Weak to Strong Pinning Crossover

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Material defects in hard type II superconductors pin the flux lines and thus establish the dissipation-free current transport in the presence of a finite magnetic field. Depending on the density and pinning force of the defects and the vortex density, pinning is either weak collective or strong. We analyze the weak to strong pinning crossover of vortex matter in disordered superconductors and discuss the peak effect appearing naturally in this context.

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Pinning of vortices by material defects is crucial in establishing the defining property of a superconductor, its ability to transport electrical current without dissipation. Collective pinning theory [1], describing the concerted action of many weak pins on the vortex system, is playing a central role in our understanding of this complex statistical mechanics problem [2]. On the other hand, first attempts describing flux pinning go back to Labusch [3], who described the interaction between vortices and strong pinning centers which introduce large (plastic) deformations in the vortex system. In this Letter, we describe how these two theories relate to one another; given the density n_p and force f_p of pinning centers, as well as the vortex density $n_\mathrm{v} = 1/a_0^2$, we identify the regimes where individual vortex lines and the bulk vortex lattice are pinned by the collective action of many weak pins or by the independent action of strong pins; see Fig. 1. We naturally recover the peak effect [4] described in the work of Larkin and Ovchinnikov [1] and establish its formal relation to the Landau theory of phase transitions.

In a type II superconductor, the field ($\mathbf{B}$) induced vortices subject to a current flow $\mathbf{j}$ experience the Lorentz force density $\mathbf{F}_\mathrm{L} = \mathbf{j} \wedge \mathbf{B}/c$, and the resulting vortex motion leads to dissipation. The superconducting response is resurrected through material inhomogeneities pinning the vortices at energetically favorable locations. The pinning force density $\mathbf{F}_\mathrm{pin}$ defines a critical current density $j_c = cF_\mathrm{pin}/B$ below which the current can flow free of dissipation. Usually, this critical current density is considerably reduced with respect to the depairing current density $j_0 \sim c\varepsilon_0/\Phi_0\xi$; here, $\Phi_0 = hc/2e$ is the flux unit, $\varepsilon_0 = (\Phi_0/4\pi\lambda)^2$ is the (line) energy scale, and λ and ξ denote the penetration depth and coherence length, respectively. Below, we focus on the most generic situation of isotropic superconductors and ignore effects due to thermal fluctuations.

When pinning is strong [1,3,5] defects act individually and the pinning force density F_p is linear in the density n_p and average pinning force $\langle f_\mathrm{pin}\rangle$ of defects. The classic arguments characterizing strong pinning go back to Labusch [3]; see also [1,5]. A strong pinning defect induces plastic deformations in the vortex lattice [6–8]), and the energy landscape $e_\mathrm{pin}(\mathbf{R})$ becomes multivalued in the vortex position $\mathbf{R}$; see Fig. 2. The averaging over defect locations then has to account for the preparation of the system. We concentrate on the critical current density and, thus, search for the force against drag; the vortex position then is parametrized through the two-component drag parameter $\mathbf{R}_\mathrm{d}$ fixing the position of the unperturbed lattice with respect to the defect. Dragging the system along the x direction, we express the drag force $-\partial_x e_\mathrm{pin}(x, y)$ integrated along x through the jump $\Delta e_\mathrm{pin}(y) > 0$ in the pinning energy and average over "impact parameters" y,

$$\langle f_\mathrm{pin}\rangle = -\int_0^{L_x} dx \int_0^{L_y} dy \frac{\partial_x e_\mathrm{pin}(x, y)}{L_x L_y} = -\int_0^{a_0} dy \frac{\Delta e_\mathrm{pin}(y)}{a_0 \tilde{a}(y)},$$

where $\tilde{a}$ denotes the distance between periodic jumps [9]. For moderately strong pins with deformations not exceeding the lattice constant, we have $\tilde{a} \approx a_0$, and assuming a maximal transverse trapping distance $t_\perp$ along the y axis

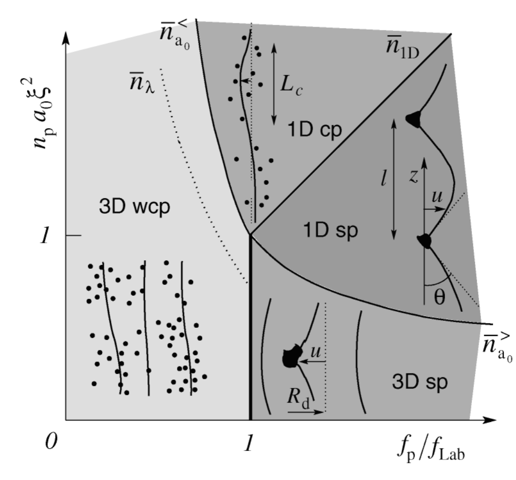

FIG. 1. Pinning diagram delineating the various pinning regimes involving collective versus individual pinning and 1D-line versus 3D-bulk pinning (f_Lab denotes the Labusch force): 3D wcp, bulk weak collective pinning; 1D cp, collective line pinning; 1D sp, strong line pinning; 3D sp, bulk strong pinning. Lines refer to crossovers.

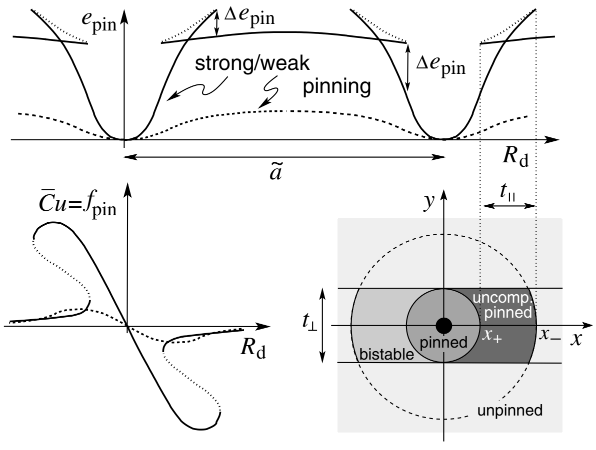

FIG. 2. Energy landscape $e_{\rm pin}$ and pinning force $f_{\rm pin}$ versus displacement $R_{\rm d}$ of the vortex lattice relative to the defect; for weak pinning these are single-valued functions in $R_{\rm d}$ (dashed lines), while strong pinning produces plastic deformations and renders $e_{\rm pin}$, $f_{\rm pin}$ multivalued (solid lines; dotted lines indicate unstable branches). Bottom right: Trapping geometry (top view) for a circularly symmetric situation.

we obtain the mean pinning force

$$\langle f_{\rm pin}\rangle \approx -\frac{t_\perp}{a_0^2}\Delta e_{\rm pin}(0) \approx -\frac{t_\perp t_\parallel}{a_0^2} f_{\rm p} \approx -\frac{S_{\rm trap}}{a_0^2} f_{\rm p}, \quad (1)$$

with the jump $\Delta e_{\rm pin}(0) \approx t_\parallel f_{\rm p}$ expressed via the typical impurity force $f_{\rm p}$ and the bistability range $t_\parallel$ of $e_{\rm pin}(x, 0)$; the product $t_\perp t_\parallel$ defines the trapping area $S_{\rm trap}$ associated with the strong pin [7]. The low impurity concentration $n_{\rm p}$ implies noninterfering defects, and we obtain a critical current density $j_c = -cn_{\rm p}\langle f_{\rm pin}\rangle/B$ linear in $n_{\rm p}$,

$$j_c \approx (c/B)n_{\rm eff} f_{\rm p} \approx j_0[n_{\rm p}\xi S_{\rm trap}]f_{\rm p}/\varepsilon_0, \quad (2)$$

with the effective impurity density $n_{\rm eff} = n_{\rm p}(S_{\rm trap}/a_0^2)$.

In order to derive a quantitative criterion for the appearance of strong pinning, we consider a single defect at the origin with a pinning potential $e_{\rm p}(\mathbf{r})$. Such a defect acts on the vortex system to produce a pinning energy density $E_{\rm p}(\mathbf{r}, \mathbf{u}) = \sum_\nu e_{\rm p}(\mathbf{r})\delta^2(\mathbf{R} - \mathbf{R}_\nu - \mathbf{u}(\mathbf{R}_\nu, z))$, with vortices positioned at $\mathbf{R}_\nu + \mathbf{u}(\mathbf{R}_\nu, z)$, $\mathbf{R}_\nu$ the equilibrium positions and $\mathbf{u}$ the displacement field. The latter follows from the solution of the implicit equation $[\mathbf{r}_\nu = (\mathbf{R}_\nu, z)]$

$$\begin{aligned} u_\alpha(\mathbf{r}_\nu) &= \int d^3r' G_{\alpha\beta}(\mathbf{r}_\nu - \mathbf{r}')[-\partial_{u_\beta}E_{\rm p}](\mathbf{r}', \mathbf{u}') \\ &= \sum_{\nu'}\int dz' G_{\alpha\beta}(\mathbf{r}_\nu - \mathbf{r}'_\nu) f_{{\rm p},\beta}(\mathbf{R}'_\nu + \mathbf{u}(\mathbf{r}'_\nu), z') \\ &= G_{\alpha\beta}(\mathbf{R}_\nu - \mathbf{R}_{\rm d}, 0) f_{{\rm p},\beta}(\mathbf{R}_{\rm d} + \mathbf{u}(\mathbf{R}_{\rm d}, 0), 0), \quad (3) \end{aligned}$$

with $G_{\alpha\beta}(\mathbf{r})$ the elastic Green's function and $\mathbf{f}_{\rm p} = -\nabla_u e_{\rm p}(\mathbf{u})$ the pinning force of the defect. In the last equation we have assumed a moderately strong pinning potential (pinning one vortex at most) of range much smaller than the lattice constant a_0 and have chosen $\mathbf{R}_{\rm d}$ as the distance to the vortex closest to the defect [10]. Evaluating (3) for $\mathbf{r}_\nu = (\mathbf{R}_{\rm d}, 0)$, we arrive at the self-consistency equation [note that $G_{\alpha\beta}(\mathbf{r} = 0)$ is diagonal]

$$\mathbf{u}(\mathbf{R}, 0) \approx \bar{C}^{-1}\mathbf{f}_{\rm p}(\mathbf{R} + \mathbf{u}(\mathbf{R}, 0), 0), \quad (4)$$

with the effective elastic constant $\bar{C}^{-1} = \int d^3k/(2\pi)^3 G_{xx}(\mathbf{k})$. For weak pinning the displacement $\mathbf{u}$ is small and the solution $\mathbf{u}(\mathbf{R}, 0) \approx \mathbf{f}_{\rm p}(\mathbf{R})/\bar{C}$ is unique. Strong pinning, however, produces multivalued functions $\mathbf{u}(\mathbf{R}, 0)$ and $e_{\rm pin}(\mathbf{R})$; cf. Fig. 2. The solution of (4) turns multivalued as the displacement collapses when $\partial_R u \to \infty$. Assuming a defect symmetric in the plane, $e_{\rm p}(\mathbf{R}, z) = e_{\rm p}(R, z)$, and dragging the lattice through the defect center along the x axis, we find $u' = f'_{\rm p}(x + u)[\bar{C} - f'_{\rm p}(x + u)]^{-1}$ (note that $x > 0$ implies $u < 0$) and arrive at the (Labusch) criterion [3] in the form

$$\partial_x f_{\rm p} = -\partial_x^2 e_{\rm p} = \bar{C}; \quad (5)$$

hence, in order to produce strong pinning the (negative) curvature of the pinning energy $e_{\rm p}$ has to overcompensate the lattice elasticity (the Labusch criterion involves the maximal negative curvature above the inflection point). Note that the Labusch criterion tests an individual pinning center and classifies it as a weak or a strong one.

When pinning is weak, the elastic forces dominate over the pinning forces and the defects compete; we then are faced with the problem of the statistical summation of individual pinning forces. For weak pins the average $\langle f_{\rm pin}\rangle$ vanishes and pinning is due to fluctuations in the pinning force density: the forces of the competing pins (with pinning force $f_{\rm p}$, density $n_{\rm p}$, and extension $r_{\rm p} \sim \xi$) add up randomly and produce the pinning energy

$$\langle \mathcal{E}_{\rm pin}^2(V)\rangle^{1/2} \approx [f_{\rm p}^2 n_{\rm p}(\xi/a_0)^2 V]^{1/2}\xi; \quad (6)$$

only vortex cores are pinned by the disorder, hence, the factor $(\xi/a_0)^2$. Within weak collective pinning theory the sublinear growth of $\langle \mathcal{E}_{\rm pin}^2(V)\rangle^{1/2}$ with volume turns linear when the displacement u increases beyond the scale ξ of the pinning potential, thus defining the collective pinning volume V_c. Each volume of size V_c is pinned independently with a pinning energy $U_c = \langle \mathcal{E}_{\rm pin}^2(V_c)\rangle^{1/2}$, and we obtain a proper pinning force density

$$F_{\rm pin} \sim U_c/V_c r_{\rm p} \sim [f_{\rm p}^2 n_{\rm p}(\xi/a_0)^2/V_c]^{1/2}; \quad (7)$$

balancing this pinning force density against the Lorentz force density jB/c, we find a finite critical current density $j_c \sim cF_{\rm pin}/B$. The remaining task is the determination of the collective pinning volume V_c; its calculation is complicated by the dispersion and anisotropy of the vortex lattice—see below and Ref. [2] for a detailed discussion.

It is instructive to compare the weak and strong pinning schemes and their dependence on dimensionality, particularly in the limit of a small defect density $n_{\rm p}$ (in the following, we assume pinning sites characterized by their force $f_{\rm p}$ and extension ξ). An isolated vortex line (1D) is always subject to strong pinning forces as the effective elastic coefficient $\bar{C}$ vanishes due to the diverging integral. At the same time, the deformation of the line due to the pins is large, and we cannot ignore their mutual competition. Comparing the elastic energy $\varepsilon_0\xi^2/L_c$ and the pinning energy $U_c = (f_\mathrm{p}^2 n_\mathrm{p} L_c \xi^2)^{1/2}\xi$, we find the collective pinning length $L_c \sim (\varepsilon_0^2/f_\mathrm{p}^2 n_\mathrm{p})^{1/3}$ and a critical current density

$$j_c \sim j_0(n_\mathrm{p}\xi^3 f_\mathrm{p}^2/\varepsilon_0^2)^{2/3}. \tag{8}$$

This result is valid as long as many pins compete within the volume $\xi^2 L_c$; the condition $n_\mathrm{p}\xi^2 L_c > 1$ defines the lower limit $\bar{n}_\mathrm{1D} \sim f_\mathrm{p}/\varepsilon_0\xi^3 < n_\mathrm{p}$ where the critical current density assumes the value $\bar{j}_c \sim j_0(f_\mathrm{p}/\varepsilon_0)^2$.

For small densities $n_\mathrm{p} < \bar{n}_\mathrm{1D}$ the pins acts individually, and we determine j_c from the force balance $(\Phi_0/c) j_c l u \sim \Delta e_\mathrm{pin} \sim f_\mathrm{p} u$, with $u \sim t_\parallel$ the displacement directed along the force. The displacement u and the length l between two subsequent pins fixing the vortex derives from an analysis of the pinned vortex geometry; see Fig. 1, inset: integrating the force equation $\varepsilon_0 \partial_z^2 u = f(z)$ [with $f(z)$ the force per unit length acting on the line] over one pinning center, we find the distortion angle $\theta = \partial_z u \sim u/l \sim f_\mathrm{p}/\varepsilon_0$ [11]. A vortex segment of length l deformed by the angle θ in the direction of the driving force encounters $\theta l^2 \xi n_\mathrm{p}$ defects (with the trapping length $t_\perp \sim \xi$). At the distance l, this number is unity; hence, $l \sim (\varepsilon_0/f_\mathrm{p} n_\mathrm{p}\xi)^{1/2}$, and we obtain the critical current density

$$j_c \sim j_0(n_\mathrm{p}\xi^3 f_\mathrm{p}^3/\varepsilon_0^3)^{1/2}. \tag{9}$$

At the crossover density $\bar{n}_\mathrm{1D} \sim f_\mathrm{p}/\varepsilon_0\xi^3$ the critical current density matches up with the weak pinning result; also, the displacement $u \sim l f_\mathrm{p}/\varepsilon_0$ is of order ξ at the crossover density $\bar{n}_\mathrm{1D}$ and, hence, matches the displacement field relevant in the collective pinning scenario. Note that collective pinning (8) dominates over the strong pinning (9) at *large* densities $n_\mathrm{p} > \bar{n}_\mathrm{1D}$.

For the vortex lattice (3D bulk pinning, and we assume $a_0 < \lambda$) the Labusch criterion (5) offers a distinction between weak and strong pinning centers; using the Green's function for the vortex lattice (see, e.g., [2]) we find $\bar{C} \sim \varepsilon_0/a_0$. According to (5) a pinning center changes from weak to strong at $f_\mathrm{p} \sim f_\mathrm{Lab} \equiv \varepsilon_0\xi/a_0$. We first review the weak pinning situation with $f_\mathrm{p} < f_\mathrm{Lab}$ (where necessary, we encode quantities in this regime with a superscript "$<$"). The determination of the anisotropic collective pinning volume $V_c = R_c^2 L_c^\mathrm{b}$ has to account for the dispersion in the tilt modulus at intermediate scales $a_0 < R_c < \lambda$ [2], producing the results

$$j_c \sim j_0 \frac{\xi^2}{a_0^2}\left[\frac{a_0}{L_c}\right]^\nu e^{-2c[L_c/a_0]^3}, \qquad R_c < \lambda, \tag{10}$$

$$j_c \sim j_0 \frac{\xi^2}{\lambda^2}\left[\frac{a_0}{L_c}\right]^6, \qquad R_c > \lambda; \tag{11}$$

we have made use of the single vortex pinning parameter $L_c/a_0 \sim (f_\mathrm{Lab}^2/f_\mathrm{p}^2 a_0\xi^2 n_\mathrm{p})^{1/3}$. The numericals c and ν follow from a two-loop renormalization group analysis [12,13]. These results are valid as long as many (competing) pins are active in the volume V_c, $n_\mathrm{p}(\xi^2/a_0^2)V_c > 1$. For $f_\mathrm{p} < f_\mathrm{Lab}$ this condition is violated in the large n_p limit. However, with increasing pinning density n_p, the collective pinning radius R_c decreases, first below λ at $\bar{n}_\lambda \sim f_\mathrm{Lab}^2/f_\mathrm{p}^2 a_0\xi^2 \ln(\lambda/a_0)$ delineating the dispersive regime, and then below a_0 at $\bar{n}_{a_0}^< \sim f_\mathrm{Lab}^2/f_\mathrm{p}^2 a_0\xi^2$ where the condition $n_\mathrm{p}(\xi^2/a_0^2)V_c > 1$ is still fulfilled. At the crossover density $\bar{n}_{a_0}^<$ the 3D weak collective pinning crosses over to the 1D weak collective pinning result (8).

Turning to strong pinning $f_\mathrm{p} > f_\mathrm{Lab}$ (encoded with a superscript "$>$") we start at high densities; as the Labusch criterion is not effective in 1D, the system remains collectively pinned for $n_\mathrm{p} > \bar{n}_\mathrm{1D}$ and crosses over to 1D strong pinning as n_p drops below $\bar{n}_\mathrm{1D}$. Decreasing n_p further, the pinning distance $l \sim a_0[(f_\mathrm{Lab}/f_\mathrm{p})/n_\mathrm{p} a_0\xi^2]^{1/2}$ increases beyond a_0 as n_p decreases below $\bar{n}_{a_0}^> \sim (f_\mathrm{Lab}/f_\mathrm{p})/a_0\xi^2$ and the system enters the 3D strong pinning regime; see Fig. 1. The calculation of the mean pinning force density $F_\mathrm{pin} \sim n_\mathrm{p}\langle f_\mathrm{pin}\rangle$ proceeds along the lines discussed above and involves the trapping area $S_\mathrm{trap} \sim t_\perp t_\parallel$ with $t_\perp \sim \xi$ and $t_\parallel \sim u \sim f_\mathrm{p}/\bar{C}$; we obtain the force density $F_\mathrm{pin} \sim n_\mathrm{p}(\xi/a_0) f_\mathrm{p}^2/\varepsilon_0$ and a critical current density

$$j_c \sim j_0 a_0 \xi^2 n_\mathrm{p} \frac{f_\mathrm{p}^2}{\varepsilon_0^2} \sim j_0 \frac{\xi^2}{a_0^2} n_\mathrm{p} a_0 \xi^2 \frac{f_\mathrm{p}^2}{f_\mathrm{Lab}^2}. \tag{12}$$

The bulk strong pinning result (12) smoothly transforms into the 1D expression (9) at $\bar{n}_{a_0}^>$ where $l \sim a_0$. On the contrary, the strong pinning expression (12) does not match up with the bulk weak collective pinning results (10) and (11) at $f_\mathrm{p} = f_\mathrm{Lab}$ (we concentrate on low impurity densities with $n_\mathrm{p} a_0\xi^2 < 1$; cf. Fig. 1). However, we have to keep in mind that our rough derivation of the strong pinning result (12) breaks down on approaching the critical force f_Lab. Indeed, since the displacement field $\mathbf{u}(\mathbf{r})$ turns single valued below f_Lab, strong pinning vanishes altogether [with a power $[f_\mathrm{p} - f_\mathrm{Lab}]^2$; see (15)], and pinning survives only in the form of weak collective pinning due to fluctuations in the impurity density. Within the approximative scheme adopted here, the sharp rise of the critical current density at $f_\mathrm{p} > f_\mathrm{Lab}$ is encoded in a jump $j_c|_\mathrm{sp}/j_c|_\mathrm{wcp} \sim (\lambda^2/a_0^2)/n_\mathrm{p} a_0\xi^2 > 1$ for $n_\mathrm{p} < \bar{n}_\lambda$ ($\sim\exp[2c/n_\mathrm{p} a_0\xi^2]$ for $n_\mathrm{p} > \bar{n}_\lambda$).

The crossover from strong to weak pinning at the Labusch condition (5) can be analyzed within a Landau type expansion: We define the free energy functional $e_\mathrm{pin}(\mathbf{u}, \mathbf{R}_\mathrm{d}) = \bar{C}u^2/2 + e_\mathrm{p}(\mathbf{R}_\mathrm{d} + \mathbf{u})$ from which the self-consistency equation (4) follows by variation. Note that the derivative $-\partial_x e_\mathrm{pin} = f_{\mathrm{p},x}(\mathbf{R}_\mathrm{d} + \mathbf{u})$ provides the force along x acting on a vortex separated from the defect by $\mathbf{R}_\mathrm{d}$ and deformed by $\mathbf{u}$; cf. Fig. 1. It is this force which has to be averaged over in the definition of $\langle f_\mathrm{pin}\rangle$.

We first concentrate on the trajectory $\mathbf{R}_\mathrm{d} = (x, 0)$ with $\mathbf{u} = (u, 0)$. The curvature $e_\mathrm{p}''(u)$ relevant in (5) assumes a maximal negative value; we denote the corresponding location and value by u_κ and $-\kappa$, respectively. Next, we expand the curvature around u_κ, $e_p''(u) \approx -\kappa + \alpha(u - u_\kappa)^2/2$; integrating in u and combining with the elastic term $\bar{C}u^2/2$, we arrive at the expansion

$$e_{\rm pin}[u, x] \approx \bar{C}\, u^2/2 + \nu\,(x + u - u_\kappa) - \kappa(x + u - u_\kappa)^2/2 + \alpha(x + u - u_\kappa)^4/24. \qquad (13)$$

This pinning energy maps to the free energy $e_{\rm mag}[\phi, h] = \tau\phi^2/2 + \alpha\phi^4/24 - h\phi$ of a one-component magnet in a magnetic field [14] if we define the "order parameter" $\phi = x + u - u_\kappa$, the "temperature difference" $\tau = \bar{C} - \kappa$, and the "magnetic field" $h = \bar{C}(x - u_\kappa - \nu/\bar{C})$. The "high-temperature" phase $\tau > 0$ describing the paramagnet corresponds to weak pinning, while the two ferromagnetic phases at "low temperatures" $\tau < 0$ stand for the pinned ($\phi < 0$) and unpinned ($\phi > 0$) states; the transition between these states is discontinuous and the associated coexistence regime extends over the "field" domain $|h| < h^* = (2/3\bar{C})\sqrt{2/\alpha}|\tau|^{3/2}$. At h^* the trapping or detrapping of the vortex from the defect produces jumps $\Delta\phi = 3\sqrt{2/\alpha}|\tau|^{1/2}$ in the "order parameter," leading to jumps $\Delta e = \Delta e_{\rm pin}/2 = (9/2\alpha)\tau^2$ in the energy.

For finite "impact parameters" y we have to determine the trapping distance $t_\perp$. Assuming rotational symmetry, the bistable regime is bounded by a circle of radius $R = x^* = u_\kappa + \nu/\bar{C}$ and, hence, $t_\perp \approx 2x^*$ (note that at $\tau = 0$ we have $h^* = 0$ but the critical drag parameter x^* does not vanish). The (uncompensated) trapping area determining the average pinning force $\langle f_{\rm pin}\rangle$ is shown in Fig. 2; combining the above results for the jump in pinning energy and the transverse trapping distance, we find the averaged pinning force [cf. (1)]

$$\langle f_{\rm pin}\rangle \approx 18(u_\kappa + \nu/\bar{C})[\bar{C} - \kappa]^2/\alpha a_0^2. \qquad (14)$$

Defining the individual force of (equal) pinning centers via $f_p = \max_u[\partial_u f_p](u)\xi = \kappa\xi$ (then $f_{\rm Lab} = \bar{C}\xi$), we can translate (14) into an expression for the critical current density j_c extending the strong pinning result (12) to the vicinity of the Labusch point,

$$j_c \sim j_0\,(\xi^2/a_0^2)n_p a_0 \xi^2\,[f_p/f_{\rm Lab} - 1]^2. \qquad (15)$$

Comparing with the weak pinning result (11), we note a sharp rise in the critical current density j_c once the strong pinning force overcomes the weak pinning result [1]. With the small parameter $\delta = (a_0/\lambda)(n_p a_0 \xi^2)^{1/2} < 1$, this crossover appears above but still close to the Labusch point as $f_{\rm Lab} \propto \bar{C}$ decreases below $f_p/(1 + \delta)$.

Another remarkable result is the interpretation of the collective pinning scenario in terms of the strong pinning picture; indeed, summing over competing pins within the collective pinning volume V_c produces the corresponding critical Labusch force. Quantitatively, we compare the force gradient $f' \sim [n_p(\xi^2/a_0^2)V]^{1/2} f_p/\xi$ accumulated within the (anisotropic) volume $V = LR^2 = (\lambda/a_0)R^3$ with the elastic parameter $\bar{C}(R) = \varepsilon_0\lambda R/a_0^3$ for smooth distortions on the scale $R > \lambda$ (nondispersive regime) and apply the Labusch criterion (5). We then find the scale $R_c = \lambda f_{\rm Lab}^2/f_p^2 n_p a_0 \xi^2$, where the accumulated pinning force overcompensates the elastic force; this length agrees with the 3D collective pinning length in the nondispersive regime [2]. The resulting bistable solutions are the signature of the alternative pinning valleys which the collective pinning volume can select beyond the scale R_c.

The above discussion sheds light on the general concept of pinning. Pinning is absent in the rigid limit. A finite but large elasticity (with $f_{\rm Lab} > f_p$) allows for only weak deformations, and individual pins cannot hold the lattice as the averaging over individual pinning forces produces a null result. Hence, pinning is due only to fluctuations in the pinning forces and thus collective. Reducing the elasticity, strong pinning defects appear when $f_{\rm Lab}$ drops below f_p; they pin the lattice individually and strong pinning, linear in the defect density n_p, outperforms collective pinning. The important role played by the curvature $e_p'' < 0$ in the pinning potential is an interesting topic for numerical studies. The crossover between weak collective and strong pinning can be realized in experiments: increasing the magnetic field towards its critical value H_{c_2} leads to a marked softening of the elastic moduli. The reduction in the elastic moduli entails a decrease of the Labusch force $f_{\rm Lab}$ and triggers the crossover from weak to strong pinning, producing the well known peak effect in the critical current density [1,4].

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