# Near-Perfect Imaging in a Focusing System Based on a Left-Handed-Material Plate

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This Letter deals with the investigation of the electromagnetic radiation focusing using a plane-parallel plate of a material with negative real parts of the permittivity and permeability. The reasons are demonstrated for the restriction of the limiting attainable resolution of the system. The possibility of obtaining the separated images of sources the distance between which is much less than the wavelength is confirmed theoretically and experimentally.

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The unique properties of materials with simultaneously negative real parts of the permittivity $\varepsilon$ and of the magnetic permeability $\mu$ (including the possibility of focusing the radiation of point sources by a plane-parallel plate) were predicted by Veselago back in the late 1960s [1]. After that, various options were suggested for the practical realization of composites [the so-called left-handed materials (LHM)] with negative values of $\varepsilon$ and $\mu$ [2–5]. Special interest was aroused by the outstanding idea of Pendry [6] about the possibility of developing a "superlens" whose resolution is not restricted by the well-known diffraction limit. According to Pendry [6], in the case of $\varepsilon = \mu = -1$, a plane-parallel LHM plate enables one to obtain an "ideal" image of a point source owing to the amplification of the evanescent components of the spectrum of spatial frequencies of electromagnetic radiation. However, a brief review [7] of recent publications on the subject shows that the possibility of overcoming the diffraction limit is still in doubt.

It is the objective of this Letter to show how the losses in the LHM plate destroy superresolution; nevertheless, the near perfect image remains achievable, and this is proved by our experiment described below.

As a result of the solution of the problem on the excitation of an infinite plane-parallel plate (Fig. 1) by a filament of electric current of amplitude $I_0$, the vector potential $A$ (the electric field amplitude proportional to this potential) is represented in the form of Fourier integral $A = (I_0/4\pi)\int_{-\infty}^{+\infty}\exp(-i\xi y)F(\xi)d\xi$, with $x > a$ (i.e., in the half-space where the focusing point is located),

$$F(\xi) = \frac{-4\mu q \exp(q_0(x_0 - x))\exp(2aq_0)}{(\mu q_0 - q)^2\exp(-2aq) - (\mu q_0 + q)^2\exp(2aq)},$$

$$q_0 = \begin{cases} i(k_0^2 - \xi^2)^{1/2}, & |\xi| \le \mathrm{Re}(k_0), \\ (\xi^2 - k_0^2)^{1/2}, & |\xi| > \mathrm{Re}(k_0), \end{cases}$$

$$q = \begin{cases} i(k^2 - \xi^2)^{1/2}, & |\xi| \le \mathrm{Re}(k), \\ (\xi^2 - k^2)^{1/2}, & |\xi| > \mathrm{Re}(k), \end{cases}$$

where $k_0 = 2\pi/\lambda$ and $k = k_0\sqrt{\varepsilon}\sqrt{\mu}$ are the wave numbers of external space and of the plate material, respectively, $\lambda$ is the wavelength, $x_0$ is the point of filament location, $2a$ is the plate thickness, the time dependence is selected in the form $\exp(i\omega t)$, and the principal values of radicals are used. Obviously, the accomplishment of ideal focusing implies that the function $F(\xi)$ must become equal to the spectral density of the vector potential of the employed linear source $f(\xi) = 1/q_0$ in some neighborhood of the focusing point (at least, within the zone of resolution of conventional systems).

Let a source be located at point $x_0 = -2a$. We treat the dependence $F(\xi)$ in the plane of intended focusing $x = 2a$ and compare it to $f(\xi) = 1/q_0$. If the losses in the plate material are vanishingly small (tend to zero), a direct substitution of $\varepsilon = \mu \to -1$ gives $F(\xi) \to f(\xi)$. This implies that an ideal image is obtained in the $x = 2a$ plane. However, if the material exhibits even minor losses $\varepsilon'' = -\mathrm{Im}(\varepsilon) > 0$, $\mu'' = -\mathrm{Im}(\mu) > 0$, the equality $F(\xi) \approx f(\xi)$ is valid only approximately, in a limited segment of the infinite region of integration with respect to $\xi$, outside of which the function $F(\xi)$ reveals rapid decreasing with increasing $|\xi|$. In particular, if we take $\mu = -1$, $\varepsilon = -1 - i\alpha$, where $\alpha \ll 1$, we can have the asymptotic representation $F(\xi) \approx f(\xi)(1 + \Delta)^{-1}$, where $\Delta = \alpha^2(2q_0)^{-4}\exp(4aq_0)$. Consequently, for any value of $\alpha > 0$ at sufficiently high values of $|\xi|$, the spectral density $F(\xi)$ exponentially tends to zero. The greater

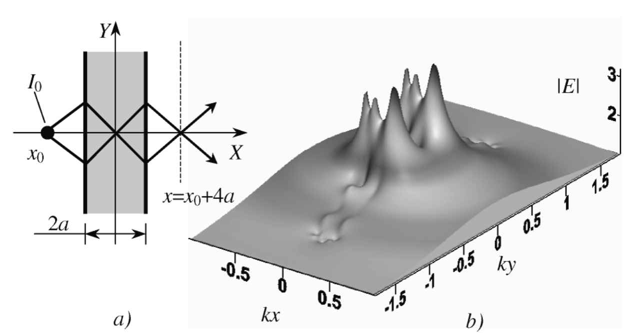


FIG. 1. Geometry of the problem on excitation of an infinite LHM plate and field intensity of two sources located in the vicinity of a thin finite-width plate.

the plate thickness $2a$, the sooner (at smaller values of $|\xi|$) one must assume that $F(\xi) \approx 0$. Needless to say, the parameters $x_0 = -2a$ and $x = 2a$ also vary with the plate thickness.

One can select the criterion of closeness of $F(\xi)$ to $f(\xi)$ (for example, the limiting relative error) and, for the given parameters of the problem, determine the number $\xi_0$ such that, at $|\xi| < \xi_0$, $F(\xi) \approx f(\xi)$. With $|\xi| > \xi_0$, the spectral density $F(\xi)$ decreases rapidly; therefore, the numbers $-\xi_0$ and $+\xi_0$ preassign the width of spectrum of spatial harmonics which are involved in image formation. The detailing of the image is associated with completeness of the spectrum of modes taken into account; in particular, the size of the focal spot is inversely proportional to the quantity $\tilde{\xi}_0 = \xi_0/|k|$. Therefore, the limiting resolution of the system will be defined by the attained value of the parameter $\tilde{\xi}_0$. The ideal image of a point source is reached at $\tilde{\xi}_0 \to \infty$. Corresponding to the case of $\tilde{\xi}_0 = 1$ is the focusing of "standard" quality, when decaying harmonics are lost. This image is formed only by oscillations of propagating types, for which $-1 < \xi/k < 1$. For all intermediate options of $1 < \tilde{\xi}_0 < \infty$, the focusing properties and resolution are improved compared to the "diffraction limit." It is such values of $\tilde{\xi}_0$ that are realized in a system with a plane-parallel plate of a left-handed material with low losses.

The calculation results demonstrate that, given a plate thickness comparable to the wavelength, very high requirements are placed on the quality of the left-handed material. For example, in the case of $\varepsilon'' = \mu'' = 0.01$ and thickness $2k_0a = 3.0$ (about half the wavelength), the value of $\tilde{\xi}_0$ is only about 1.5. In order to attain the value of $\tilde{\xi}_0 \approx 3$ for the same geometry of the problem, the losses must be reduced to $\varepsilon'' = \mu'' = 0.000\,05$, which is unlikely to be achieved. With a greater plate thickness, even lower values of $\tilde{\xi}_0$ are attained. Note that minor losses in the dielectric do not result in such a dramatic effect on the amplitudes of propagating harmonics; therefore, with the treated plate thickness, it is easier to register the effect of the focusing of standard quality. On the other hand, the analysis of expressions for $F(\xi)$ and $\Delta$ indicates that the use of thin LHM plates brings about a drastic *reduction of loss requirements* and leads one to hope for the possibility of practical utilization of the effect of near perfect resolution.

We used other methods as well to test the inferences about the physical matter and quality of image focusing of a point source. In particular, the excitation of a plate of *finite width* by two parallel filaments of cophase electric current was treated. The parameters $\varepsilon$ and $\mu$ of the material (possibly, anisotropic) were included in the expressions of polarization currents in the plate volume. Then, a set of volume integral equations was composed relative to field components in dielectric using the Green function of free space. This set of equations was solved numerically. As a result, the basic inferences could be confirmed without involving the concepts of the LHM wave number and exponentially decaying or rising wave modes. In addition, the need to choose the desired sheet of Riemann surface was obviated. Finally, a model of a finite width plate is more realistic.

The results of calculations for a plate with a relatively large thickness of the order of half the wavelength at $\varepsilon = \mu = -1 - i0.01$ confirmed the absence of superresolution. Zones of local concentration of electromagnetic field energy arise within the plane-parallel plate and in the vicinity of this plate in the neighborhood of the points of crossing of refracted rays, and the direction of energy transport by and large corresponds to the ray representations. However, the size of these "spots" and the achieved resolution (the extremely small distance between the sources, at which their images are discernible separately) showed no advantages over conventional focusing systems.

Much better performance is exhibited by a system based on a thin LHM plate. In particular, even in the case of relatively high losses in the left-handed material, when $\varepsilon'' = \mu'' = 0.1$, one can obtain a sharp separable image of sources separated by a distance of one-tenth of the wavelength. This is confirmed by the results of calculation by the method of volume integral equations [Fig. 1(b)] obtained for the plate thickness $2k_0a = 0.2$, width $2k_0b = 2.4$, and distance between the sources $k_0d = 0.6$ (i.e., $d \approx \lambda/10$). Note the peaks in the figures that correspond to the maximal values of the field amplitude on the front and rear faces of the plate due to the accumulation of the energy of evanescent modes (the peaks on the front face disappear at $\varepsilon'' = \mu'' = 0$, $\varepsilon = \mu = -1$).

The condition $F(\xi) \approx f(\xi)$ provides for a good resolution only when the "image" is registered in a plane parallel to the plate $x = \text{const}$. For ideal focusing in the orthogonal plane $y = \text{const}$, one must provide for a maximum of spectral density of exponentially decaying harmonics along the coordinate $x$. However, as $x$ increases (at $x > a$), the spectral density of decaying harmonics monotonically decreases. Therefore, the superlens does not enable one in principle to reproduce a scene of finite depth with ideal resolution on all coordinates. In view of this, note that, when a source (filament) is placed at the point $|x_0| = 3a$ and the image arises directly on the "unilluminated" face of the plate, special conditions for focusing are developed. Here, a "sharp" vertex of the field amplitude distribution function along both coordinates $x$ and $y$ may be obtained, because, first, a local maximum of the spectral density of decaying harmonics is attained, and, second, the phase velocity of propagating waves at any angle is directed away from the focusing point. The results of field calculations in the vicinity of and within the plate support this inference.

An experiment was performed to check the possibility of attaining the near perfect image in practice. The basic components of the experimental facility are shown in Fig. 2 (the geometric dimensions are given in mm). The facility includes, vertically arranged, two transmitting antennas, 1 and 2, and one receiving antenna, 3 (parallel half-wave vibrators), as well as an LHM plate, 4. The scheme of their arrangement copies the geometry of the treated problem. The microwave image of the transmitting antennas is registered by measuring the level of the signal received by antenna 3 during its movement in the horizontal direction parallel to the plate.

The LHM plate is made of a styrofoam-based composite material. The artificial magnetic properties of the composite are provided by resonance inclusions in the form of spirals, similar to the media suggested and investigated in [3,4]. For the composite not to show the effect of chirality (in this case, the emergence of the horizontal component of the electric field because of the electric moment of the spirals), equal numbers of left- and right-hand spirals are used, staggered as in [4] [Fig. 2(b)]. Note that bifilar spirals may be used for the same purposes [3]; in the particular case of one turn and zero pitch, these spirals reduce to split-ring resonators [8]. The choice of standard spirals was governed by the simplicity of their individual adjustment to one and the same frequency (which is important for the success of the experiment) by shortening the wire or slightly changing the distance between turns. Each spiral is 5 mm in diameter, contains approximately two turns of insulated copper wire 0.74 mm in diameter, and is tuned to the resonant frequency of 1.6 GHz.

Different ways exist to ensure the effective dielectric properties of a composite $\varepsilon' < 0$. A highly homogeneous medium may be produced using elements of one type, namely, spirals arranged in a special manner [4]. However, for the purposes of our experiment, it is quite sufficient to use the simplest system of parallel conductors arranged vertically between holders [Fig. 2(b)]; the resultant structure turns out to be close to that described by Smith *et al.* [2]. We used conductors made of copper wire 0.74 mm in diameter, tuned to the resonance frequency of 1.6 GHz (the length of each conductor, 89 mm).

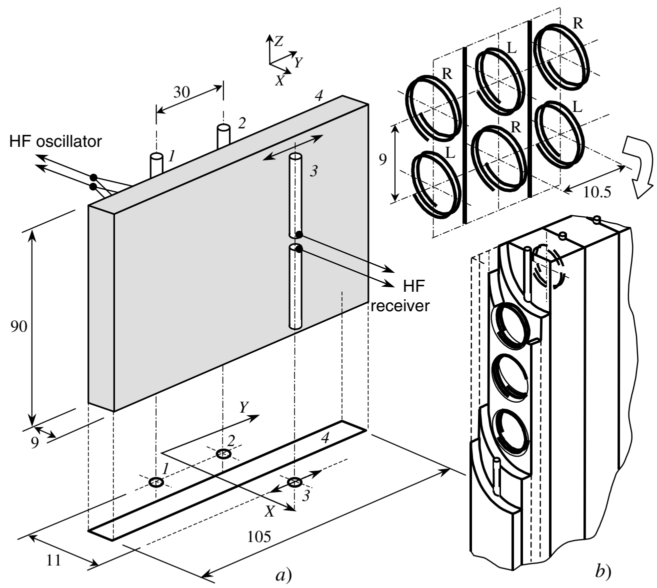


FIG. 2. Schematic of the experimental facility.

Since $\varepsilon'$ and $\mu'$ can reach negative values simultaneously in a narrow frequency band, the measurements were carried out in the bandwidth sufficiently wide to include that frequency region. Figure 3 gives the levels of recorded signal (in dB) as a function of frequency (in GHz) and location of the receiving antenna (coordinates $y$, mm). The following options were examined: (a) there is no plate between the antennas [Fig. 3(a)], (b) the LHM plate described above is located between the transmitting antennas and the receiving antenna [Fig. 3(b)], and (c) a plate of quartz glass 6 mm thick is placed between the transmitting antennas and the receiving antenna (standard dielectric). In the first and third cases, two sources cannot be resolved by readings of the receiving antenna. A signal maximum is observed at point $y = 0$ equidistant from both sources. The frequency at which the signal is maximal is determined by resonant properties of the receiving and transmitting antennas. The presence of a quartz plate somewhat reduces this frequency; however, the pattern does not change qualitatively. Entirely different results are observed when the LHM plate is used. Three characteristic frequency ranges may be identified

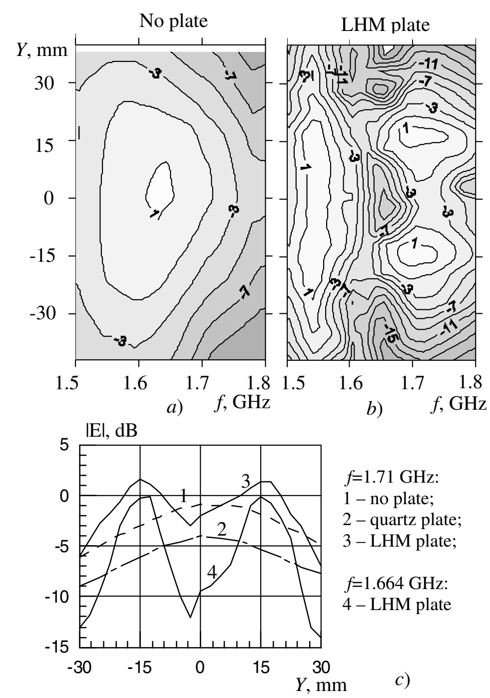


FIG. 3. The results of measurements of the field amplitude on different frequencies, with the receiving antenna moving parallel to the plate.

in the diagram [Fig. 3(b)]. At frequencies below approximately 1.62 GHz, the superresolution is absent again, because, in this frequency range (below the resonance frequency of the composite elements), the effective $\varepsilon'$ and $\mu'$ are positive [2–4]. Between the frequencies of 1.62 and 1.65 GHz, the signal level in the receiving antenna drops abruptly, which is attributed to high values of the effective $\varepsilon''$ and $\mu''$. Finally, at frequencies of approximately 1.65 to 1.8 GHz, two signal maxima are registered whose coordinates $y$ exactly correspond to the position of the sources. So we observe superresolution of two sources separated by the distance of $\lambda/6$. For different distances between the sources, measurements likewise reveal this correspondence. Characteristic sections of the diagram at fixed frequencies are given in Fig. 3(c). Curves 1, 2, and 3 are given for options (a), (b), and (c), respectively, at a frequency $f = 1.71$ GHz on which the signal in the receiving antenna at $y = \pm 15$ mm is maximal and several dB higher than the level recorded in the absence of a plate. In the vicinity of point $y = 0$, curve 3 exhibits a dip up to 4.5 dB deep (with respect to the maximum). One can select such a frequency on which the dip is more pronounced and, consequently, better conditions are provided for separate observation of the sources (see curve 4 in Fig. 3(c), frequency of 1.664 GHz). A minor asymmetry in the shape of the curves is associated with the natural imperfection of the experimental sample of the LHM plate.

Worthy of note is that neither the set of spirals nor the wire grid used separately (to result in $\mu' < 0$ or $\varepsilon' < 0$ correspondingly) revealed the possibility to achieve superresolution at the given thickness of the LHM plate sample (both in computer modeling and experiment). On frequencies where the effect of "superresolution" shows up, the manufactured plate sample, indeed, exhibits the properties of left-handed material (in particular, with respect to the negative phase velocity of the wave in such a medium). To confirm this, a plate, first of quartz glass and then of LHM composite, was introduced into the spacing between the field source and receiving antenna. The phase advance $\Delta\varphi$ of the received signal was registered in the process of plate travel. The emergence of a plate of standard material (quartz) between the source and receiving antenna resulted in a positive phase increment. The presence of the LHM plate caused a decrease in the phase advance, which confirms the realization of negative effective $\varepsilon'$ and $\mu'$.

Note that a significant value of negative phase advance of over 100° was reached when using the manufactured LHM plate. This is indicative of high values of effective $|\varepsilon|$ and $|\mu|$ of the composite (an approximate estimate is $\sqrt{\varepsilon\mu} \approx 8.5$) and, as was demonstrated by the calculation results, is important from the standpoint of ensuring a high resolution when the left-handed material is not isotropic. The composite employed in the experiment has the $xx$ component of the tensor of effective magnetic permeability $\mu'_{xx} \approx +1$, because the resonators are hardly excited by the $x$ component of magnetic field. Computer simulation revealed that, at $\mu'_{xx} = +1$ and $\varepsilon'_{xx} = \varepsilon'_{yy} = \mu'_{yy} = -1$, the image quality of two filamentary sources greatly deteriorates [compared to the results given in Fig. 1(b)]: the field amplitudes in the focusing zones are low, and the dip between the images of the sources is almost unnoticeable. Nevertheless, the image quality may be sharply improved by increasing the values of parameters, for example, to $\varepsilon'_{xx} = \varepsilon'_{yy} = -10$, $\mu'_{yy} = -6$ (other relationships between $\varepsilon$ and $\mu$ may be selected as well). Therefore, in designing technical devices involving the use of left-handed materials, there is no need to develop an isotropic medium and exactly satisfy the condition $\varepsilon = \mu = -1$. Acceptable results may be attained using anisotropic samples which are simpler to manufacture. Note that high values of $|\varepsilon|$ and $|\mu|$ in the case of an isotropic left-handed material disturb the focusing properties of the system (in particular, with $\varepsilon = -10 - i0.1$, $\mu = -6 - i0.1$, source images are no longer separately observed); however, small deviations from $\varepsilon = \mu = -1$ (for example, $\varepsilon = -1.5$, $\mu = -0.7$) may be permitted depending on the requirements placed on the image quality.

Therefore, theoretical and experimental data demonstrate that one can use thin-layer left-handed materials to develop a system with an improved (compared to conventional devices) resolution which is not restricted by the diffraction limit.

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