Competition-Driven Network Dynamics: Emergence of a Scale-Free Leadership Structure and Collective Efficiency

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Using the minority game as a model for competition dynamics, we investigate the effects of interagent communications across a network on the global evolution of the game. Agent communication across this network leads to the formation of an influence network, which is dynamically coupled to the evolution of the game, and it is responsible for the information flow driving the agents' actions. We show that the influence network spontaneously develops hubs with a broad distribution of in-degrees, defining a scale-free robust leadership structure. Furthermore, in realistic parameter ranges, facilitated by information exchange on the network, agents can generate a high degree of cooperation making the collective almost maximally efficient.

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In a competitive environment with seriously limited resources, an individual is able to make the most gains if he avoids the crowds and finds strategies that place him into the distinguished class of the elites, or of the "few." Even though this class forms a *minority group* when compared to the whole agent society, it can largely influence the dynamics of the entire society for the simple reason that the elites hold the best strategies in the given situation, and thus they become key target nodes for others to communicate with and follow. For our purposes, an agent is a leader if at least one agent is following and, thus, acting on his advice. The influence of a leader is measured by the number of followers he has. Agents who are not leaders are simply coined "followers." However, leaders can follow other leaders, thereby creating a leadership structure. Certainly, the leadership structure, and even which particular agents are leaders at all, is often very dynamic (mostly because the success of a certain strategy is determined by the context of the strategies used by the other agents).

One of the most ubiquitous mechanisms guiding people in deciding whom or what to follow is reinforcement learning [1], which is a mechanism for statistical inference created through repeated interactions with the environment. For example, in iterated situations or games, it can be argued that we all monitor our social circle and "score" our acquaintances, including ourselves, based on past performance (success measure). We then take more seriously, and often follow, those with a higher score (success rate) [2].

In order to study the scenario described above, in this Letter we use a well known multiagent model of competition, the minority game (MG) [3–5], which we modify to include interagent communications or influences across a social network. The two main questions we address here are as follows: (1) What type of leadership structure is generated? (2) Can the effects of interagent communications aggregate up to the level of the collective and affect its behavior?

The original MG is an abstraction of a market played by agents with bounded rationality, inspired by the El Farol bar problem introduced by Arthur [6]. In this iterated game, at every step, N agents must choose between two different options, symbolized by A and B, e.g., "buy" and "sell." Only agents in the minority group get a reward. The agents have access to global information, which is the identity of the minority group for the past m rounds. Each agent bases his choice on a set of S strategies available to them. A strategy, which is an agent's "way of thinking," is a prediction [6] for outcome A or B, in response to *all* possible histories of length m. The strategies are distributed randomly among agents, and thus, in general, each agent has a different set of S strategies. Each agent makes his next choice in the game using reinforcement learning: every agent keeps a score for each of the S strategies which he then increments by one each round if that strategy correctly predicted the minority outcome (regardless of usage). The strategy used to make the new choice is the one with the best score up to that time. If two or more strategies share the best score, then one of those strategies is picked randomly. Previously, the effects of local information in the MG were studied both with the reinforcement learning type [7] and the nonreinforcement learning type [8–10] of agent communication mechanisms on Kauffman networks [11] and with the nonreinforcement learning type of mechanisms on linear chains [9,10].

In our model, a social network of agents is described by a graph with vertices representing the agents, and edges representing acquaintanceship between pairs of agents.

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This network of acquaintances forms the substrate network (**G**), or skeleton for interagent communications [2,7–10,12]. An edge ab in **G** means that agents a and b may exchange game-relevant information. However, it does not indicate whether the exchanges influence the action by any of the involved agents. That information is modeled by a second network, the influence network (**F**), which is a directed subset of **G**, and in which an edge ab, pointing from a to b, means that agent a *acts* on the advice of agent b when deciding the minority choice. In the competitive environment of the stock market, Kullman, Kertész, and Kaski, by studying time-dependent cross correlations have recently shown the existence of such a directed network of influence among companies (**F**) based on data taken from the New York Stock Exchange [13]. We do not, in general, know the precise topology of the social networks. However, it is known that social networks have a small-world character [14–16]. Here we take **G** to be an Erdős-Rényi (ER) random graph with link probability p. An ER random graph shows the small-world effect, since the diameter of the graph increases only logarithmically with the number of vertices [17], and the nodes also have a well defined average degree, pN, which results from cognitive limitation [16]. Studies using other types of network topologies, which are more suited to describe social networks (one drawback of ER is its low clustering coefficient [14]) will be presented in future publications. Just as in the original MG, in our model, in order to make his next decision, each agent uses his best performing strategy to predict what the next minority choice will be. However, he does not necessarily act on that prediction. Instead, the prediction simply constitutes the agent's opinion, which he then shares with all his first neighbors on the substrate network **G**. This is done by all agents simultaneously, and thus every agent obtains as information the predictions of all his first neighbors. Each agent then uses this information to make his final choice, via reinforcement learning, and each keeps scores of the prediction performance of all his first neighbors and himself and updates the scores after every round by incrementing the scores of the agents whose prediction was correct. Each agent then acts on the prediction or opinion of the neighboring agent with the highest score. Of course, if one has a higher score than any of his neighbors, then he acts on his own prediction.

The game is initialized by fixing at random S strategies for each agent, an arbitrary initial history string, and a fixed instance of the substrate network **G**. After initial transients (which we took to be 10^5 iterations), the game evolution becomes insensitive to the particular initial history string for the average quantities presented below. However, it may remain sensitive to the disorder in the strategy space of the NS strategies that are used, and to the disorder associated with the particular social network chosen. Thus, there are four relevant parameters in this game: N, S, m, and $p \in [0, 1]$. Of course, in reality the substrate network can also change (we make new friends and others fade away). However, we assume its dynamics to be much slower than that of **F**, and therefore it is neglected here. As defined previously, an agent i is a leader if it has at least one follower, j, and thus agent j follows through action what agent i suggests. For this to happen, i has to have the largest prediction score among the acquaintances of j, which are defined as the k_j edges j has in **G**. In an ER graph, the number of k_j links has a Poisson distribution with an average value at $\lambda = pN$, and an exponential tail. An agent j follows only one agent's opinion to decide his action, and thus its number of outlinks is always one, $k_j^{(\mathrm{out})} = 1$. However, the number of inlinks for agent j, $k_j^{(\mathrm{in})}$, can be any number between 0 and k_j, according to the number of agents acting on his advice. Figure 1 shows the in-degree distribution for various numbers of agents N, network connectivity p, and memory length m. These values were obtained after averaging over both network and strategy space disorders. The first striking observation from Fig. 1(a) is that over a wide range of parameters the inlink distribution is described by a power law with a sharp cutoff. Thus, the average number of leaders with k followers, N_k, is a scale-free distribution [18]. This happens in spite of the fact that the

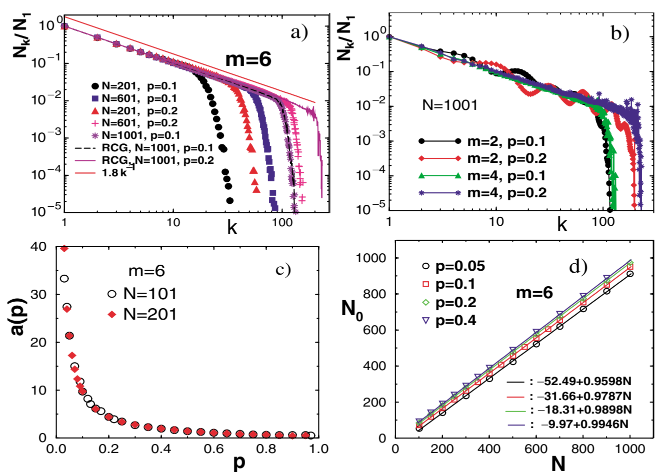

FIG. 1 (color online). Leaders and followers. (a),(b) The average of the number of leaders with k followers normalized by the average number of leaders with exactly one follower N_1. The symbols correspond to varying system sizes and link probabilities, $p = 0.1$ and $p = 0.2$, respectively, while the dashed and thin continuous lines correspond to the same quantity for the random choice game on the ER substrate. Next to the curves, the thick continuous line has a slope of -1. (b) The same quantity for small memories, $m = 2$ and $m = 4$, with $S = 2$ for $p = 0.1$ and $p = 0.2$. The curves oscillate around the same $1/k$ law. For all curves in (a) and (b) the averages were taken over 17 runs, which was sufficient, because of the fast convergence of the quantities. (c) $a(p) \equiv N_1$ with good approximation is independent of the system size N. (d) The number of followers as a function of the system size N. For all cases $S = 2$.

substrate network, which is an ER graph is not a scale-free network, and therefore it was not introduced *a priori* into the underlying structure. The scale-free character of the influence network **F** is selected by the reinforcement learning nature of the agent-agent interaction rules. The fact that a broad scale-free structure is selected on the back of a Poisson distributed network seriously limits the size of the leadership. Indeed, Fig. 1(d), which shows the nonleaders, or followers, expresses this fact: the pure followers constitute over 90% of the population for the cases presented in Fig. 1(a).

Plotting N_k/N_1, all the curves can be collapsed in the scaling regime up to their cutoffs, indicating that $N_k(N, m; p) \propto k^{-\beta} N_1(N, m; p)$. The power of the decay, β is very close to unity, which means that kN_k is independent of k and the other parameters in the scaling regime. Since k is the influence of a leader with k followers, kN_k represents the total influence of the kth layer in the leadership hierarchy. The above observation therefore means that all layers of the hierarchy are equally influential; influence is evenly distributed among all levels of the leadership hierarchy. This result is robust and insensitive to the particular parameters m, p, and N, for N large (>50) and $pN > 5$, even in the low m (memory) regime. Here, however, oscillations build up around the $1/k$ behavior which still serves as a backbone for the leadership structure, but it becomes less obvious as m is decreased; see Fig. 1(b). Another important observation is that $N_1(N, m; p)$ depends strongly only on p and not on N or m, thus $N_1(N, m; p) = a(p)$, as shown in Fig. 1(c). Therefore, we have

$$N_k(N, m; p) = k^{-\beta} a(p) f_k(N, m; p). \tag{1}$$

The fact that $N_1(N, m; p)$ is virtually independent of N means that if the number of agents is increased, the leadership structure and size *in the scaling regime* will not change. What changes though, is the number of the "sheep" or followers, which is N_0. It grows in proportion to N, as seen in Fig. 1(d). Also, the cutoff at the high-k end of the distribution occurs at larger k as N is increased. The deviation of the function $f_k(N, m; p)$ from a constant accounts for the fluctuations in the leadership structure which vanish (the fluctuations) with increasing m. This is because of the fact that the strategy space suffers a combinatorial explosion as m is increased (there are in total 2^{2^m} strategies), and the agents' strategies therefore become highly uncorrelated [3–5].

This suggests that the results for large m can be reproduced if the agents simply play a random choice game (RCG) on the network. In a RCG, agents do not use strategies, but instead only toss a coin when making predictions. Indeed, Fig. 1(a) shows that the RCG on the ER network produces the same scale-free backbone of the leadership structure. Thus, in our model the closeness to the scale-free backbone is determined by the level of mutual decorrelation of agents' strategies. This is to say that increased trait diversity (strategy space) leads to stable scale-free leadership structure.

Although the leadership structure is stable for large m, the position of an individual agent in the leadership hierarchy is not. By computing the time correlations present in the number of inlinks, we can show that the average lifetime of an agent in a particular leadership position is short for large m, as detailed in Ref. [19]. In contrast, at low m values, leaders become frozen in their positions. In other words, in the low m regime, where trait diversity is small, as in a dictatorship, where agents' action space is severely limited, leaders "live" longer in their positions.

Next, we briefly study the global performance of the collective on the network. Consider choice A as the reference option, and denote by $A(t)$ the attendance, or the number of agents choosing option A at time t. One of the most frequently used measures for a "world utility" function for the collective [20] is the variance σ of the fluctuations in the time series of $A(t)$. In the language of economics, it is the volatility of the market, and from a systems design point of view [20], it is the quantity that we ultimately want to minimize.

As mentioned before, this game has two types of quenched disorder embedded into it. A natural question then is if one can find or evolve networks that achieve zero or almost zero volatility given a group and its strategies or, alternatively, if one can find strategies that achieve zero, or near zero volatility, given a particular substrate network. To answer this question, we performed simple random searches in one of the quenched disorder spaces (network or strategy), keeping the other quench disorder fixed (strategy or network). An example with $m = 2$ and $m = 8$ is displayed in Fig. 2(a) as a function of connectivity p. The first conclusion is that overall, the collective does worse with "smart agents" (large m) on highly connected networks if they exchange information about their strategies. However, in the low m regime ($m = 2$), the system efficiency can improve not only beyond that of the standard MG, but also beyond that of the RCG without network (line labeled RCG), and even beyond the standard MG's best performance (which is at a different value of $m = 6$ for these parameters). Thus, a networked, low trait diversity system can be more effective as a collective than a sophisticated group. Note that the optimal p values are still much larger than the critical value for the giant component in the ER network, which is $1/N$, and thus we need well connected single component graphs in order to observe the collective efficiency emerge from the agent-agent interactions. However, the optimal values are actually in the realistic range for social networks, giving for the average number of contacts $\lambda = pN \simeq 10$–20. If N is varied, the optimum range for p shifts such that the optimum value of pN remains constant. Figure 2(b) shows a sample time series from the

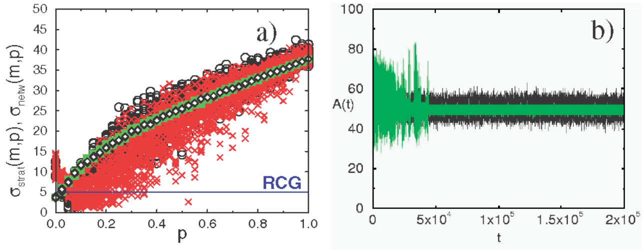

FIG. 2 (color online). Collective efficiency. (a) The time-averaged volatility (over 5×10^5 steps) of the market as a function of the substrate network connectivity parameter, p. The empty circles ($m = 2$) and the solid squares ($m = 8$) are obtained by fixing the strategy space disorder and taking randomly 50 network samples, while the crosses ($m = 2$) and the diamonds ($m = 8$) are obtained with the network space disorder fixed for 50 strategy disorders. Here $S = 2$ and $N = 101$. (b) A sample time series for one of the low lying points in (a) at $p = 0.1$, $m = 2$. The black time series corresponds to a run for the ordinary MG at minimum volatility which is at $m = 6$, $S = 2$. The black curve has a variance of 2.36, while the other curve has a variance of 1.07.

optimal connectivity region. Notice the low volatility compared to the best performance of the MG (in the background). In the standard MG the variations in σ at the best performance point are low, and even an extended search (average 500 samples) in the strategy disorder space could not generate σs lower than 2.0, while, in contrast, time series such as the one in Fig. 2(b) are easily generated in average within 50 random samples in the optimal connectivity region. This emerging collective efficiency can be understood in terms of the crowd-anticrowd description of the MG, as introduced by Johnson, Hart, and Hui [5]. In the MG, low m means that only a small number of different strategies are possible; thus, many agents are forced to use the same strategy and behave as a crowd or a group. This grouping effect generates the large volatility in the ordinary MG. When the game is played on a network, however, an agent, even if he shares the same strategy as the others in a large group, now has the possibility to listen to some other agents, and possibly even other groups. Thus, he is no longer forced to behave the same way as his own group, thereby breaking the grouping behavior. If, however, p is too large, there is a grouping behavior appearing due to the network, because an agent has too many followers if his score is the highest, creating a group on the network. The two crowding effects compete and a balance between them is reached in the optimum connectivity region.

In summary, we have shown that the evolution of multiagent games can strongly depend on the nature of the agent's information resources, including local information gathered on the social network, a network whose structure in turn is influenced by the fate of the game itself. In our study, we allowed for this dynamic coupling between the game and the network by using reinforcement learning as an ubiquitous mechanism for interagent communications. Our observations are as follows: (1) If reinforcement learning is used, a scale-free leadership structure can be created, even on the backbone of non-scale free networks. (2) In low trait diversity collectives, enhanced collective efficiency may appear, making this effect worthwhile for systems design studies [20].

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