Determination of the Strange Form Factors of the Nucleon from νp, $\bar{\nu} p$, and Parity-Violating $\vec{e} p$ Elastic Scattering

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A new method of obtaining the strange form factors of the nucleon is presented, in which forward-angle parity-violating $\vec{e} p$ elastic scattering data are combined with νp and $\bar{\nu} p$ elastic scattering data to extract all three strange form factors: electric, magnetic, and axial (G_E^s, G_M^s, and G_A^s). In this Letter, νp and $\bar{\nu} p$ data from the Brookhaven E734 experiment are combined with the Jefferson Laboratory HAPPEX $\vec{e} p$ data to obtain two distinct solutions for the strange form factors at $Q^2 = 0.5$ GeV^2. More generally, combining the neutrino elastic scattering data from E734 with the existing and upcoming $\vec{e} p$ data will yield the strange form factors of the nucleon for Q^2 of 0.45–1.05 GeV^2. Measurement of G_A^s is crucial to the determination of the strange quark contribution to the nucleon spin Δs.

In the past 15 years, a program of parity-violating polarized electron elastic scattering experiments has evolved, with the goal of measuring the strange form factors of the nucleon. These experiments build on the numerous existing (and ongoing) measurements of the neutron and proton vector electromagnetic form factors, G_E^n, G_M^n, G_E^p, and G_M^p. They gain sensitivity to the strangeness contribution to these form factors by exploiting the interference between electromagnetic and weak processes. This interference produces a helicity-dependent parity-violating asymmetry in the elastic scattering. This program of measurements was sparked by the seminal papers of Kaplan and Manohar [1], McKeown [2], and Beck [3]. The SAMPLE experiment at the MIT-Bates Laboratory and the HAPPEX experiment at Jefferson Laboratory have already made measurements of these asymmetries, and a tremendous amount of additional data will be collected on these asymmetries at Mainz [4] and Jefferson Laboratory [5–8] in the next 3–4 years.

This program of experiments has been designed to measure the strange electric and magnetic form factors. It is desirable to measure also the strange axial form factor G_A^s, but there is a complication. The parity-violating asymmetry observed in these experiments, when the target is a proton, can be expressed as [9]

$$A = \left[\frac{-G_F Q^2}{4\pi\alpha\sqrt{2}} \right]$$
$$\times \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1 - 4\sin^2\theta_W)\epsilon' G_M^\gamma G_A^e}{\epsilon(G_E^\gamma)^2 + \tau(G_M^\gamma)^2}, \quad (1)$$

where $G_{E(M)}^\gamma$ are the traditional electric (magnetic) form factors of the proton and $G_{E(M)}^Z$ are their weak (Z-exchange) analogs, $\tau = Q^2/4M_p^2$, M_p is the mass of the proton, $\epsilon = [1 + 2(1 + \tau)\tan^2(\theta/2)]^{-1}$, θ is the electron-scattering angle, and $\epsilon' = \sqrt{\tau(1 + \tau)(1 - \epsilon^2)}$.

Last, G_A^e is the effective axial form factor seen in electron scattering:

$$G_A^e = -G_A^{CC} + G_A^s + G_A^{ewm}.$$

Here G_A^{CC} is the charged-current axial form factor, which has been measured in neutrino scattering and electroproduction of pions [10]: at $Q^2 = 0$ it equals the axial-vector coupling constant $g_A = 1.267 \pm 0.0035$, and its Q^2 dependence is well described by a dipole form $g_A/(1 + Q^2/M_A^2)^2$, where $M_A = 1.026 \pm 0.021$ GeV is the axial mass. The term G_A^s is the strange axial form factor: at $Q^2 = 0$ it equals Δs, the first moment of the spin-dependent strange quark momentum distribution measured in polarized deep-inelastic scattering, from which data it is estimated [11] that $\Delta s = -0.14 \pm 0.03$. The Q^2 dependence of G_A^s is unknown. Last, G_A^{ewm} represents electroweak mixing contributions to G_A^e; this includes radiative corrections to Z-exchange diagrams. This last term G_A^{ewm} must be calculated in order to allow extraction of G_A^s from a measurement of G_A^e in elastic electron scattering. The difficulties in calculating this term have been noted already [9,12–14] and have played a role in recent experimental work. The SAMPLE experiment, which combined backward-angle electron-scattering data on proton and deuteron targets to extract G_M^s and G_A^e at $Q^2 = 0.091$ GeV^2, originally reported [15] values of $G_M^s = 0.14 \pm 0.29 \pm 0.31$ and $G_A^e = 0.22 \pm 0.45 \pm 0.39$, which deviated from the calculated value [14] of G_A^e by 1.5σ and had the opposite sign. A new analysis of the SAMPLE data at $Q^2 = 0.091$ GeV^2, along with a new measurement at $Q^2 = 0.038$ GeV^2 [16], now supports the theoretical results of Zhu *et al.* [14] at these very low values of Q^2, while not significantly changing the value of G_M^s at $Q^2 = 0.091$ GeV^2. However, it is important to note that the value of Δs used in the interpretation of these data is still only an estimate from polarized deep-inelastic scattering experiments. Better experimental information on the strange axial form factor and Δs is

needed for a final determination of the strange magnetic form factor from these data.

The G^0 experiment [5] at Jefferson Laboratory will circumvent this difficulty with the axial term by combining three measurements: forward scattering of protons from $\vec{e}p$ collisions, backward scattering of electrons from $\vec{e}p$ collisions, and backward scattering of electrons from $\vec{e}d$ collisions. In this way, they will extract G_M^s, G_E^s, and G_A^e separately, so that their results for G_M^s and G_E^s will not be contaminated by uncertain contributions to G_A^e. These data will cover a range of Q^2 from 0.1 to 1.0 GeV^2 and thereby test the calculations of Zhu *et al.* [14] over this range.

Another technique for avoiding the axial term is to observe the parity-violating asymmetry in scattering from a spinless, isoscalar target, such as ^{4}He, as proposed by Musolf and Donnelly [13]. In this case, only the electric form factors contribute to the asymmetry. Two measurements at Jefferson Laboratory will make use of this idea, at $Q^2 = 0.1$ and 0.6 GeV^2, to measure G_E^s. The low Q^2 experiment [6] will measure the slope of G_E^s, while the experiment at moderate Q^2 [7] will measure the actual value of G_E^s.

Elastic scattering of neutrinos from protons does not suffer the difficulties described above, as there is no G_A^{ewm} form factor in neutrino scattering. This Letter explores what can be learned about the strange axial form factor by combining existing $\nu p \to \nu p$ and $\bar{\nu} p \to \bar{\nu} p$ data with the existing and upcoming data on parity-violating $\vec{e}p$ scattering.

The HAPPEX Collaboration has measured the forward-angle parity-violating asymmetry in $\vec{e}p$ elastic scattering at $Q^2 = 0.477$ GeV^2. They report a combination of G_E^s and G_M^s [17]:

$$G_E^s + 0.392 G_M^s = 0.025 \pm 0.020 \pm 0.014.$$

Their result contains a contribution as well from G_A^e, but their sensitivity to G_A^e is ~4% of that to G_E^s and G_M^s because their measurement is at a very forward angle. They used a value of G_A^e from Ref. [14]. The calculation of Ref. [14] is not yet confirmed at this value of Q^2, but even a large error in it would produce only a small additional uncertainty in the interpretation of the HAPPEX result.

The only major measurement to date of νp and $\bar{\nu} p$ elastic scattering cross sections took place at Brookhaven National Laboratory (BNL), Experiment E734 [18], using wideband neutrino and antineutrino beams of average kinetic energy 1.25 GeV incident upon a large liquid scintillator target-detector system. Table I summarizes the results of this experiment. Several attempts have been made to extract the strange axial form factor from these data [18–20]. In all cases, there was an assumption made that G_A^s had a dipole Q^2 dependence — that assumption will not be made here.

TABLE I. Differential cross section data from BNL E734 [18]. The uncertainties shown are total; they include statistical, Q^2-dependent systematic, and Q^2-independent systematic contributions, all added in quadrature. The extra row at the bottom (0.50 GeV^2) lists the cross sections averaged between the 0.45 and 0.55 GeV^2 points.

Q^2 GeV^2	$d\sigma/dQ^2(\nu p)$ 10^{-12} (fm/GeV)^2	$d\sigma/dQ^2(\bar{\nu} p)$ 10^{-12} (fm/GeV)^2
0.45	0.165 ± 0.033	0.0756 ± 0.0164
0.55	0.109 ± 0.017	0.0426 ± 0.0062
0.65	0.0803 ± 0.0120	0.0283 ± 0.0037
0.75	0.0657 ± 0.0098	0.0184 ± 0.0027
0.85	0.0447 ± 0.0092	0.0129 ± 0.0022
0.95	0.0294 ± 0.0074	0.0108 ± 0.0022
1.05	0.0205 ± 0.0062	0.0101 ± 0.0027
0.50	0.137 ± 0.023	0.0591 ± 0.0102

Many years ago Llewellyn Smith [21] noted the usefulness of measuring the difference in the differential cross sections of the charged-current reactions $\bar{\nu} n \to \mu^- p$ and $\nu p \to \mu^+ n$, as this yields a simple relation between G_A^{CC} and the magnetic form factors of the proton and neutron. In the present discussion G_A^{CC} is regarded as known and the difference in the cross sections of the neutral current processes $\nu p \to \nu p$ and $\bar{\nu} p \to \bar{\nu} p$ is used to relate G_M^s and G_A^s. The cross section for νp and $\bar{\nu} p$ elastic scattering is given by [19]

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2}{2\pi} \frac{Q^2}{E_\nu^2} (A \pm BW + CW^2),$$

where the $+$ ($-$) sign is for ν ($\bar{\nu}$) scattering, and

$$W = 4(E_\nu/M_p - \tau), \qquad \tau = Q^2/4M_p^2,$$

$$A = \tfrac{1}{4}\{(G_A^Z)^2(1+\tau) - [(F_1^Z)^2 - \tau(F_2^Z)^2](1-\tau) + 4\tau F_1^Z F_2^Z\},$$

$$B = -\tfrac{1}{4}G_A^Z(F_1^Z + F_2^Z),$$

$$C = \frac{1}{64\tau}[(G_A^Z)^2 + (F_1^Z)^2 + \tau(F_2^Z)^2].$$

Here E_ν is the neutrino beam energy, and F_1^Z, F_2^Z, and G_A^Z are, respectively, the neutral weak Dirac, Pauli, and axial form factors. Taking the difference of these two cross sections,

$$\Delta \equiv \frac{d\sigma}{dQ^2}(\nu p) - \frac{d\sigma}{dQ^2}(\bar{\nu} p) = -\frac{G_F^2}{4\pi} \frac{Q^2}{E_\nu^2} G_A^Z (F_1^Z + F_2^Z) W,$$

produces a relation between the Sachs magnetic form factors (in $F_1^Z + F_2^Z$) and the axial form factors (in G_A^Z). By making use of charge symmetry, one may show that

the sum of the weak neutral Dirac and Pauli form factors is

$$F_1^Z + F_2^Z = \tfrac{1}{2}[(1 - 4\sin^2\theta_W)G_M^p - G_M^n - G_M^s].$$

Also recalling that

$$G_A^Z = \tfrac{1}{2}(-G_A^{CC} + G_A^s),$$

then the expression involving Δ may be written as

$$[G_A^{CC}]G_M^s - G_A^s G_M^s + [(1 - 4\sin^2\theta_W)G_M^p - G_M^n]G_A^s +$$

$$\left\{ \frac{16\pi}{W} \frac{E_\nu^2}{Q^2} \frac{\Delta}{G_F^2} - G_A^{CC}[(1 - 4\sin^2\theta_W)G_M^p - G_M^n] \right\} = 0.$$

This relationship has the form

$$aG_M^s - G_A^s G_M^s + bG_A^s + c = 0,$$

where one may easily identify the factors a, b, and c from the previous equation. Using the dipole form for the magnetic form factors, $G_M^{p,n} = \mu_{p,n}/(1 + Q^2/M_V^2)^2$ (where $M_V = 0.843$ GeV is the vector mass and $\mu_p = 2.793$ and $\mu_n = -1.913$ are the proton and neutron magnetic moments) and using the cross sections from E734, one may calculate a relation between G_M^s and G_A^s, shown in Fig. 1 for $Q^2 = 0.5$ GeV^2. There is an asymptote in G_M^s when $G_A^s = a$ and, similarly, an asymptote in G_A^s when $G_M^s = b$. Since the absolute values of G_M^s and G_A^s are unlikely to be very large, this relation rules out a range of moderate positive values of both G_M^s and G_A^s.

If one now *adds* the ν and $\bar{\nu}$ cross sections together, the dependence on G_A^Z may be eliminated using the expression derived from the difference of the cross sections, leaving a relation between F_1^Z and F_2^Z and, hence, a relation between G_E^s and G_M^s:

$$\Sigma \equiv \frac{d\sigma}{dQ^2}(\nu p) + \frac{d\sigma}{dQ^2}(\bar{\nu}p) = \frac{G_F^2}{4\pi}\frac{Q^2}{E_\nu^2}\left[\left(-1 + \tau + \frac{W^2}{16\tau}\right)(F_1^Z)^2 + \left(+1 + \tau + \frac{W^2}{16\tau}\right)\frac{(4\pi)^2\Delta^2}{G_F^4}\frac{E_\nu^4}{Q^4}\frac{1}{W^2(F_1^Z + F_2^Z)^2}\right.$$

$$\left. + \left(+1 - \tau + \frac{W^2}{16\tau}\right)\tau(F_2^Z)^2 + 4\tau F_1^Z F_2^Z\right].$$

This relation can be expressed as a fourth-order polynomial in G_E^s and G_M^s. The solutions to this expression are contours in the (G_E^s, G_M^s) plane. A set of contours, using the E734 data at $Q^2 = 0.5$ GeV^2, is shown in Fig. 2. The dipole and Galster [22] forms are used for the electric form factors of the proton and neutron, respectively:

$$G_E^p = \frac{1}{(1 + Q^2/M_V^2)^2}, \qquad G_E^n = -\frac{\mu_n \tau}{1 + 5.6\tau} G_E^p.$$

These two relationships, one between G_M^s and G_A^s and another between G_M^s and G_E^s, need some additional input from another experiment before any actual values of G_E^s,
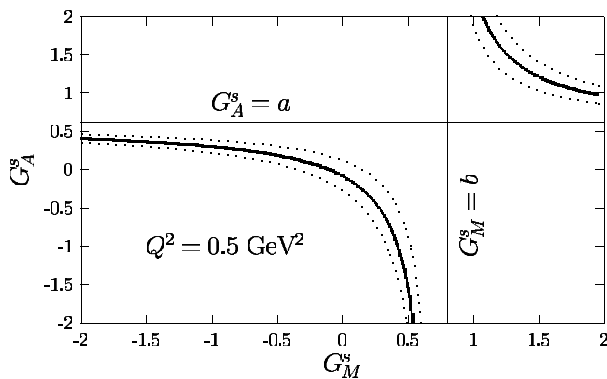
G_M^s, and G_A^s can be determined. The only existing additional experimental information in this Q^2 range is the HAPPEX result [17]. As displayed in Figs. 2 and 3, combining the HAPPEX results at $Q^2 = 0.477$ GeV^2 with the E734 data at $Q^2 = 0.5$ GeV^2 gives two solutions for G_E^s, G_M^s, and G_A^s, listed in Table II. This is the first determination of G_E^s and G_A^s for $Q^2 \neq 0$. These two solutions are very different from each other, and in a few years one of them will be selected with a measurement at a nearby Q^2, as will be done in the G^0 and E91-004 experiments at Jefferson Laboratory. However, there are
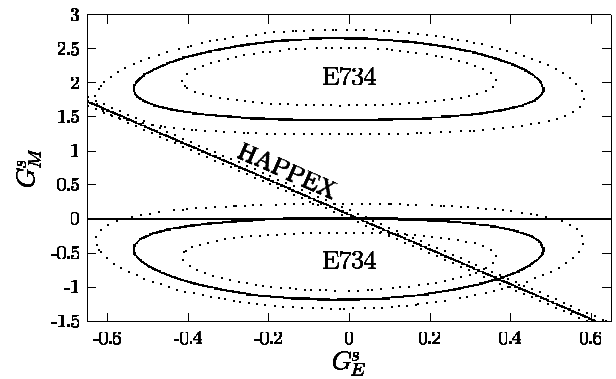

FIG. 1. Relation between G_A^s and G_M^s at $Q^2 = 0.5$ (GeV)^2. The solid line is the central value, while the dotted lines correspond to the total uncertainties in the E734 data. The asymptotes occurring for $G_A^s = a$ and $G_M^s = b$ are indicated; see text for details.

FIG. 2. The solid contours show the relation between G_M^s and G_E^s at $Q^2 = 0.5$ GeV^2, using the E734 data. The dotted contours correspond to the total E734 uncertainties. The straight (solid and dotted) lines show the HAPPEX result at $Q^2 = 0.477$ GeV^2.
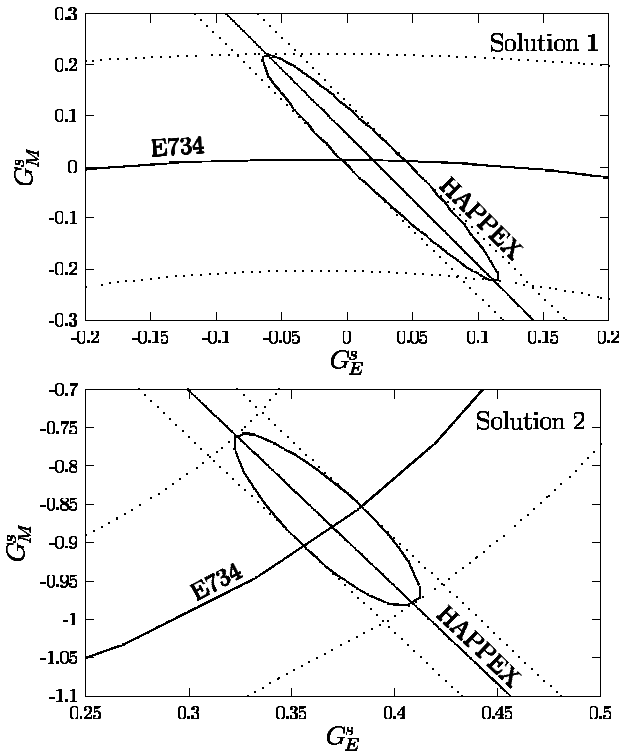
FIG. 3. Two solutions for G_E^s and G_M^s from the HAPPEX and E734 data. The dotted lines correspond to the total errors of each experiment. The solid contours bound the allowed regions of G_E^s and G_M^s.

TABLE II. Two solutions for the strange form factors at $Q^2 = 0.5$ GeV^2 produced from the E734 and HAPPEX data.

	Solution 1	Solution 2
G_E^s	0.02 ± 0.09	0.37 ± 0.04
G_M^s	0.00 ± 0.21	-0.87 ± 0.11
G_A^s	-0.09 ± 0.05	0.28 ± 0.10

Fig. 1 shows. These data on G_A^s will be crucial in nailing down the value of Δs, which has been one of the goals of hadronic spin physics for many years.

already good reasons to favor solution 1 over solution 2. First of all, the value of G_A^s in solution 1 is consistent with the estimated value of $G_A^s(Q^2 = 0) = \Delta s = -0.14 \pm 0.03$ from deep-inelastic data [11], whereas that found in solution 2 is much larger and of a different sign. Similarly, the value of G_M^s in solution 1 is consistent with that measured by SAMPLE, whereas the value in solution 2 is much larger in magnitude. However, the final determination must come from the additional data to be collected at Jefferson Laboratory in the next few years.

The extensive program of measurements to be done by the G^0 Collaboration, when combined with the BNL E734 data, will give the Q^2 dependence of G_A^s in the range 0.45–0.95 GeV^2. The first phase of the G^0 experiment (in 2004) will measure forward scattering of protons (very similar to the HAPPEX measurement at $Q^2 = 0.477$ GeV^2 and similarly insensitive to G_A^e) for Q^2 in the range 0.1–1.0 GeV^2. That will yield two solutions (in a similar fashion as Figs. 2 and 3) for each of G_E^s, G_M^s, and G_A^s for several Q^2 points in the range 0.45–0.95 GeV^2. The second phase of G^0 (in 2005–2006) will observe backward scattered electrons, and those data will select the set of solutions to use. The uncertainty in the extraction of G_A^s from the E734 and G^0 data will be between ± 0.03 and ± 0.10, depending on the value of G_M^s — these two are highly correlated, as

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