Palatini Form of $1/R$ Gravity

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It has been suggested that the Universe's recent acceleration is due to a contribution to the gravitational action proportional to the reciprocal of the Ricci scalar. Although the original version of this theory disagrees with solar system observations, a modified Palatini version, in which the metric and connection are treated as independent variables, has been suggested as a viable model of the cosmic acceleration. We show that this theory is equivalent to a scalar-tensor theory in which the scalar field kinetic energy term is absent from the action. Integrating out the scalar field gives rise to additional interactions among the matter fields of the standard model of particle physics at an energy scale of order 10^{-3} eV (the geometric mean of the Hubble and the Planck scales), and so the theory is excluded by, for example, electron-electron scattering experiments.

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The observed acceleration of the Universe's expansion [1,2] is normally attributed to so-called dark energy, that is, an additional source of gravity such as a cosmological constant or a quintessence field. However, it has recently been suggested that the acceleration is due instead to a modification of gravity at cosmological distance scales [3,4]. In particular, Carroll *et al.* [4] suggested a gravitational action of the form

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R - \frac{\mu^4}{R}\right], \tag{1}$$

where R is the Ricci scalar, $\kappa^2 = 8\pi G$, and μ is a mass scale of order the Hubble scale; see also Refs. [5–9]. Chiba [6] showed that the theory (1) is equivalent to a scalar-tensor theory [10] with a very light scalar field that couples to matter with gravitational strength. This theory is therefore ruled out by solar system experiments [6].

However, a modified version of the theory (1) can be obtained by treating the metric and the connection as independent dynamical variables in the variational principle, as suggested by Vollick [5]. For general relativity, this Palatini or first-order variational principle is equivalent to the more usual variational principle where the connection is taken to be determined by the metric. However, for actions that are nonlinear functions of the Ricci scalar, the Palatini variational principle and the standard variational principle give rise to inequivalent theories. The Palatini version of the theory (1), like the original version, can explain the recent acceleration of the Universe's expansion [5,8], but, unlike the original form, has not been shown to be in conflict with solar system experiments.

In this Letter we show that the Palatini form of the theory (1) is equivalent to a type of scalar-tensor theory in which the scalar field kinetic energy term is absent from the action. Integrating out the scalar field gives rise to additional interactions among the matter fields of the standard model at an energy scale of order 10^{-3} eV, and so the theory is excluded by particle physics experiments.

We start by reviewing the equivalence of higher-order gravity theories of the form (1) to scalar-tensor theories [11]. Consider an action of the form

$$S[\bar{g}_{\mu\nu}, \psi_m] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}f(\bar{R}) + S_m[\bar{g}_{\mu\nu}, \psi_m]. \tag{2}$$

Here $\bar{g}_{\mu\nu}$ is the metric (which we have barred for later notational convenience), $\bar{R}$ is its Ricci scalar, and f is any function. The second term is the matter action S_m, which is some functional of the matter fields ψ_m and of the metric. We use units in which $\hbar = c = 1$, and we use the sign conventions of Ref. [12].

We introduce an extra scalar field φ into the theory by defining a modified action [11]

$$\tilde{S}[\bar{g}_{\mu\nu}, \varphi, \psi_m] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}[f(\varphi) + (\bar{R} - \varphi)f'(\varphi)] + S_m[\bar{g}_{\mu\nu}, \psi_m]. \tag{3}$$

The φ equation of motion obtained from this action is $\varphi = \bar{R}$, as long as $f''(\varphi) \neq 0$, and thus the theory (3) is classically equivalent to the original theory (2). Next, we define a conformally rescaled metric $g_{\mu\nu}$ by

$$\bar{g}_{\mu\nu} = e^{2\alpha(\varphi)}g_{\mu\nu}, \tag{4}$$

where α is the function of φ given by

$$e^{2\alpha(\varphi)}f'(\varphi) = 1, \tag{5}$$

and we introduce the canonically normalized scalar field

$$\Phi = -\frac{\sqrt{6}}{\kappa}\alpha(\varphi) = \frac{\sqrt{6}}{2\kappa}\ln f'(\varphi). \tag{6}$$

The action now simplifies to

$$\tilde{S}[g_{\mu\nu}, \Phi, \psi_{\rm m}] = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\nabla\Phi)^2 - V(\Phi)\right] + S_{\rm m}[e^{2\alpha(\Phi)}g_{\mu\nu}, \psi_{\rm m}]. \quad (7)$$

This has the form of a scalar-tensor theory [10], written in terms of the Einstein-frame metric $g_{\mu\nu}$. The potential for the scalar field is

$$V = \frac{\varphi f'(\varphi) - f(\varphi)}{2\kappa^2 f'(\varphi)^2}, \quad (8)$$

and the coupling function $\alpha(\Phi)$ is given by

$$\alpha(\Phi) = -\frac{\kappa}{\sqrt{6}}\Phi. \quad (9)$$

In this class of theories, the scalar field couples to matter with essentially the same strength as does gravity [6]. A measure of the ratio of the scalar coupling to the gravitational coupling is [10]

$$1 - \gamma = \frac{4(d\alpha/d\Phi)^2/\kappa^2}{1 + 2(d\alpha/d\Phi)^2/\kappa^2} = \frac{1}{2}, \quad (10)$$

where γ is the parametrized post-Newtonian parameter. Solar system experiments show that $|\gamma - 1| \leq 3 \times 10^{-4}$ [13], and thus the theory (7) is ruled out unless the potential $V(\Phi)$ is such that the field Φ is massive and short ranged [6]. For the model (1), the potential (8) is [4,6]

$$V(\Phi) = \frac{\mu^2}{\kappa^2}\exp\left[-2\sqrt{\frac{2}{3}}\kappa\Phi\right]\sqrt{\exp\left[\sqrt{\frac{2}{3}}\kappa\Phi\right] - 1}. \quad (11)$$

For this theory to explain the cosmic acceleration, we require $\mu \sim H_0$, where $H_0 \sim 1.5 \times 10^{-33}$ eV is the Hubble scale, and the resulting present-day effective mass $\sqrt{V''(\Phi)}$ of the scalar field is $\sim H_0$ at $\kappa\Phi \sim 1$. Thus this theory is not viable [6].

Turn now to the Palatini form of the theory (2). We first review the formulation of this theory given by Vollick [5]. The action is a function of the Jordan-frame metric $\bar{g}_{\mu\nu}$, a symmetric connection $\hat{\nabla}_\mu$, and the matter fields $\psi_{\rm m}$. We can use instead of the connection $\hat{\nabla}_\mu$ the tensor field $H^\lambda_{\mu\nu}$ defined by $\hat{\nabla}_\mu v^\lambda - \bar{\nabla}_\mu v^\lambda = H^\lambda_{\mu\nu}v^\nu$ for any vector field v^λ, where $\bar{\nabla}_\mu$ is the connection determined by the metric $\bar{g}_{\mu\nu}$. The action takes the form [5]

$$S[\bar{g}_{\mu\nu}, H^\mu_{\nu\lambda}, \psi_{\rm m}] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}f(\hat{R}) + S_{\rm m}[\bar{g}_{\mu\nu}, \psi_{\rm m}], \quad (12)$$

where $\hat{R} = \bar{g}^{\mu\nu}\hat{R}_{\mu\nu}$ and $\hat{R}_{\mu\nu}$ is the Ricci tensor of the connection $\hat{\nabla}_\mu$, given by

$$\hat{R}_{\mu\nu} = \bar{R}_{\mu\nu} + \bar{\nabla}_\lambda H^\lambda_{\mu\nu} - \bar{\nabla}_\mu H^\lambda_{\lambda\nu} + H^\lambda_{\lambda\sigma}H^\sigma_{\mu\nu} - H^\lambda_{\mu\sigma}H^\sigma_{\lambda\nu}. \quad (13)$$

Varying the action with respect to $H^\lambda_{\mu\nu}$ gives an equation of motion whose unique solution is [5]

$$H^\lambda_{\mu\nu} = \delta^\lambda_{(\mu}\bar{\nabla}_{\nu)}\ln f'(\hat{R}) - \tfrac{1}{2}\bar{g}_{\mu\nu}\bar{g}^{\lambda\sigma}\bar{\nabla}_\sigma \ln f'(\hat{R}). \quad (14)$$

Equation (14) is equivalent to the statement that the connection $\hat{\nabla}_\mu$ is compatible with the metric $f'(\hat{R})\bar{g}_{\mu\nu}$. Varying the action with respect to the metric and simplifying using Eq. (14) yields $f'(\hat{R})\hat{R}_{\mu\nu} - f(\hat{R})\bar{g}_{\mu\nu}/2 = \kappa^2\bar{T}_{\mu\nu}$, where $\bar{T}_{\mu\nu}$ is the stress-energy tensor. The traceless part of this equation can be written as

$$\left[\bar{R}_{\mu\nu} - \bar{\nabla}_\mu\bar{\nabla}_\nu \ln f'(\hat{R}) + \frac{1}{2}\bar{\nabla}_\mu \ln f'(\hat{R})\bar{\nabla}_\nu \ln f'(\hat{R})\right]_{\rm TL} = \frac{\kappa^2}{f'(\hat{R})}[T_{\mu\nu}]_{\rm TL}, \quad (15)$$

where TL means "the traceless part of." The trace gives the algebraic relation

$$f'(\hat{R})\hat{R} - 2f(\hat{R}) = \kappa^2\bar{T}, \quad (16)$$

which can be used to solve for $\hat{R}$ in terms of $\bar{T} = \bar{g}^{\mu\nu}\bar{T}_{\mu\nu}$. The Ricci scalar of the metric $\bar{g}_{\mu\nu}$ is then given by the contraction of Eq. (13): $\bar{R} = \hat{R} + 3\bar{\Box}\ln f'(\hat{R}) + 3[\bar{\nabla}\ln f'(\hat{R})]^2/2$. These equations replace the usual Einstein equations, and the corresponding Friedmann-Robertson-Walker cosmological models have been studied by Refs. [5,8].

We now show that the theory (12) is equivalent to a type of scalar-tensor theory. We make the ansatz that the connection $\hat{\nabla}_\mu$ is compatible with the metric $e^{2\chi}\bar{g}_{\mu\nu}$ for some scalar field χ, which implies

$$H^\lambda_{\mu\nu} = 2\delta^\lambda_{(\mu}\bar{\nabla}_{\nu)}\chi - \bar{g}_{\mu\nu}\bar{g}^{\lambda\sigma}\bar{\nabla}_\sigma\chi. \quad (17)$$

From Eq. (14) this ansatz is always satisfied by solutions of the equations of motion. The action (12) now becomes

$$S[\bar{g}_{\mu\nu}, \chi, \psi_{\rm m}] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}f[\bar{R} - 6(\bar{\nabla}\chi)^2 - 6\bar{\Box}\chi] + S_{\rm m}[\bar{g}_{\mu\nu}, \psi_{\rm m}]. \quad (18)$$

All stationary points of the action (12) will also be stationary points of the action (18), by construction. However, the converse is not true, since solutions of the equations of motion of the theory (18) are stationary only under a restricted set of variations of $\delta H^\mu_{\nu\lambda}$ of the connection. We discuss this point further below.

We next use the technique discussed above of introducing an auxiliary scalar field φ. The modified action is

$$\tilde{S}[\bar{g}_{\mu\nu}, \chi, \varphi, \psi_{\rm m}] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\bar{g}}\{f(\varphi) + [\bar{R} - 6(\bar{\nabla}\chi)^2 - 6\bar{\Box}\chi - \varphi] \times f'(\varphi)\} + S_{\rm m}[\bar{g}_{\mu\nu}, \psi_{\rm m}]. \quad (19)$$

As before the φ equation of motion is

$$\varphi = \bar{R} - 6(\bar{\nabla}\chi)^2 - 6\bar{\Box}\chi, \tag{20}$$

so the theories (18) and (19) are classically equivalent. We next transform to the Einstein conformal frame $g_{\mu\nu}$ using the relations (4) and (5). We also define the canonically normalized version Φ of the field φ by Eq. (6) as before, and we use instead of the field χ the quantity

$$\Psi = \sqrt{6}[\alpha(\varphi) + \chi]/\kappa. \tag{21}$$

The resulting action is

$$\tilde{S}[g_{\mu\nu}, \Phi, \Psi, \psi_{\rm m}] = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\nabla\Psi)^2 - V(\Phi)\right] + S_{\rm m}[e^{2\alpha(\Phi)}g_{\mu\nu}, \psi_{\rm m}], \tag{22}$$

where the potential $V(\Phi)$ and coupling function $\alpha(\Phi)$ are given by the same expressions (8) [or (11)] and (9) as before.

We now address the issue of spurious solutions of the equations of motion of the theory (22) that do not satisfy the equations of motion of the original theory (12). We show that the appropriate subclass of solutions of the theory (22) are those solutions with $\Psi = \text{const}$. To see this, note that for solutions of the original theory (12), the connection is compatible with the metric $f'(\hat{R})\bar{g}_{\mu\nu}$, by Eq. (14). Using Eqs. (20), (5), and (21) this metric can be written as

$$f'(\varphi)\bar{g}_{\mu\nu} = e^{-2\alpha}\bar{g}_{\mu\nu} = e^{-2\kappa\Psi/\sqrt{6}}(e^{2\chi}\bar{g}_{\mu\nu}). \tag{23}$$

Comparing with the ansatz (17) shows that on shell we must have $\Psi = \text{const}$, as claimed. It is also possible to show that this is the only restriction on solutions.

Therefore, the final action can be obtained simply by deleting the field Ψ from the action (22):

$$\tilde{S}[g_{\mu\nu}, \Phi, \psi_{\rm m}] = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - V(\Phi)\right] + S_{\rm m}[e^{2\alpha(\Phi)}g_{\mu\nu}, \psi_{\rm m}]. \tag{24}$$

This action has exactly the same form as the scalar-tensor form (7) of the standard-variation version of the theory, except that the kinetic energy term for the field Φ has been deleted. (Similar theories in which a scalar field is nondynamical have been considered in attempts to avoid cosmological singularities [14].)

The equations of motion of this formulation of the theory are significantly simpler than those of the original formulation. They are

$$\frac{1}{\kappa^2}G_{\mu\nu} = -V(\Phi)g_{\mu\nu} + e^{2\alpha(\Phi)}\bar{T}_{\mu\nu}, \tag{25}$$

and

$$V'(\Phi) = \alpha'(\Phi)e^{4\alpha(\Phi)}\bar{T}, \tag{26}$$

where $\bar{T}_{\mu\nu} = -(2/\sqrt{-\bar{g}})\delta S_{\rm m}/\delta\bar{g}^{\mu\nu}$ is the Jordan-frame stress-energy tensor and $\bar{T} = \bar{g}^{\mu\nu}\bar{T}_{\mu\nu}$. Note that Eq. (26) implies that Φ is an algebraic function of $\bar{T}$.

We next discuss the nature of the solutions of the algebraic Eq. (26) for Φ. We restrict attention to the model (11) of Carroll *et al.*, and to the case of negative $\bar{T}$. We define $\rho_{\rm c} = \mu^2/\kappa^2$, which is a critical energy density of order the present cosmological energy density. When $|\bar{T}| \ll \rho_c$, we have $\Phi \approx \Phi_{\rm max} = \sqrt{3/2}\ln(4/3)/\kappa$ (where $\Phi_{\rm max}$ is the value of Φ at the local maximum of the potential), $V(\Phi) \approx V(\Phi_{\rm max}) = 3\sqrt{3}\rho_c/16$, and $e^{2\alpha(\Phi)} \approx 3/4$. When $|\bar{T}| \gg \rho_c$, on the other hand, the solution is $\kappa\Phi \sim (\rho_{\rm c}/|\bar{T}|)^2\sqrt{3/2} \ll 1$, corresponding to $V(\Phi) \sim \rho_c^2/|\bar{T}|$ and $e^{2\alpha(\Phi)} \sim 1$. Thus, up to fractional corrections of order $\rho_c^2/|\bar{T}|^2$, the equation of motion (25) reduces to Einstein's equation.

However, in terrestrial and astrophysical environments, these conclusions are valid only when the stress-energy tensor $\bar{T}_{\mu\nu}$ is interpreted to be the true, microscopic stress energy, and not the macroscopic, spatially averaged stress-energy tensor. What this means is as follows. The true, microscopic stress-energy tensor $\bar{T}_{\mu\nu}$ will vary over atomic scales in matter, from $|\bar{T}| \gg \rho_c$ inside atoms, to $|\bar{T}| \ll \rho_c$ in between atoms. Let us write $\bar{T}_{\mu\nu} = \langle\bar{T}_{\mu\nu}\rangle + \delta\bar{T}_{\mu\nu}$, $g_{\mu\nu} = \langle g_{\mu\nu}\rangle + \delta g_{\mu\nu}$, where $\langle\bar{T}_{\mu\nu}\rangle$ and $\langle g_{\mu\nu}\rangle$ are the spatial averages of the stress-energy tensor and the metric over some length scale large compared to atomic scales, and $\delta T_{\mu\nu}$ and $\delta g_{\mu\nu}$ are the fluctuations due to the microscopic structure of matter. For general relativity, we have $\delta g_{\mu\nu} \ll 1$, and so the fluctuations can be treated as a linear perturbation in Einstein's equation. This guarantees that Einstein's equation continues to hold to a good approximation with $\bar{T}_{\mu\nu}$ replaced by $\langle\bar{T}_{\mu\nu}\rangle$ and with $g_{\mu\nu}$ replaced by $\langle g_{\mu\nu}\rangle$.

However, this property does not hold for the theory (24). If we write $\Phi = \langle\Phi\rangle + \delta\Phi$, then since the energy density varies from scales $\gg \rho_c$ inside atoms to scales $\ll \rho_c$ in between, it follows that $\kappa\delta\Phi$ is of order unity. Hence, $\delta\Phi$ cannot be treated as a linear perturbation in Eq. (25); for example, one cannot make the replacement $\langle e^{2\alpha(\Phi)}\rangle = e^{2\alpha(\langle\Phi\rangle)}$. Thus Eqs. (25) and (26) are not valid with the fields $\bar{T}_{\mu\nu}$, $g_{\mu\nu}$ and Φ are all replaced by their spatial averages. In particular, this invalidates the Friedmann-Robertson-Walker cosmological models obtained from Eqs. (25) and (26) [or equivalently Eqs. (15) and (16)] with $\bar{T}_{\mu\nu}$ taken to be that of pressureless matter [5,8].

The strong dependence of solutions of the equations of motion on the microphysical structure of matter suggests that one should treat the matter source in terms of quantum field theory. In this context, one can integrate out the field Φ by solving its equation of motion and backsubstituting into the action (24). One then finds that additional interactions are generated among the various matter fields of the standard model of particle physics that are sufficiently large to be in severe violation of experimental bounds.

We illustrate this effect by taking the matter action to be the Dirac action for free electrons:

$$S_{\mathrm{m}}[\bar{g}_{\mu\nu}, \Psi_{\mathrm{e}}] = \int d^4x\sqrt{-\bar{g}}\bar{\Psi}_{\mathrm{e}}[i\bar{\gamma}^{\mu}\nabla_{\mu} - m_{\mathrm{e}}]\Psi_{\mathrm{e}}. \quad (27)$$

Here Ψ_{e} is a Dirac spinor, m_{e} is the electron mass, and $\bar{\gamma}_{\mu}$ are the Dirac matrices associated with the metric $\bar{g}_{\mu\nu}$, satisfying $\bar{\gamma}^{\mu}\bar{\gamma}^{\nu} + \bar{\gamma}^{\nu}\bar{\gamma}^{\mu} = -2\bar{g}^{\mu\nu}$. Substituting into Eq. (24) gives for the total action

$$\tilde{S}[g_{\mu\nu}, \Phi, \Psi_{\mathrm{e}}] = \int d^4x\sqrt{-g} \times \left[\frac{R}{2\kappa^2} - V(\Phi) + ie^{3\alpha(\Phi)}\bar{\Psi}_{\mathrm{e}}\gamma^{\mu}\nabla_{\mu}\Psi_{\mathrm{e}} - e^{4\alpha(\Phi)}m_{\mathrm{e}}\bar{\Psi}_{\mathrm{e}}\Psi_{\mathrm{e}}\right], \quad (28)$$

where $\gamma^{\mu} = e^{-\alpha}\bar{\gamma}^{\mu}$ are the Dirac matrices associated with the Einstein-frame metric $g_{\mu\nu}$. The equation of motion for Φ is $V'(\Phi) = 3\alpha'(\Phi)e^{3\alpha(\Phi)}i\bar{\Psi}_{\mathrm{e}}\gamma^{\mu}\nabla_{\mu}\Psi_{\mathrm{e}} - 4\alpha'(\Phi)e^{4\alpha(\Phi)}m_{\mathrm{e}}\bar{\Psi}_{\mathrm{e}}\Psi_{\mathrm{e}}$. Solving this equation for Φ using Eqs. (9) and (11) yields

$$\kappa\Phi = \kappa\Phi_{\max} - \frac{1}{\sqrt{2}}\mathcal{M} - \sqrt{\frac{3}{8}}\mathcal{K} + O(\mathcal{K}^2, \mathcal{M}^2, \mathcal{K}\mathcal{M}), \quad (29)$$

where $\mathcal{K}$ and $\mathcal{M}$ are the dimensionless quantities $\mathcal{K} = i\kappa^2\bar{\Psi}_{\mathrm{e}}\gamma^{\mu}\nabla_{\mu}\Psi_{\mathrm{e}}/\mu^2$, $\mathcal{M} = \kappa^2 m_{\mathrm{e}}\bar{\Psi}_{\mathrm{e}}\Psi_{\mathrm{e}}/\mu^2$. Substituting the solution (29) back into the action (28) and rescaling $g_{\mu\nu} \to (4/3)g_{\mu\nu}$ gives

$$\tilde{S}[g_{\mu\nu}, \Psi_{\mathrm{e}}] = \int d^4x\sqrt{-g}\left[\frac{R}{2\tilde{\kappa}^2} - \Lambda + i\bar{\Psi}_{\mathrm{e}}\gamma^{\mu}\nabla_{\mu}\Psi_{\mathrm{e}} - m_{\mathrm{e}}\bar{\Psi}_{\mathrm{e}}\Psi_{\mathrm{e}} - \frac{3\sqrt{3}}{16m_*^4}(i\bar{\Psi}_{\mathrm{e}}\gamma^{\mu}\nabla_{\mu}\Psi_{\mathrm{e}})^2 - \frac{1}{\sqrt{3}}\frac{m_{\mathrm{e}}^2}{m_*^4}(\bar{\Psi}_{\mathrm{e}}\Psi_{\mathrm{e}})^2 + \sqrt{\frac{3}{4}}\frac{m_{\mathrm{e}}}{m_*^4}(i\bar{\Psi}_{\mathrm{e}}\gamma^{\mu}\nabla_{\mu}\Psi_{\mathrm{e}})(\bar{\Psi}_{\mathrm{e}}\Psi_{\mathrm{e}}) + \ldots\right], \quad (30)$$

where $\tilde{\kappa} = \sqrt{4/3}\kappa$, $m_* = \sqrt{\mu/\kappa}$, and $\Lambda = \mu^2/(\sqrt{3}\kappa^2)$ is the induced cosmological constant.

The last three terms in the action (30) are corrections to the standard model. These corrections are characterized by the mass scale m_*, which is roughly the geometric mean of the Planck and the Hubble scales, of order 10^{-3} eV. Since this energy scale is so small, it is clear that the action (30) is in severe conflict with atomic physics and particle physics experiments, for example, electron-electron scattering. Thus the original gravitation theory (12) is ruled out.

A simple physical explanation for this effect is the following. Consider a region of space where the energy density ρ is of order ρ_c and which varies over a length scale $\mathcal{L} \ll H_0^{-1}$, where H_0 is the Hubble scale. Then, the corresponding Einstein-frame gravitational field will be negligible, but the quantity $e^{2\alpha}$ and the perturbation to the Jordan-frame metric will be of order unity, from Eq. (26). Therefore the gravitational acceleration experienced by test particles will be $\sim c^2/\mathcal{L}$, of order the gravitational acceleration produced by a black hole of size $\sim \mathcal{L}$ near its horizon. Such gravitational accelerations are significantly larger than in general relativity. Although the theory (12) was designed to produce deviations from general relativity only at very large length scales $\sim \mu^{-1} \sim H_0^{-1}$, in fact deviations from general relativity can be manifest at much smaller distance scales, as long as the local radius of curvature of spacetime $\sim\sqrt{c^3/(G\rho)}$ is of order μ^{-1}, and $\mathcal{L} \ll \mu^{-1}$. Since the density scale $\rho \sim \rho_c$ can be achieved in the scattering of sufficiently low energy elementary particles, such particles experience the nonconventional large gravitational forces discussed above, and the particle scattering cross sections are therefore affected.

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