Nonlinear Band Gap Transmission in Optical Waveguide Arrays

Ramaz Khomeriki*

Department of Physics, Tbilisi State University, 3 Chavchavadze Avenue, Tbilisi 0128, Republic of Georgia

(Received 15 September 2003; published 13 February 2004)

The effect of nonlinear transmission in coupled optical waveguide arrays is theoretically investigated and a realistic experimental setup is suggested. The beam is injected in a single boundary waveguide, linear refractive index of which (n_0) is larger than refractive indexes (n) of other identical waveguides in the array. Particularly, the effect holds if $\omega(n_0 - n)/c > 2Q$, where Q is a linear coupling constant between array waveguides, ω is a carrier wave frequency, and c is a light velocity. Numerical experiments show that the energy transfers from the boundary waveguide to the waveguide array above a certain threshold intensity of the injected beam. This effect is due to the creation and the propagation of gap solitons in full analogy with a similar phenomenon in sine-Gordon lattice [F. Geniet and J. Leon, Phys. Rev. Lett. **89**, 134102 (2002)].

DOI: 10.1103/PhysRevLett.92.063905 PACS numbers: 42.65.Wi, 05.45.–a, 42.25.Gy, 63.20.Pw

Nonlinear phenomena in a large diversity of physical systems have a close relation with each other because the nonlinear dynamics can be described only within a few theoretical models [1]. Thus there exists a possibility to predict novel effects in realistic physical systems via modeling similar processes in simple hypothetical systems, namely, chains of coupled nonlinear oscillators could be used for this purpose. For instance, as recently discovered by Geniet and Leon [2], *nonlinear supratransmission* phenomenon takes place in the discrete sine-Gordon lattice. This means that by driving harmonically and continuously one end of the lattice with frequencies within a band gap there is no energy flow through the lattice for low amplitude driving, while above a definite driving amplitude threshold a sudden energy flow takes place. This nontrivial effect has been explained by means of the direct soliton creation at the end of the lattice; in other words the sudden energy flow occurs when the driving adjusts the internal oscillations of the localized object. It was also noted the possibility of the existence of a similar mechanism of a gap soliton generation in photonic band gap materials. It should be especially mentioned that nonlinear supratransmission has been detected not only by making numerical simulations for the model system of the discrete sine-Gordon lattice, but it is also experimentally realized on a mechanical pendulums chain driven at one end at band gap frequencies [2].

The present Letter aims to analyze whether a similar scenario takes place in the case of the discrete nonlinear Schrödinger (DNLS) equation and then to make predictions concerning the corresponding nonlinear processes in coupled optical waveguide arrays [3]. The experimental conditions are suggested for which the optical waveguide array becomes transparent with respect to the beam injected into the single boundary waveguide if the beam's intensity exceeds a certain threshold (see Fig. 1). This effect is opposite to the ordinary case when for low intensities (linear regime) the light injected into the single waveguide spreads to other waveguides, while in the nonlinear case the light is trapped into several neighboring waveguides thus leading to the spatial discrete optical breather creation (see, e.g., the most recent experimental papers on the subject [4]). It should be mentioned that the longitudinal space dimension in an optical waveguide array plays a role of a time variable, and one should take special note of this when suggesting real experiments on waveguide arrays.

Let us begin by considering the boundary driven DNLS equation which could be written in the following form ($j = 1, \dots, N$):

$$i\frac{\partial \psi_j}{\partial z} + \psi_{j+1} + \psi_{j-1} + 2|\psi_j|^2\psi_j = 0; \qquad \psi_0 = Ae^{i\Delta z}. \tag{1}$$

Here the z variable stands for time; Δ and A are the driving frequency and amplitude of the boundary, respectively. The initial condition reads as $\psi_j(0) = 0$; it is supposed that the driving is turned on adiabatically, e.g., $A = A_0[1 - \exp(-z/\tau)]$ in order to avoid the appearance

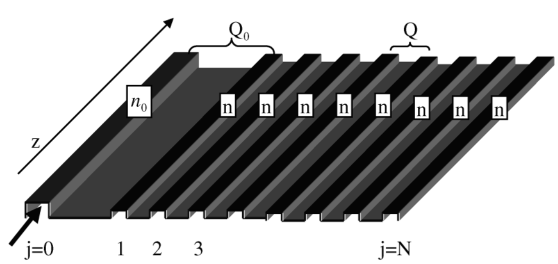

FIG. 1. Suggested experimental setup: The beam is injected into the boundary waveguide numbered as $j = 0$, Q_0 is a linear coupling between boundary and first waveguides, and Q is a coupling constant between the waveguides in the array. n_0 and n are linear refractive indexes of boundary and array waveguides, respectively; z is a longitudinal space dimension playing a role of time in the boundary driven DNLS equation (1).

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of the perturbations from the initial shock. In simulations $\tau = 10$ and the nonlinear dynamics is monitored up to time scales 10^4; thus in a stationary regime ($z \gg \tau$) one has $A = A_0$. The damping also has been applied at the right end of the waveguide array in order to suppress edge reflection. Note that in the absence of driving the sum of intensities $\sum_j^N |\psi_j|^2$ is a conserved quantity, thus the nonlinear dynamics across the array could be described via intensity flux J_j through the site j:

$$J_j = i(\psi_j \psi_{j+1}^* - \psi_j^* \psi_{j+1}), \tag{2}$$

where the intensity and intensity flux at site j are connected with each other via the discrete continuity condition $d|\psi_j|^2/dz + (J_j - J_{j-1}) = 0$.

The numerical simulations have been performed choosing different values of the boundary driving parameters Δ and A. From the numerical experiments it follows that boundary driving leads to the perturbation of all sites if the driving frequency Δ is located within the linear phonon band $-2 < \Delta < 2$, i.e., there is a nonzero intensity flux for any driving amplitudes (see Fig. 2). On the other hand if the driving frequency is in an upper band gap, particularly $\Delta > 2$, for low driving amplitudes only several neighboring sites are excited and the intensity flux to remote sites is zero. The energy starts to flow only if the driving amplitude exceeds a certain threshold (see Fig. 3). Note that if the driving frequencies are within a lower band gap there is no intensity flow for any driving amplitudes. Now it is time to analyze the mechanisms for that effect.

As far as boundary driving is applied it is natural to expect that localized solutions will excite. This statement is in full accordance with the consideration of a similar process in discrete sine-Gordon type models where the nonlinear supratransmission has been discovered [2]. Thus one can assume that the nonzero intensity flux will appear when the boundary driving excites moving localized solutions. It is easy to derive a semidiscrete approximate envelope soliton solution substituting the ansatz $\psi_j = \Psi(j)\exp\{i(\beta z - \chi j)\}$ into the DNLS equation (1). Assuming that the envelope $\Psi(j)$ varies smoothly along the lattice and expanding $\Psi(j \pm 1)$ about the site j, we then get the following approximate one soliton solution (see, e.g., Ref. [5] for details of a similar derivation):

$$\psi_j = \frac{|\psi_j|_{\max}}{\cosh[|\psi_j|_{\max}(j - Vz)]} e^{i(\beta z - \chi j)}, \tag{3}$$

with a following nonlinear dispersion relation for the carrier wave of the envelope soliton (χ varies from 0 to 2π)

$$\beta = 2\cos\chi + |\psi_j|^2_{\max}, \tag{4}$$

and $V = \partial\beta/\partial\chi = -2\sin\chi$ is a soliton's group velocity.

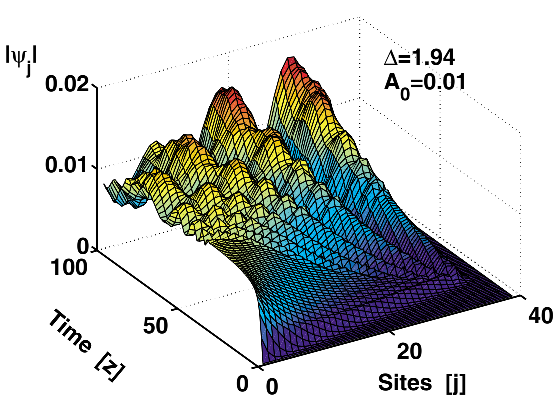

FIG. 2 (color online). Three dimensional plot of time evolution of the boundary driven DNLS equation for inband driving frequency $\Delta = 1.94$ and the very small driving amplitude $A_0 = 0.01$. As seen, the intensity transmits to remote sites.

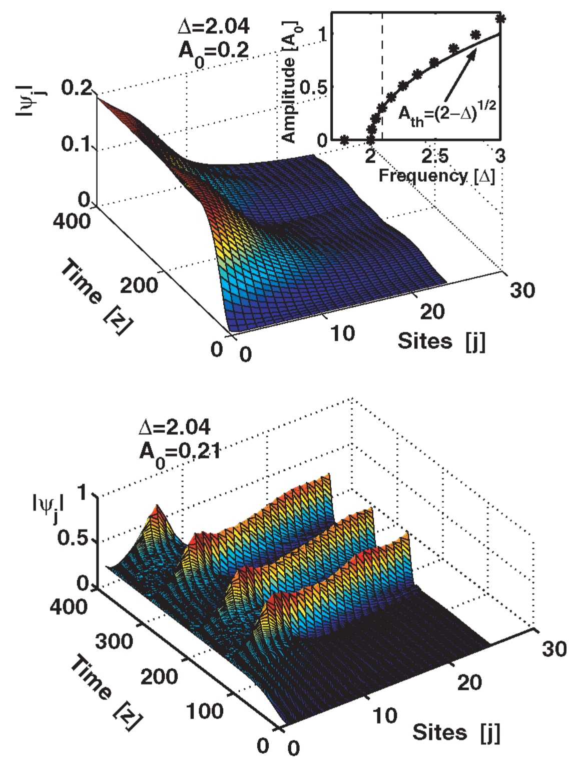

FIG. 3 (color online). Same as in Fig. 2 for band gap driving $\Delta = 2.04$. For driving amplitudes below the threshold ($A_{\rm th} = 0.202$) the pattern in the upper graph could be described by the standing breather solution (7), while for the driving amplitudes above the threshold (lower graph) the transmission could be described by a train of gap solitons (3). The inset shows the dependence of the driving threshold upon the driving frequency; the asterisks are the results of the numerical simulations on boundary driven DNLS equation (1) and the solid line represents an analytical curve (6). The dashed line divides the range of A_0 (left-hand side) for which an analytical approximate semidiscrete approach is given by formulas (3) and (7). Moreover within the same range above a threshold, low amplitude semi-discrete envelope solitons participate in band gap transmission (see lower graph). At the right side of the dashed line in the inset high amplitude moving breathers are excited, which are further trapped by the lattice (see Fig. 4), which leads to the suppression of band gap transmission.

Note that the assumption that the soliton envelope varies smoothly along the lattice puts the following restriction on the soliton amplitude $|\psi_j|^2_{\max} \ll 1$.

It is expected that intensity flux appears in the system if driving adjusts the nonlinear dispersion relation (4), i.e., $\beta = \Delta$ and $|\psi_j|_{\max} = A_0$. In other words, flux is nonzero only if one can find such χ that the following condition is fulfilled: $\Delta = 2\cos\chi + A_0^2$. Therefore for inband driving $-2 < \Delta < 2$ the nonzero flux appears even for very low driving amplitudes A_0, while in an upper band gap

$$\Delta > 2, \tag{5}$$

there exists a certain amplitude threshold

$$A_{\mathrm{th}} = \sqrt{\Delta - 2} \tag{6}$$

below which ($A_0 < A_{\mathrm{th}}$) there is no intensity flux into the system. Instead, only several sites are excited and that pattern could be described [6] by a static breather solution of Eq. (1). This solution could be derived from the general one (3) for zero velocity $V = -2\sin\chi = 0$, i.e., $\chi = 0$ and requires the fulfillment of the boundary condition $\psi_0 = A_0 \exp(i\Delta z)$ in the stationary state ($z \gg \tau$):

$$\psi_j = \frac{\sqrt{\Delta - 2}}{\cosh[(j + x_0)\sqrt{\Delta - 2}]} e^{i\Delta z}, \tag{7}$$

where $x_0 = \mathrm{acosh}[\sqrt{\Delta - 2}/A_0]/\sqrt{\Delta - 2}$.

The results of numerical simulations are fully explained by the above consideration. The Fig. 3 inset presents the comparison between numerical experiments on Eq. (1) and the analytical formula (6) for the driving threshold above which ($A_0 > A_{\mathrm{th}}$) a nonzero intensity flux appears in the system. For driving frequencies Δ close to 2 there is a perfect agreement, but this agreement becomes worse for larger driving frequencies. The point is that for driving frequencies sufficiently larger than 2 threshold amplitudes become comparable with unity according to the relation (6). But for such amplitudes the continuum envelope approximation (3) is invalid. Moreover, as numerical simulations show large amplitude excitations are trapped by the lattice (see Fig. 4); as a result the localizations do not move and intensity flux becomes zero. Thus the band gap transmission effect exists if there are moving solutions in the system. As a result in the case of the DNLS equation the discovered phenomenon is observable for driving frequencies $2 < \Delta < 2.09$.

Now let us discuss how the obtained results could be applied to describe nonlinear transmission processes in the system of coupled optical waveguides. To realize a band gap driving it is suggested (see Fig. 1) to inject a beam into the boundary waveguide with the linear refractive index n_0 larger than the refractive index n of other waveguides forming the array. Let us introduce a linear coupling constant between array waveguides as Q, while the coupling between boundary ($j = 0$) and first waveguides is defined as Q_0. Besides that, let us suppose

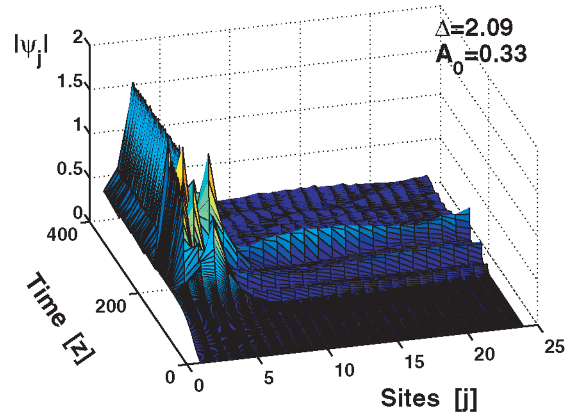

FIG. 4 (color online). Trapping of large amplitude moving gap soliton. For relatively large driving amplitudes the soliton starts to move but further the lattice traps it. This in itself stops the transmission process.

for simplicity that the nonlinear refractive index (Kerr nonlinearity) in the $j = 0$ waveguide is equal to zero and the onsite nonlinear refractive index in array waveguides is D. Thus the wave envelopes in waveguides could be described by a set of the following equations ($j = 2, \ldots, N$):

$$\begin{aligned} i\frac{\partial\psi_0}{\partial z} + \frac{\omega}{c}n_0\psi_0 + Q_0\psi_1 &= 0,\\ i\frac{\partial\psi_1}{\partial z} + \frac{\omega}{c}n\psi_1 + Q_0\psi_0 + Q\psi_2 + D|\psi_1|^2\psi_1 &= 0,\\ i\frac{\partial\psi_j}{\partial z} + \frac{\omega}{c}n\psi_j + Q(\psi_{j+1} + \psi_{j-1}) + D|\psi_j|^2\psi_j &= 0, \end{aligned} \tag{8}$$

where ω is a carrier wave frequency and c is a light velocity. The last equation from the set (8) is a well-known representation of the infinite waveguide array by the DNLS equation [3], while the first two equations describe the influence of the boundary waveguide (with a different linear refraction index) on the semi-infinite array (see also Ref. [7]). After the appropriate rescaling

$$\psi_j = \psi_j' e^{iz\omega n/c}\sqrt{2Q/D} \quad \text{for } j = 1, \ldots, N; \tag{9}$$

$$\psi_0 = \psi_0' e^{iz\omega n/c}(Q/Q_0)\sqrt{2Q/D}; \qquad z = z'/Q.$$

Equations (8) obtain a simpler form [$\Delta \equiv \omega(n_0 - n)/Qc$]:

$$\begin{aligned} i\frac{\partial\psi_0'}{\partial z'} + \Delta\psi_0' + \frac{Q_0^2}{Q^2}\psi_1' &= 0,\\ i\frac{\partial\psi_j'}{\partial z'} + (\psi_{j+1}' + \psi_{j-1}') + |\psi_j'|^2\psi_j' &= 0 \end{aligned} \tag{10}$$

($j = 1, \ldots, N$) which reduces to the boundary driven DNLS (1) with the boundary condition $\psi_0' = \psi_0'(0)\exp(i\Delta z')$ in the limit $(Q_0/Q) \to 0$; therefore for $Q_0/Q \ll 1$ one can use the results derived for the case

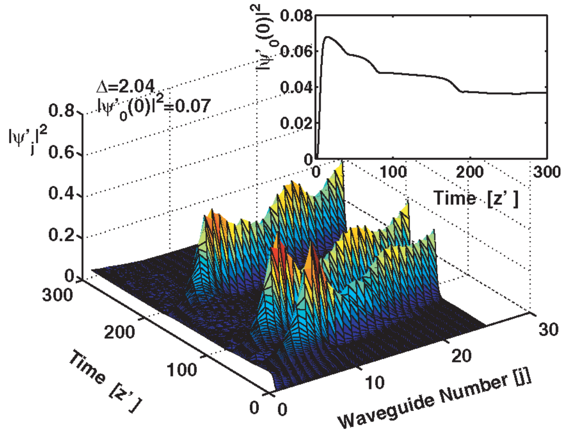

FIG. 5 (color online). Rescaled waveguide intensity $|\psi_j'|^2$ in the band gap transmission regime. The inset shows longitudinal dimension z' dependence of rescaled beam intensity in the boundary waveguide. In creating the solitons the intensity from the boundary waveguide is transferred to other waveguides and as a result the transmission stops when the beam intensity in the boundary waveguide goes below the threshold. The simulations are done for $Q_0/Q = 0.1$.

of the boundary driven DNLS (5) and (6). Particularly, for

$$\Delta \equiv \omega(n_0 - n)/Qc > 2, \tag{11}$$

the localized excitations (3) and (7) form with the propagation constant located in the upper band gap $\beta = \Delta > 2$. Thus if the injected intensity in the boundary is below the threshold

$$|\psi_0(0)|^2_{\text{th}} = \frac{2Q^3}{DQ_0^2}|\psi_0'(0)|^2_{\text{th}} \simeq \frac{2Q^2}{DQ_0^2}\left[\frac{(n_0 - n)\omega}{c} - 2Q\right], \tag{12}$$

one has a static breather solution (7) and the intensity flux to the array waveguides is equal to zero, while above the threshold energy transmission begins via gap solitons (3).

The above expression for the threshold (12) becomes exact in the limit $(Q_0/Q) \to 0$. For small but nonzero (Q_0/Q) the beam intensity in the boundary waveguide could be considered almost a constant quantity irrespective of the spread of energy in the nonlinear band gap transmission regime, because according to the rescaling beam, intensity in the boundary waveguide is Q/Q_0 times larger than the amplitude of the propagating soliton through the array. In the case when Q_0 becomes comparable with Q almost all intensity in the boundary waveguide is needed to form a soliton. As a result the intensity in the boundary waveguide sharply decreases and therefore a much larger threshold intensity is needed [than given by relation (12)] to develop the nonlinear transmission in the band gap regime. In Fig. 5 is presented the picture for the case $Q_0/Q = 0.1$. As can be seen the beam intensity in the boundary waveguide is above the threshold at the origin $z' = 0$, and it produces gap solitons causing the band gap transmission. However, this process itself causes the decrease of the beam intensity in the boundary waveguide; the intensity after the creation of several solitons goes below the threshold and the transmission process is not observable for large z'.

As discussed earlier the nonlinear band gap transmission regime holds if one has low amplitude solitons. Large amplitude solitons tend to pin and energy transfer becomes much less effective. However, this is true only in the case of the DNLS equation where only on-site nonlinearities are taken into account. As shown recently [8], considering also the terms describing intersite nonlinearities, one has moving breather solutions even at large excitation amplitudes. High intensity moving breathers have also been detected on the recent experiments [4]. The numerical simulations have been undertaken adding to the DNLS equation the terms with intersite nonlinearities. In this case the energy transfer via the moving breathers takes place even for large excitation amplitudes, and as a result the optical transparency regime is observable in the whole range of $\Delta > 2$.

Summarizing, it should be noted again that the novel scenario of the nonlinear band gap transmission in optical waveguide arrays is predicted and a simple experimental setup is suggested for its realization. The suggested experimental setup would serve also for the generation of optical gap solitons propagating across the waveguide array.

Finally I would like to thank all the participants of the NATO advanced research workshop at Erice (Italy, 2003) for inspiring discussions. Special thanks are offered to Yuri Kivshar, Stefano Lepri, Stefano Ruffo, and Lasha Tkeshelashvili for suggestions. The work is supported by USA CRDF Award No. GP2-2311-TB-02 and NATO reintegration Grant No. FEL.RIG.980767.

*Electronic address: khomeriki@hotmail.com

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