^2 n)^{2/3}$. For the same reason, the transition temperature T_c of a Fermi superfluid must scale as $T_F = \mathcal{E}_F/k_B$, i.e., $T_C = \gamma T_F$, where γ is a universal constant. That γ can be of order 1 was suggested by Holland *et al.* [1]. Current estimates of T_c range from 0.5 to $0.2T_F$ [1,4,5]. The possibility of such a high T_c has made Feshbach resonance a focus of attention in the current race for achieving fermion superfluidity. On the other hand, the normal state is equally intriguing, for it contains the same nonperturbative effects.

The universal properties of a Fermi gas in unitarity regime have been demonstrated recently by a sequence of beautiful experiments [6–9]. John Thomas's group [6] has pointed out that the interaction energy $E_{\rm int}$ of a Fermi gas of ^{6}Li near Feshbach resonance is of the form $\beta\mathcal{E}_F^o$, $\beta \approx -0.25$ for $0.1 < T/T_F < 0.2$. Direct measurement of this energy was performed by Salomon's group [7]. They found that $E_{\rm int}$ remains *smooth* across the resonance despite the diverging scattering length, and saturates at a value of the order of Fermi energy $\mathcal{E}_F$, with $\beta \sim -0.3$ at $0.5 < T/T_F < 1$. Similar saturation is also observed in rf spectroscopy by Ketterle's group [8] and Jin's group [9]. The observed relation between $E_{\rm int}$ and $\mathcal{E}_F^o$ is a support for UH. Further experiments on other alkali fermions will help to verify its validity. The sign of $E_{\rm int}$ and its continuity across the resonance [7], however, require additional physics and are related to formation of molecules [3]. Since the Duke experiments cover the temperature range above and below the estimated T_c, it raises the question of how superfluidity is affected by unitarity, and their signature in the unitarity regime.

In this Letter, we perform a systematic study of the thermodynamics of quantum gases at unitarity using UH. We show that (i) At unitarity, the thermodynamic potentials acquire universal forms depending only on the nature of the thermodynamic phase. (ii) The properties of a degenerate Fermi gas near resonance (be it normal or superfluid) are characterized by only a few universal numbers. (iii) Universal thermodynamics provides a simple way to determine the density profile of a trapped fermion superfluid near resonance. (iv) It also allows one to calculate the hydrodynamic modes of a fermion superfluid at finite temperature. (v) Bose gas in the unitarity limit, if stable, will have a *fermionic* energy density.

Before proceeding, we first make clear what quantities UH describes. Let us consider the Hemholtz free energy density $f = f(T, n, B, \{r_i\})$, which is a function of T, n, external magnetic field (which controls a_{sc}), and other interaction lengths such as effective range, etc.

(collectively denoted as $\{r_i\}$). While a_{sc} diverges at resonance (say, at $B = B_o$), it is assumed that $\{r_i\}$ remains atomic size so that $x_i = r_i n^{1/3} \ll 1$. If f is smooth across the resonance (as indicated in Ref. [7]), then in the neighborhood of B_o it is well approximated by its value at resonance $f(T, n; B_o, \{x_i\})$. Moreover, if f has an asymptotic expansion in x_i, we then have

$$f(T, n; B_o, \{x_i\}) = f(T, n; B_o, 0)(1 + 0(x_i)). \qquad (1)$$

UH describes the first term Eq. (1)—that it has only two energy scales, $\mathcal{E}_F^o(n)$ and $k_B T$; and is independent of B_o for it would otherwise introduce additional energy scales. In this Letter, quantities of ideal gas will be denoted by a superscript "^{o}".

Universal thermodynamics near resonance.—To be efficient, we consider the grand potential instead of the Hemholtz free energy. The former is defined as $\Omega(T, \mu_\uparrow, \mu_\downarrow, V) = -k_B T \ln \mathrm{Tr} e^{-(H-\mu_\uparrow N_\uparrow - \mu_\downarrow N_\downarrow)/k_B T}$ for a two component quantum gas (labeled as $\uparrow$ and $\downarrow$) with mass M, chemical potential $\mu_\uparrow$, $\mu_\downarrow$, and volume V. We first consider $\mu_\uparrow = \mu_\downarrow$ which implies $n_\uparrow = n_\downarrow = n/2$, $m = (n_\uparrow - n_\downarrow)/2 = 0$. According to UH, all microscopic scales are absent at resonance. The only energy scales are $k_B T$ and μ. The corresponding density scales are λ^{-3} and $n_f(\mu)$, where $\lambda = h/\sqrt{2\pi M k_B T}$ is the thermal wavelength, and $n_f(\mu) \equiv (3\pi^2)^{-1}(2M\mu/\hbar)^{3/2}$. Since pressure $P = -\Omega/V$, dimensional analysis implies

$$P(T, \mu) = \frac{2k_B T}{5\lambda^3} \mathcal{W}_0(x^{-1}) = \frac{2\mu n_f(\mu)}{5} \mathcal{G}_0(x), \qquad (2)$$

where $x = k_B T/\mu$, and ($\mathcal{W}_0$, $\mathcal{G}_0$) are dimensionless scaling function. These two forms are useful in Boltzmann and degenerate regimes, respectively, since their arguments are small in these cases. Using the well-known relation $\epsilon = Ts + \mu n - P$ and $dP = nd\mu + sdT$, where ϵ and s are energy density and entropy density, we have

$$n = n_f(\mu)[\mathcal{G}_0(x) - (2/5)x\mathcal{G}_0'(x)], \qquad (3)$$

$$s = (2/5)k_B n_f(\mu)\mathcal{G}_0'(x), \qquad \epsilon = 3P/2. \qquad (4)$$

Since universality hypothesis makes no reference to the thermodynamic phase, Eqs. (2)–(4) *apply to both normal and superfluid phases*, which of course have different scaling functions. The scaling functions, however, are constraint by the positivity of s and n, as well as stability conditions $\partial^2 P/\partial T^2 = \partial s/\partial T > 0$ and $\partial^2 P/\partial \mu^2 = \partial n/\partial \mu > 0$. The density profile $n(\mathbf{x})$ in a nonuniform potential $V(\mathbf{x})$ can be readily determined from Eq. (3) within local density approximation (LDA) by replacing μ with $\mu(\mathbf{x}) \equiv \mu - V(\mathbf{x})$.

When $m \neq 0$, universality hypothesis implies that

$$P(T, \mu_\uparrow, \mu_\downarrow) = \frac{2\mu n_f(\mu)}{5} \mathcal{G}(x; \nu/\mu), \qquad (5)$$

where $\mu = (\mu_\uparrow + \mu_\downarrow)/2$, $\nu = \mu_\uparrow - \mu_\downarrow$, and $\mathcal{G}(x; \nu/\mu)$ is a scaling function even in ν due to the invariance of Ω under spin exchange. For small ν, we have

$$\mathcal{G}(x; \nu/\mu) = \mathcal{G}_0(x) + \frac{\mathcal{G}_2(x)}{2}\left(\frac{\nu}{\mu}\right)^2 + \cdots, \qquad (6)$$

where $\mathcal{G}_2$ is another dimensionless function. Defining magnetic susceptibility χ and specific heat c at constant μ as $m = (n_\uparrow - n_\downarrow)/2 = \partial P/\partial \nu = \chi \nu$ and $c = T(\partial s/\partial T)_\mu$, Eqs. (5) and (6) then imply $c/(T\chi) = k_B^2 \mathcal{G}_0''(x)/\mathcal{G}_2(x)$, which is a universal function of $x = k_B T/\mu$.

Boltzmann limit.—This is the limit where the fugacities $z_i = e^{\mu_i/k_B T}$, $i = \uparrow, \downarrow$ are small, and UH can be proved rigorously [3]. Expanding Ω in z_i for a Fermi gas, we have $P(T, \mu_\uparrow, \mu_\downarrow) = P^{(o)}(T, \mu_\uparrow, \mu_\downarrow) + 2\sqrt{2} b_2 (k_B T/\lambda^3) z_\uparrow z_\downarrow$, where $P^{(o)}(T, \mu_\uparrow, \mu_\downarrow) = \sum_{i=\uparrow,\downarrow} k_B T \lambda^{-3}(z_i - 2^{-5/2} z_i^2) + O(z_i^3)$ is the pressure of the an ideal Fermi gas, and b_2 is the second virial coefficient which is a function of temperature only [10]. Using the relations $\epsilon = Ts + \mu_\uparrow n_\uparrow + \mu_\downarrow n_\downarrow - P$ and $dP = \sum_i n_i d\mu_i + sdT$, we have

$$n_{\uparrow(\downarrow)}\lambda^3 = z_{\uparrow(\downarrow)}(1 + 2\sqrt{2} b_2 z_{\downarrow(\uparrow)}) - 2^{-3/2} z_{\uparrow(\downarrow)}^2, \qquad (7)$$

$$s = \frac{5}{2}\frac{P}{T} - \frac{\mu_\uparrow n_\uparrow + \mu_\downarrow n_\downarrow}{T} + 2\sqrt{2}\frac{k_B T}{\lambda^3}\frac{\partial b_2}{\partial T} z_\uparrow z_\downarrow, \qquad (8)$$

$$\epsilon = \frac{3P}{2} + 2\sqrt{2}\frac{k_B T}{\lambda^3} z_\uparrow z_\downarrow T \frac{\partial b_2}{\partial T}. \qquad (9)$$

Since $b_2 = 1/2$, and $\partial b_2/\partial T = 0$ at resonance [3,10], we recover the universal thermodynamics in **I**.

It is also useful to use (T, n) instead of (T, μ) as variables. The following relations are applicable to all scattering lengths and can be compared with experiments. Eliminating z_i in Eqs. (7)–(9), we have

$$P = k_B T(n + [2^{-5/2}(n_\uparrow^2 + n_\downarrow^2) - 2\sqrt{2} b_2 n_\uparrow n_\downarrow]\lambda^3), \qquad (10)$$

$$\epsilon = \frac{3}{2}k_B T\left[n + \frac{(n_\uparrow^2 + n_\downarrow^2)\lambda^3}{2^{5/2}} - \phi n_\uparrow n_\downarrow\right], \qquad (11)$$

where $\phi = 2\sqrt{2}\lambda^3(b_2 - \frac{2}{3}T\partial b_2/\partial T)$. From Eqs. (7) and (10), it is easy to calculate isothermal compressibility $\kappa_T = n^{-1}(\partial n/\partial P)_T$ and isothermal spin susceptibility $\chi_T = (\partial m/\partial \nu)_T$. Their deviations from ideal gas values are $\Delta\kappa_T = \sqrt{2} b_2 n\lambda^3/(nk_B T)$, $\Delta\chi_T = -(n/k_B T)(\sqrt{2} b_2 n \lambda^3/4)$, hence $n^2\Delta\kappa_T/\Delta\chi_T = -4$. It is easy to derive the same results for the Bose gas, which is Eqs. (10) and (11) with a minus sign in the n_i^2 terms.

Degenerate Normal Gas.—This is the case $x = k_B T/\mu \ll 1$. For small spin polarization, the pressure can be expanded in x and ν/μ as $P = P^{(n)}(T, \mu, \nu)$,

$$P^{(n)} = \frac{2\mu n_f(\mu)}{5} A^{3/2}\left[1 + \frac{5\pi^2(Bk_B T)^2}{8(A\mu)^2} + \frac{15(C\nu)^2}{32(A\mu)^2} + \ldots\right], \qquad (12)$$

where coefficients A, B, C are universal numbers [11]. They are written in this form to simplify later discussions. The absence of linear T term is due to vanishing entropy at $T = 0$. For ideal Fermi gas, $A = B = C = 1$. Stability conditions $\partial s/\partial T$, $\partial n/\partial \mu > 0$ imply that $A, B > 0$; and $C > 0$ unless the system is ferromagnetic. From Eq. (3) and the relation $m = \partial P/\partial \nu$, we have $n = n^{(n)}(T, \mu, \nu)$,

$$n^{(n)} = n_f(A\mu)\left[1 + \frac{\pi^2(Bk_BT)^2}{8(A\mu)^2} + \frac{3}{32}\left(\frac{C\nu}{A\mu}\right)^2 + \cdots\right], \tag{13}$$

and $m = [3c/(8A)][n_f(A\mu)/(A\mu)](C\nu)$, where $n_f(A\mu) = A^{3/2}n_f(\mu)$. These two equations for n and m readily give the density profile $n_i(\mathbf{x})$ ($i = \uparrow, \downarrow$) in nonuniform potentials $V_i(\mathbf{x})$ within LDA by substituting $\mu_i(\mathbf{x}) = \mu_i - V_i(\mathbf{x})$. Note, however, that these relations are valid only for $\mu_i(\mathbf{x}) \gg k_BT$ and $\nu \ll \mu$. As one approaches the surface of the cloud, density decreases and the system will switch to Boltzmann regime in surface regions where $e^{\mu_i(\mathbf{x})/k_BT} \ll 1$, with densities given by Eq. (7).

To find an accurate formula interpolating between degenerate and Boltzmann limits, we note that for $\nu = 0$ and deep in Boltzmann regime (i.e., $z = e^{\mu/k_BT} \ll 1$), Eq. (7) is simply $n = z/\lambda^3$ and is the high temperature limit of the ideal gas relation $n = n_{id}(\mu, T)$, $n_{id}(\mu, T) = \lambda^{-3}f_{3/2}(z)$, where $f_{3/2}(z)$ is the Fermi integral [12]. On the other hand, Eq. (13) is precisely the low temperature expansion of the ideal gas relation $n_{id}(\mu, T)$ with the substitution $(\mu, T) \rightarrow (A\mu, BT)$. The desired interpolation will then be of the form

$$n^{(n)}(T, \mu) = n_{id}(A(x)\mu, B(x)T), \qquad x = \mu/k_BT, \tag{14}$$

where $(A(x), B(x))$ are functions of x (as required by UH) such that $(A(x), B(x)) \rightarrow (A, B)$ as $x \gg 1$, and $(A(x), B(x)) \rightarrow (1, 1)$ as $z = e^x \ll 1$. Since the switching from degenerate to Boltzmann regime takes place at $\mu \sim k_BT$, a simple extrapolation is $A(x) = (Ae^x + 1)/(e^x + 1)$, $B(x) = (Be^x + 1)/(e^x + 1)$. The density profile in a trap calculated within LDA using Eq. (14) and these expressions of $A(x)$ and $B(x)$ is shown as the dashed curve in the lower figure in Fig. 1.

To derive relations related to experiments, we invert the relations $n = n(T, \mu, \nu)$, $m = m(T, \mu, \nu)$ to express μ, ν, and hence $\epsilon = 3P/2$ in terms of (T, n, m). To the lowest order in $k_BT/\mathcal{E}_F^o$, we have $\mu = \mu^{(n)}(T, \mu, \nu)$,

$$\mu^{(n)} = \mathcal{E}_F^o(n)(1 - W)/A, \qquad \frac{C\nu}{A\mu} = \frac{8A}{3C}\frac{m}{n}, \tag{15}$$

$$\epsilon^{(n)} = [3n\mathcal{E}_F^o(n)/5][1 + 5W]/A, \tag{16}$$

where $W = (\pi^2/12)(Bk_BT/\mathcal{E}_F^o)^2 + (2Am/3Cn)^2$. Alternatively, we can write $\mu^{(n)} = \mathcal{E}_F^o(1 + \beta_\mu)$ and $\epsilon^{(n)} = (3n\mathcal{E}_F^o/5)(1 + \beta_\epsilon)$, where β_μ and β_ϵ are related to the interaction parameters measured in Refs. [6,7], respectively [13]. In the degenerate limit, the W term will be small. We then have $A = (1 + \beta_\mu)^{-1} \sim 1.3$ since $\beta_\mu \sim$

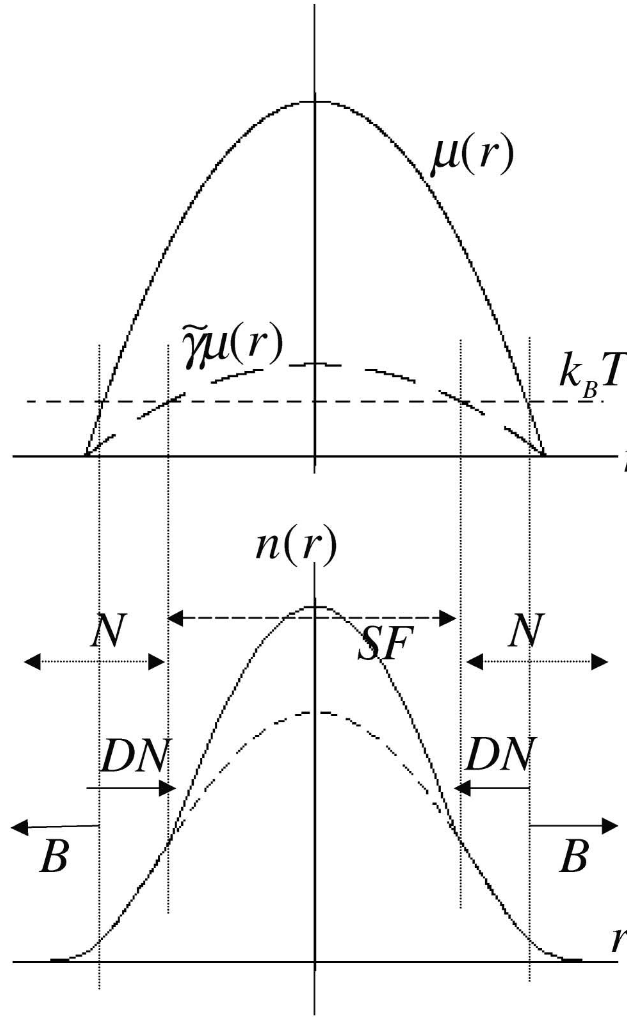

FIG. 1. The upper plot determines the superfluid (SF) and normal (N) regions. The densities, Eqs. (14) and (21) , are depicted as solid and dashed curves in the lower figure. The arrows in (N) indicate increasing degenerate (D) and Boltzmann (B) behavior. We use $\mu(r) = \mu - M\omega^2r^2/2$, $A = 1.3$, $B = 1$, $D = 1.2$, $\gamma = 0.2$, $\mu = 32\hbar\omega$, $k_BT = 5\hbar\omega$, hence $\tilde{\gamma} = \gamma A = 2.6$, $k_BT/\mu = 0.16$, $k_BT/(\gamma A\mu) = 0.65$.

-0.25 from Ref. [6]. Equations (15) and (16) imply that β_μ and β_ϵ have opposite temperature and spin polarization corrections (i.e., the W/A term), differing from each other by a factor of 5.

Superfluid at resonance.—This is the case where universal thermodynamics proves very useful. We shall consider superfluids with zero spin polarization (hence $\nu = 0$). Within Ginzburg-Landau theory, the difference in grand potential between a superfluid with order parameter $\Psi(\mathbf{r}) = \langle\psi_\uparrow\psi_\downarrow\rangle$ and a normal fluid nrar superfluid transition is $\Omega[\Psi] - \Omega^{(n)} = \int d\mathbf{r}\omega[\Psi(\mathbf{r})]$,

$$\omega[\Psi] = K|\nabla\Psi|^2 - r_2|\Psi|^2 + r_4|\Psi|^4/2, \tag{17}$$

where $K, r_4 > 0$, and r_2 vanishes at transition. The equilibrium potential is $\Omega = \Omega[\Psi_o]$, where Ψ_o is the minimum of Eq. (17). According to universality hypothesis, K,

r_2, and r_4 are functions of $k_B T$ and μ only. The condition for transition $r_2(T = T_c, \mu) = 0$ implies $T_c = T_c(\mu)$. Using dimensional analysis and expanding r_2, r_4, and K about T_c, we have $k_B T_c(\mu) = \gamma(A\mu)$, $K = \xi\hbar^2/(2M)$; and to the lowest order of $1 - T/T_c$, $r_2 = \alpha_2\mu(1 - T/T_c)$, $r_4 = \alpha_4\mu/n_f(A\mu)$, where $(\gamma, \xi, \alpha_2, \alpha_4)$ are universal numbers characterizing the superfluid state near T_c.

When $T > T_c$, the system is normal, $\Psi_o = 0$, and

$$P = P^{(n)}(T, \mu, 0), \qquad n = n^{(n)}(T, \mu, 0). \tag{18}$$

For $T < T_c$, we have $|\Psi_o|^2 = r_2/r_4$, and $P = P^{(n)}(T, \mu) + r_2^2/(2r_4)$. Explicitly, we have $|\Psi_o|^2 = \alpha n_f(A\mu)(1 - T/T_c)$, $\alpha = \alpha_2/\alpha_4$, and $P = P^{(s)}(T, \mu)$,

$$P^{(s)} = P^{(n)}(T, \mu, 0) + \frac{2\mu n_f(A\mu)D}{5}\left(1 - \frac{x}{\gamma}\right)^2, \tag{19}$$

where $D = 5\alpha_2^2/(4\alpha_4)$, and $x = k_B T/\mu$. Equation (19) then implies $n = n^{(s)}(T, \mu)$,

$$n^{(s)} = n^{(n)}(T, \mu, 0) + n_f(A\mu)D\left(1 - \frac{x}{\tilde{\gamma}}\right)\left(1 - \frac{x}{5\tilde{\gamma}}\right), \tag{20}$$

where $\tilde{\gamma} = \gamma A$, $x = k_B T/\mu$. Equations (14) and (20) provide a simple method to construct the density profile of a trapped superfluid within LDA: We first plot $\mu(\mathbf{r}) = \mu - V(\mathbf{r})$ and $\tilde{\gamma}\mu(\mathbf{r})$ as a function of $\mathbf{r}$. (See Fig. 1.) The regions where $k_B T < \tilde{\gamma}\mu(\mathbf{r})$ and $\tilde{\gamma}\mu(\mathbf{r}) < k_B T$ correspond to superfluid (SF) and normal (N) region. The latter is further separated into degenerate normal regime (DN), $\tilde{\gamma}\mu \gg k_B T$; and Boltzmann regime (B), $e^{\mu/k_B T} \ll 1$. The densities in (SF) and (N) are given by Eqs. (20) and (14), respectively. In Fig. 1 we use $A = 1.3$ as mentioned before. The values of μ and $\hbar\omega$ are taken to be those in typical experiments. All other parameters are taken to be of order 1 for the purpose of demonstration. It is interesting to note that the "superfluid bump" in Fig. 1 is also obtained in Ref. [1]. Here, we show that *it is a necessary consequence of the universality hypothesis.*

To express quantities in terms of T and n, we invert Eq. (20) and then obtain $\mu = \mu^{(s)}(T, n)$, $\epsilon = \epsilon^{(s)}(T, n)$. If superfluid transition takes place in degenerate regime, $\epsilon \equiv (\pi B k_B T_c)^2/[8(A\mu)^2] = (\pi k_B \gamma)^2/8 \ll 1$. In that case, it is simple to show that

$$\mu^{(s)} = \mu^{(n)}(T, n, 0) - \zeta\mathcal{E}_F^o\left(1 - \frac{y}{\gamma}\right)\left(1 - \frac{y}{5\gamma}\right), \tag{21}$$

$$\epsilon^{(s)} = \epsilon^{(n)}(T, n, 0) - \frac{3n\mathcal{E}_F^o\zeta}{5}\left(1 - \frac{y^2}{\gamma^2}\right), \tag{22}$$

where $y = k_B T/\mathcal{E}_F^o(n)$, $k_B T_C = \gamma A\mu = \gamma\mathcal{E}_F^o(n)$, $\zeta = 2D/(3A) = 5\alpha_2^2/(6A\alpha_4)$. Equations (16) and (22) imply a universal specific heat jump across T_c

$$[(c_s - c_n)/c_n]_{T_c} = 12\zeta A/(5\pi^2\gamma^2 B^2). \tag{23}$$

In the superfluid phase, a "second" sound mode u_2 must exist in addition to the first (or ordinary) sound u_1. In the collisional limit, their velocities are $u_1 = \sqrt{(\partial P/(M\partial n)_\sigma}$ and $u_2 = \sqrt{\sigma^2\rho_s/[\rho_n(\partial\sigma/\partial T)_P]}$, respectively [14], where $\sigma = s/(Mn)$ is the entropy per unit mass, ρ_s is the superfluid mass density which can be obtained easily from the gradient term of Eq. (17) to be $\rho_s = \tilde{\xi}Mn(1 - T/T_c)$, $\tilde{\xi} = 4\xi\sigma\alpha$ [15]; ρ_n is normal fluid mass density, $\rho_s + \rho_n = Mn$. We then have

$$u_1^2 = 2\mathcal{E}_F^o/(3MA), \qquad u_2/u_1 = Q\sqrt{1 - T/T_c}, \tag{24}$$

where $Q^2 = \frac{3}{4}(\gamma B)^2\tilde{\xi}\{1 + (12\zeta/5)(A/\pi\gamma B)^2\}^{-1}$ [16].

Bosons in unitarity limit.—Since universality hypothesis makes no reference on statistics, Eq. (12) applies to *stable* Bose systems at unitarity, and predicts that they will have a *fermionic* energy density. This prediction is consistent with the well-known fermionic energy density of 1D Bose gas with infinite repulsion (or Tonk gas). On the other hand, universality hypothesis does not guarantee superfluidity. This is also illustrated by 1D Tonk gas which exhibits no superfluidity.

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[11] The universality hypothesis gives the general form of the thermodynamic functions but not the specific values of the universal numbers in these functions.

[12] $f_{3/2}(z) = (4/\pi^{1/2})\int_0^\infty dx x^2/(1 + e^{x^2}/z) = -\sum_{\ell=1}^\infty(-z)^\ell/\ell^{3/2}$.

[13] In Ref. [6], the interaction energy is deduced from the chemical potential. The relation between β_ϵ and the results in Ref. [7], as well as the determination of A and B from experimental data will be discussed elsewhere.

[14] I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity* (W. A. Benjamin, Inc., New York, 1965), Chap. 4.

[15] We write $K|\Psi|^2 = \rho_s\mathbf{v}_s^2/2$, where $\mathbf{v}_s = \hbar\nabla\theta/(2M)$, and θ is the phase Ψ, $\Psi = |\Psi|e^{i\theta}$.

[16] Since $\rho_s \propto (1 - T/T_c)$, to find u_2 near T_c, it is sufficient to evaluate σ at T_c and take $\rho_n = Mn$. Moreover, for Fermi systems at low temperatures, one can replace $(\partial\sigma/\partial T)_P$ by $(\partial\sigma/\partial T)_n$ and $(\partial P/\partial T)_\sigma$ by $(\partial P/\partial T)_n$.