Evidence for the Existence of the [2 0 2]3/2 Deformed Band in Mirror Nuclei ^{25}Mg and ^{25}Al

Y. Fujita,[1,*] I. Hamamoto,[2] H. Fujita,[3] Y. Shimbara,[1] T. Adachi,[1] G. P. A. Berg,[3,4] K. Fujita,[3] K. Hatanaka,[3] J. Kamiya,[3,†] K. Nakanishi,[3] Y. Sakemi,[3] Y. Shimizu,[3] M. Uchida,[5] T. Wakasa,[3,‡] and M. Yosoi[5]

[1]*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[2]*Division of Mathematical Physics, LTH, University of Lund, P.O. Box 118, S-22100 Lund, Sweden*
[3]*Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan*
[4]*Kernfysisch Versneller Instituut, Zernikelaan 25, 9747 AA Groningen, The Netherlands*
[5]*Department of Physics, Kyoto University, Sakyo, Kyoto 606-8502, Japan*

(Received 15 October 2003; published 13 February 2004)

After 50 years of its prediction, the highest-lying [2 0 2]3/2 orbit among the six Nilsson single-particle orbits originating from the *sd* shells in prolately deformed nuclei and the rotational band on this orbit were identified. The band members were observed in ^{25}Al at excitation energies of 6–7.5 MeV in a high-resolution ^{25}Mg(^{3}He, t) charge-exchange reaction at 0° having a strong selectivity for Gamow-Teller transitions. In the comparison with the analogous $M1$ transitions in ^{25}Mg, the $J^\pi = 3/2^+$ bandhead state and the excited $5/2^+$ and $7/2^+$ members were clearly assigned.

DOI: 10.1103/PhysRevLett.92.062502 PACS numbers: 21.10.Re, 21.10.Pc, 25.55.Kr, 27.30.+t

In axially symmetric deformed nuclei the Nilsson orbits [1] are labeled by using asymptotic quantum numbers $[Nn_z\Lambda]\Omega$, where N is the total oscillator quantum number and n_z the number of quanta along the z axis. The projections of the orbital and total angular momenta of the single particle along the z axis are denoted by Λ and Ω, respectively. In the presence of axial symmetry about the z axis, the z component K of the total spin J is a good quantum number, and $K = \Omega$ holds for the rotational band based on the Nilsson orbit.

In the middle of the *sd* shell, odd-mass nuclei with mass number $19 \leq A \leq 25$ have a strong prolate deformation [2], and their low-lying states form rotational bands based on intrinsic configurations of single neutron or proton Nilsson orbits. Of the six Nilsson orbits originating from *sd* shells, five orbits have been known for a long time [2]. The lowest [2 2 0]1/2 orbit and the next [2 1 1]3/2 orbit form the ground-state bands in the $A = 19$ mirror nuclei (^{19}F and ^{19}Ne) and both $A = 21$ (^{21}Ne and ^{21}Na) and $A = 23$ nuclei (^{23}Na and ^{23}Mg), respectively. The [2 0 2]5/2 orbit, the only $K^\pi = 5/2^+$ Nilsson orbit in the *sd*-shell region, forms the ground-state band in the $A = 25$ mirror nuclei ^{25}Mg and ^{25}Al studied here, and the [2 1 1]1/2 and [2 0 0]1/2 bands form the excited bands [2].

The highest-lying [2 0 2]3/2 orbit, on the other hand, has never been properly identified, although it was already predicted in the middle of the 1950s [1,3]. This orbit would lie at the Fermi level for nuclei with neutron or proton numbers N or $Z \approx 19$. Therefore, the expectation was that this orbit would not be observed, because nuclei with these N or Z are not deformed near the stability line due to the shell closure at Z and/or $N = 20$. This scenario has changed with the development of intense beams of nuclei far from the β-stability line, making nuclear spectroscopy of these nuclei possible. Experimental evidence suggests that the nucleus ^{32}Mg with $N = 20$ is deformed [4]. Then the ground state (g.s.) of neighboring odd-N nuclei may exhibit the configuration of the [2 0 2]3/2 orbit (see, for example, Ref. [5]).

The quadrupole moment of the first 2^+ state of the even-even $N = Z$ nucleus ^{24}Mg is $Q_{2^+} \approx -18$ fm^2 [6]. This suggests that ^{24}Mg has a prolate deformation with a deformation parameter $\delta \approx 0.4$–0.5, one of the largest values of all deformed *sd*-shell nuclei. Low-lying states of $A = 25$ mirror nuclei ^{25}Mg and ^{25}Al are well described by the particle-rotor model [2,7,8]. The study of intraband and interband Gamow-Teller (GT) and $M1$ transitions shows that the K-selection rules work very well [9], suggesting a good axially symmetric shape of these nuclei. Since higher mass *sd*-shell nuclei are less deformed or spherical [2], the last opportunity to find the [2 0 2]3/2 rotational band without waiting for the study of exotic nuclei is to survey the higher excitation region of the $A = 25$ nuclei.

Recently a (^{3}He, t) reaction measurement at 0° and at an intermediate incident energy of 140 MeV/nucleon with a high energy resolution of 35 keV [10] made it possible to study individual GT ($\Delta L = 0$, $\Delta J^\pi = 1^+$) excitations. The approximate proportionality between the 0° cross sections and the GT transition strengths B(GT) was shown for values of $B(\mathrm{GT}) \geq 0.04$ in the studies of analogous GT transitions in $A = 27$ mirror nuclei ^{27}Al and ^{27}Si [11], and $A = 26$ nuclei ^{26}Mg, ^{26}Al, and ^{26}Si [12]. Absolute B(GT) values can be obtained by calibrating to β-decay data. In addition, $\Delta J^\pi = 1^+$ and $\Delta K^\pi = 1^+$ selectivities were clearly observed in the excitations of low-lying rotational bands of ^{25}Al [9] and ^{23}Mg [13]. Since the $J^\pi = 5/2^+$ g.s. of ^{25}Mg is the bandhead state of the $K^\pi = 5/2^+$, [2 0 2]5/2 band, $J^\pi = 3/2^+, 5/2^+$, and $7/2^+$ members of $K^\pi = 3/2^+$ bands in ^{25}Al are selectively excited in this reaction. The $K^\pi = 3/2^+$, [2 0 2]3/2 band is expected at higher excitation energies $E_x > 6$ MeV (see Fig. 5-1 of Ref. [2]). Therefore, the "direct" nature of the reaction study has the advantage of avoiding a complicated level reconstruction at high excitation energies in a γ-decay study usually used for the search of rotational bands.

An experiment of the ^{25}Mg(^{3}He, t)^{25}Al reaction was performed at the high-resolution facility of RCNP, consisting of the "WS course" [14] and the Grand Raiden spectrometer [15] using a ^{3}He beam from the $K = 400$ Ring Cyclotron. A thin self-supporting ^{25}Mg target with a thickness of 0.93 mg/cm^2 and an isotopic enrichment of 98.3% was used. The outgoing tritons were momentum analyzed within the full acceptance of the spectrometer and detected with a focal-plane detector system allowing for particle identification and track reconstruction in horizontal and vertical directions. Good angle resolution of about 8 mrad (FWHM) was achieved by applying the *angular dispersion matching* technique [16] and the "overfocus mode" of the spectrometer [17]. The acceptance of the spectrometer was subdivided into scattering-angle regions in the analysis by using the track information. An energy-resolution far better than the energy spread of the beam was realized by applying *dispersion matching* and *focus matching* techniques [16]. For fast and efficient beam tuning, the "faint beam method" [10] was applied. With the achieved 35 keV (FWHM) resolution, discrete states were observed up to $E_x = 8.5$ MeV in the "0° spectrum" [Fig. 1(a)] showing events for scattering angles $\Theta \leq 0.8°$. Above the proton separation energy of 2.27 MeV particle decay can occur. The different widths of states in the expanded spectrum [Fig. 1(b)] suggest different configurations of these states.

The E_x values of the states in ^{25}Al (see Table I) were determined with ≈ 5 keV accuracy with the help of kinematic calculations using the known states in ^{13}N, ^{16}F, and ^{26}Al as the calibration standard (for details, see Refs. [9,13]). These values agree within errors with those given in Ref. [18]. The widths Γ and J^π values are from Refs. [9,18]. The counts of individual peaks were obtained by peak fitting using the shape of the well separated $J^\pi = 7/2^+$ GT state at 1.613 MeV as standard. Obvious broadening is seen only for peaks at and above 6.117 MeV. These states were fitted with the standard shape folded with the Breit-Wigner function representing the decay width.

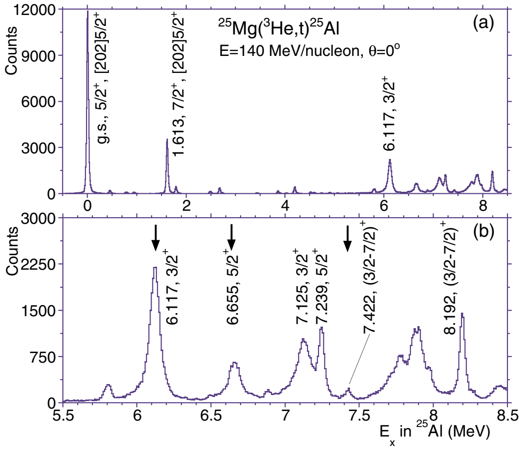

FIG. 1 (color online). The ^{25}Mg(^{3}He, t)^{25}Al reaction spectra of (a) the range up to the excitation energy of 8.5 MeV and (b) expanded 5.5–8.5 MeV region. The members of the new rotational band are indicated by arrows.

The B(GT) value of 0.165 ± 0.007 obtained in the β decay from the ^{25}Al g.s. to the 1.612 MeV state of ^{25}Mg was used for the B(GT) calibration. Assuming the symmetry of transitions in $A = 25$ mirror nuclei, this value was assigned to the analogous transition to the 1.613 MeV state in ^{25}Al in the ^{25}Mg(^{3}He, t) reaction. The B(GT) values for other excited GT states were determined by using their proportionality to the counts of peaks at 0°. The kinematical effect as a function of excitation energy was corrected by using the result of a distorted-wave Born approximation calculation (for details, see Ref. [9]).

It is known that the low-lying states in ^{25}Mg and ^{25}Al form rotational bands based on Nilsson orbits. Band structures shown by full lines in Fig. 2 were proposed for ^{25}Al [2,7,8], and similar ones for the mirror nucleus ^{25}Mg. It was suggested that the ground states of these nuclei have almost pure [2 0 2]5/2 nature [2].

An interesting feature of the ^{25}Al spectrum shown in Fig. 1(a) is that only two strong states are seen up to $E_x = 6$ MeV. All other states are very weakly excited although several GT states were reported [18]. It was found [9] that most of these weak states are the members of $K^\pi = 1/2^+$ bands. Therefore, the transitions from the g.s. with $K^\pi = 5/2^+$ are of $\Delta K = 2$ nature and are strongly suppressed. The two strong low-lying states are the members of the $\Delta K = 0$ g.s. band. This fact shows that $A = 25$ nuclei are well deformed and have an axially symmetric shape with K a good quantum number. The first pronounced peak above $E_x = 6$ MeV is the 6.117 MeV, $J^\pi = 3/2^+$ state, which is the strongest even in the region up to $E_x = 8.5$ MeV [see Fig. 1(b)]. Exactly the same feature was reported for the mirror nucleus ^{25}Mg in the (e, e') reaction at 180° [19], which also have the $\Delta K = 1$ selectivity. Above the two low-lying states the first pronounced state, corresponding to our 6.117 MeV state, was at 5.77 MeV (5.747 MeV in the latest compilation [18]). They suggested that this 5.77 MeV state and the twice stronger state at 7.03 MeV may be the $J^\pi = 3/2^+$ and $5/2^+$ members of the [2 0 2]3/2 band, respectively. As will be discussed, the $5/2^+$ member should be weaker, and thus the 7.03 MeV state cannot be the member. Here, the important fact is that the clear observations of both the 6.117 MeV, $J^\pi = 3/2^+$ state in the (^{3}He, t) reaction and its isobaric analog state in the (e, e') reaction suggest that they are the band-head state of an allowed $K^\pi = 3/2^+$ band.

Assuming prolate deformation, a simple picture for the g.s. of ^{25}Mg is that the [2 2 0]1/2 and [2 1 1]3/2 configurations are filled with two protons and two neutrons,

TABLE I. Clearly observed $J^\pi = 3/2^+$, $5/2^+$, and $7/2^+$ states in ^{25}Al and ^{25}Mg and the B(GT) and $B(M1)$ strengths to these states. The ratios $R_{\rm ISO}$ calculated from these $B(M1)$ and B(GT) values are given, where $R_{\rm MEC} = 1.25$ is assumed. Excitation energies are in units of MeV, and the $B(M1)$ values are in units of μ_N^2.

States in ^{25}Al				States in ^{25}Mg						
		(^{3}He, t)				γ decay			(e, e')	
E_x	$2J^\pi$	B(GT)	Γ (keV)	E_x	$B(M1)\uparrow$	$B^R(M1)$	$R_{\rm ISO}$	$B(M1)\uparrow$	$B^R(M1)$	$R_{\rm ISO}$
0.0	5^+	0.408(2)[a]		0.0						
1.613	7^+	0.165(7)[a]		1.612	0.83(12)	0.63(9)	3.1(8)	1.2(3)	0.87(22)	4.2(11)
6.117	3^+	0.217(18)	51(2)	5.747	0.11(5)	0.08(4)	0.30(16)	0.27(13)	0.20(10)	0.7(4)
6.655	5^+	0.083(12)	58(9)							
7.125	3^+	0.136(12)	117(4)	7.03				0.55(25)	0.41(19)	
7.239	5^+	0.056(5)	19(4)							
7.422	$(3\text{–}7)^+$	0.006(3)								

[a] B(GT) value from β-decay measurement.

while the last odd neutron occupies the [2 0 2]5/2 configuration. There are three possible configurations with $K^\pi = 3/2^+$ that can be excited via simple single-step processes in the intermediate-energy (^{3}He, t) reaction. These are the ν[2 1 1]3/2 hole band made from the ν[2 1 1]3/2 → π[2 0 2]5/2 transition, the π[2 0 2]3/2 particle band from the ν[2 0 2]5/2 → π[2 0 2]3/2 transition, and the band caused by the ν[2 1 1]3/2 → π[2 1 1]1/2 transition. In the third case, the two configurations form $K^\pi = 1^+$ at first and then $K^\pi = 3/2^+$ by coupling with the ν[2 0 2]5/2 configuration. In order to distinguish these three candidates, we use the fact that the GT transitions are caused by the spin operator, while analogous $M1$ transitions are caused by both the spin and orbital operators that can interfere differently depending on the configurations.

The matrix elements for the orbital operators $\ell_\pm$ and spin operators $s_\pm$ can be calculated by using the wave functions given in Table 5-9 of Ref. [2], where a deformation $\delta = 0.4$ is assumed. For the single-particle transition from the [2 0 2]5/2 to [2 0 2]3/2 configurations, we get

$$\langle 2\ 0\ 2\ 3/2 | \ell_- | 2\ 0\ 2\ 5/2 \rangle = -0.447, \tag{1}$$

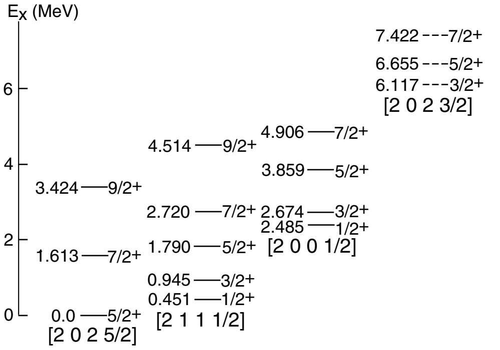

FIG. 2. The band structures of the low-lying positive-parity states in ^{25}Al shown by full lines [2], and the states of the newly identified [2 0 2]3/2 band (dotted lines). Each state is denoted by its excitation energy (in MeV) and J^π value.

$$\langle 2\ 0\ 2\ 3/2 | s_- | 2\ 0\ 2\ 5/2 \rangle = 0.975. \tag{2}$$

Having different signs of the orbital and spin terms means that they interfere destructively in $M1$ transitions, since the g factors for the orbital and spin operators $(g_\ell - g_R)$ and $(g_s - g_R)$ have the same sign, positive for protons and negative for neutrons (see, e.g., Ref. [13]). On the other hand, in the transitions from the [2 1 1]3/2 to the [2 0 2]5/2 and [2 1 1]1/2 configurations, we get, respectively,

$$\langle 2\ 0\ 2\ 5/2 | \ell_+ | 2\ 1\ 1\ 3/2 \rangle = 1.950, \tag{3}$$

$$\langle 2\ 0\ 2\ 5/2 | s_+ | 2\ 1\ 1\ 3/2 \rangle = 0.224, \tag{4}$$

and

$$\langle 2\ 1\ 1\ 1/2 | \ell_- | 2\ 1\ 1\ 3/2 \rangle = 0.282, \tag{5}$$

$$\langle 2\ 1\ 1\ 1/2 | s_- | 2\ 1\ 1\ 3/2 \rangle = 0.884. \tag{6}$$

Having the same sign of the orbital and spin terms means that they interfere constructively in $M1$ transitions. For reference, the ℓ_z and s_z matrix elements for the g.s. band have the same sign and values of 2.0 and 0.5, respectively.

The experimental B(GT) and $B(M1)\uparrow$ values are compared in Table I, where the unit and the coupling constant of the isovector spin term in $B(M1)\uparrow$ are renormalized in the $B^R(M1)$ values [13,20] so that it can be compared directly with the B(GT) values. The value $R_{\rm ISO}$ is defined by $(1/R_{\rm MEC})[B^R(M1)/B({\rm GT})]$, which is larger than unity if the spin and orbital terms in the $M1$ transition are constructive and vice versa [20]. The ratio $R_{\rm MEC}$ shows different contributions of the meson exchange currents in the GT and $M1$ transitions and its value is about 1.25 for sd shell nuclei [20,21]. Both the γ-decay data [18] and the (e, e') data [19] show constructive interference ($R_{\rm ISO} > 1$) for the $M1$ transition to the member of the g.s. band at 1.612 MeV in ^{25}Mg, as expected from the same sign of the orbital and spin matrix elements. On the other hand, a destructive interference ($R_{\rm ISO} < 1$) is seen for the $M1$ transition to the $J^\pi = 3/2^+$, 5.747 MeV state. Therefore, this state and its analog state at 6.117 MeV in ^{25}Al are most probably the bandhead state of the [2 0 2]3/2, $K^\pi = 3/2^+$ rotational band.

In the GT transitions from the $J_1 = 5/2$ state of the $K_1 = 5/2$ rotational band to the J_2 members of the $K_2 = 3/2$ rotational band, the B(GT) values are proportional to the squared values of the Clebsch-Gordan (CG) coefficient $(J_1\ K_1\ 1\ K_2 - K_1|J_2\ K_2)$ [see Eq. (4-91) of Ref. [2]], where the squared values are 2/3, 2/7, and 1/21 for the transitions to the $J_2 = 3/2$, 5/2, and 7/2 states, respectively. On this base, we assign the 6.655 MeV state as the $5/2^+$ member. The experimental ratio of 2.6 ± 0.3 of the B(GT) values for the 6.117 MeV, $J = 3/2$ state and the 6.655 MeV, $J = 5/2$ state is in agreement with the expected ratio of $(2/3)/(2/7) = 2.33$. In addition, they have a similar width Γ (see Table I).

A very small B(GT) value ≈ 0.016 is expected for the $J = 7/2$ member from the ratio of the CG coefficients. Therefore, we use the energy systematics of the rotational spectrum $E(K, J)$ for the assignment of the $J = 7/2$ member. A simplest form with a single-particle energy E_K and a moment of inertia $\mathcal{I}$

$$E(K, J) = E_K + AJ(J + 1) = E_K + \frac{\hbar^2}{2\mathcal{I}} J(J + 1) \quad (7)$$

gives $A = \hbar^2/2\mathcal{I} = 0.108$ MeV for the [2 0 2]3/2 band from the observed energies of the $J = 3/2$ and 5/2 states. By using Eq. (7), we expect the $J = 7/2$ member at $E_x = 7.41$ MeV. The only candidate in our spectrum is the weakly but clearly excited 7.422 MeV state with possible J^π values of $(3 - 7)^+$ [18]. It has a B(GT) value of 0.006 and a width similar to those of the other band members at 6.117 and 6.655 MeV.

The wave functions given in Table 5-9 of Ref. [2] allow one to calculate the theoretical B(GT) values for $\delta = 0.4$. The so-called "quenching" of the GT strength is a common phenomenon [22], and it is of interest how much strength is observed experimentally. For the transitions to the $K^\pi = 5/2^+$ band members (g.s. and 1.613 MeV states), we observe $\approx 58\%$ of the expected strengths, which corresponds to a quenching factor of 0.76. This is in agreement with the widely accepted value of 0.76 ± 0.03 [23]. On the other hand, the transitions to the newly identified $K^\pi = 3/2^+$ band in the higher E_x region are much suppressed. A quenching factor of 0.40–0.43 was obtained.

In summary, we have identified the [2 0 2]3/2 orbit, the highest-lying orbit among the six Nilsson orbits in the prolately deformed sd-shell nuclei, nearly 50 years after its prediction. The bandhead state of the orbit was observed at $E_x = 6.117$ MeV in ^{25}Al by the charge-exchange reaction ^{25}Mg(^{3}He, t) at 0°. The high resolution allowed to obtain B(GT) values for members of the band. The ratios of B(GT) values among the members and the reduction of the analogous $M1$ transition strength compared to the GT strength were the decisive elements of the identification. The J and K selectivities of the reaction also played an important role in the identification.

The ^{25}Mg(^{3}He, t)^{25}Al experiment was performed at RCNP, Osaka University under the Experimental Program E158. The authors are grateful to the accelerator group of RCNP for providing a high-quality ^{3}He beam. They also thank the target laboratory of GSI, Darmstadt, for preparing a thin ^{25}Mg target.

---

*Email address: fujita@rcnp.osaka-u.ac.jp
†Present address: JAERI, Tokai, Ibaraki 319-1195, Japan.
‡Present address: Department of Physics, Kyushu University, Higashi, Fukuoka 812-8581, Japan.

[1] S. G. Nilsson, Mat. Fys. Medd. Dan. Vid. Selsk. **29**, No. 16 (1955).
[2] A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2, and references therein.
[3] B. R. Mottelson and S. G. Nilsson, Mat. Fys. Skr. Dan. Vid. Selsk. **1**, No. 8 (1959).
[4] T. Motobayashi *et al.*, Phys. Lett. B **346**, 9 (1995).
[5] G. Klotz *et al.*, Phys. Rev. C **47**, 2502 (1993).
[6] P. Raghavan, At. Data Nucl. Data Tables **42**, 189 (1989).
[7] A. E. Litherland *et al.*, Can. J. Phys. **36**, 378 (1958).
[8] F. Heidinger *et al.*, Z. Phys. A **338**, 23 (1991).
[9] Y. Shimbara *et al.*, Eur. Phys. J. A **19**, 25 (2004).
[10] H. Fujita *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **484**, 17 (2002).
[11] Y. Fujita *et al.*, Phys. Rev. C **59**, 90 (1999).
[12] Y. Fujita *et al.*, Phys. Rev. C **67**, 064312 (2003).
[13] Y. Fujita *et al.*, Phys. Rev. C **66**, 044313 (2002).
[14] T. Wakasa *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **482**, 79 (2002).
[15] M. Fujiwara *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **422**, 484 (1999).
[16] Y. Fujita *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. B **126**, 274 (1997), and references therein.
[17] H. Fujita *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **469**, 55 (2001).
[18] P. M. Endt, Nucl. Phys. **A521**, 1 (1990); **A633**, 1 (1998), and references therein.
[19] L.W. Fagg *et al.*, Phys. Rev. **187**, 1384 (1969).
[20] Y. Fujita *et al.*, Phys. Rev. C **62**, 044314 (2000), and references therein.
[21] A. Richter *et al.*, Phys. Rev. Lett. **65**, 2519 (1990).
[22] F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992), and references therein.
[23] B. A. Brown and B. H. Wildenthal, At. Data Nucl. Data Tables **33**, 347 (1985).