

## Effect of Measurement on the Periodicity of the Coulomb Staircase of a Superconducting Box

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We report on the effect of the backaction of a single Cooper pair transistor electrometer (**E**) on the charge state of a superconducting box (**B**). The charge is  $e$  periodic in the gate bias of **B** when **E** is operated near voltages  $2\Delta/e$  or  $4\Delta/e$ . We show that this is due to quasiparticle poisoning of **B** at a rate proportional to the number of quasiparticle tunneling events in **E** per second. We are able to eliminate this backaction and recover  $2e$ -charge periodicity using a new measurement method based on switching-current modulation of **E**.

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The superconducting box, e.g., a small island of Al film very weakly coupled to the outside circuitry by Josephson junctions, has shown considerable promise as a qubit for quantum information processing where the two states can be represented by superpositions of 0 or 1 excess Cooper pairs in the box [1–4]. Measurement of the quantum state of this so-called charge qubit without inducing unwanted decoherence is a significant problem as is quasiparticle poisoning, i.e., the introduction of an unpaired electron (quasiparticle) into the box. At temperatures of 10 mK or so, where experiments are commonly done, the number of quasiparticles should, in principle, be negligible. However, such quasiparticle poisoning, due perhaps to the measurement process itself, is commonly observed. A manifestation of this is seen in the so-called Coulomb staircase. When a charge,  $q_{g,B}$ , is capacitively induced on the box, one expects Cooper pairs to tunnel resonantly into or out of the box at  $q_{g,B} = n_o e$ , where  $n_o$  is an odd integer, to maintain the lowest energy charge state of the box. This results in the Coulomb staircase of the charge in the box  $Q_B(q_{g,B})$  with period  $2e$  in  $q_{g,B}$ . On the other hand, if there are quasiparticles in the system, then maintaining the lowest charging energy state also leads to quasiparticle tunneling. This gives rise to splitting of the steps in the Coulomb staircase, which shifts toward  $e$  periodicity as the number of quasiparticles increases [5]. As a result, the lowest energy state of the box at  $q_{g,B} = n_o e$  no longer corresponds to a resonant state of the Cooper pair tunneling. For the box qubit, this means that relaxation does not bring the system back to its computational ground state at its operating point. Since the ability to prepare the initial state of the qubit is an absolutely necessary condition for quantum computing, quasiparticle poisoning has been a serious roadblock for groups working to build a charge qubit [6–8]. Solving this problem in charge qubits is essential.

The purpose of this Letter is to investigate the effects of measurement (i.e., backaction) on the measured charge in the box and to develop approaches to minimize these effects. For this study, we use two capacitively coupled single Cooper pair transistors (SCPTs), one of which acts

as an electrometer (**E**) and the other as a superconducting box (**B**). The latter gives a good representation of the box and at the same time allows us to study quasiparticle poisoning effects without needing to operate **E**. The SCPT electrometer can be operated in several different modes when measuring the charge of **B** [9,10]. Commonly, the charge measurements of **B** are done by operating **E** in the voltage modulation mode (VM). In this mode, **E** is biased at a sufficiently high voltage that quasiparticles are generated, so it effectively functions as a single electron transistor where the source drain voltage is modulated by  $q_{g,E}$  with a period of  $e$ . However, it is known that the switching current of an SCPT, i.e., the current at which it switches hysteretically from the low voltage or phase-diffusion branch to  $eV > 2\Delta$ , is also charge sensitive [6] and can be used for charge measurement. We refer to this as the switching-current mode (SW). The SW mode of operation has been analyzed [11], but until now no measurements of the charge on the island of a box using a SW mode of the electrometer have been reported. We present the results of the measurements of the island charge in **B** by **E** operated in either the VM or SW mode. The results demonstrate that measurement-induced poisoning, which leads to an  $e$ -periodic Coulomb staircase when using **E** in the VM mode, can be eliminated in the SW mode.

The parameters of the sample [Fig. 1(a)] are as follows: **E** has a normal state resistance  $R_{n,E} = 61.4$  k $\Omega$ , charging energy  $E_{c,E} = 33$   $\mu$ eV, and Josephson energy  $E_{J,E} = 21.2$   $\mu$ eV. For **B** these parameters are  $R_{n,B} = 63.0$  k $\Omega$ ,  $E_{c,B} = 47$   $\mu$ eV, and  $E_{J,B} = 20.6$   $\mu$ eV.  $E_J$  is the average of the Josephson energies of two junctions as determined from the values of  $R_n$  and the superconducting gap  $\Delta \approx 200$   $\mu$ eV by using the Ambegaokar-Baratoff formula.  $E_c$  is determined from the amplitude of maximum voltage modulation of the devices. The coupling capacitance between **E** and **B** is determined to be 80 aF from their measured coupling of 4.8%. All devices are made using standard two angle shadow evaporations without having normal metal quasiparticle traps close to junctions. The sample is placed in a microwave tight copper can located

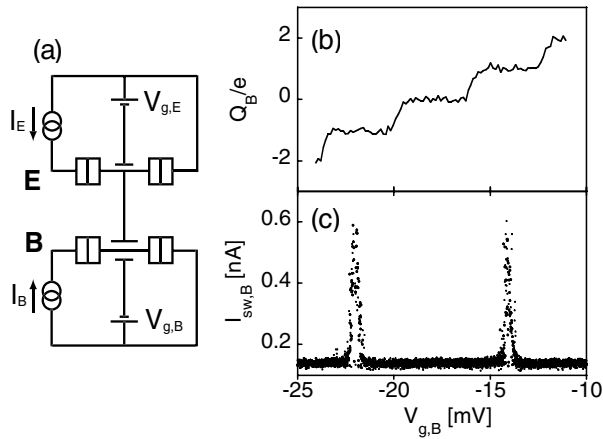


FIG. 1. (a) Schematic of the measurement showing the SCPT electrometer (**E**) capacitively coupled to the second SCPT (**B**). Tunnel junctions are represented by double box symbols. (b) Average charge of **B** as a function of its gate voltage when the electrometer operates in the VM mode. (c) Switching current of **B** as a function of its gate voltage. There is an undetermined shift in gate voltage between measurements in (b) and in (c) because of drift in the background charge.

on a temperature regulated stage of a dilution refrigerator having a base temperature of 6 mK. All the measurement leads are filtered by low temperature microwave filters [12] that are thermally anchored to the mixing chamber.

Figure 1(b) shows the charge on the island of **B** measured with **E** in the VM mode. During this measurement, the source and drain leads of **B** are at a common potential with respect to its gate. Figure 1(c) presents the switching-current modulation of **B** measured with the bias current through **E**,  $I_E$ , set equal to zero. In this and the following measurements, the bias current of **B**,  $I_B$ , is ramped at a rate of  $\approx 140$  nA/s. As can be seen, the switching-current modulation of **B** is  $2e$  periodic as expected at low temperature, but the charge of **B**, measured by the electrometer, is  $e$  periodic. Similar dependences of  $Q_B$  on  $q_{g,B}$  were measured with **E** biased in either of its voltage sensitive regions, i.e., near the gap where  $V_E \approx 4\Delta/e$  or near the Josephson-quasiparticle peak where  $V_E \approx 2\Delta/e$ . To determine if the  $e$  periodicity of  $Q_B(q_{g,B})$  is due to the backaction of **E** on **B**, the quasiparticle poisoning rate of **B**,  $\gamma_B$ , was measured for a range of bias conditions of **E**. In addition to this, we studied how the biases of two other SCPTs located on the same chip, but coupled more weakly to **B**, affected  $\gamma_B$ .

The details of the technique for determining  $\gamma_B$  from switching-current distributions have been reported previously [13]. Briefly, as  $I_B$  is linearly ramped in time, the switching-current histogram of **B**, with its gate biased near  $q_{g,B} = n_o e$ , exhibits two peaks if the number of quasiparticles on the island changes during the measurement of histogram. One peak,  $I_{\text{even}}$ , which is close to the maximum of the switching-current characteristic

$I_{\text{sw},B}(q_{g,B})$ , occurs when the island has an even number of electrons (even state) and the other—much lower current—peak,  $I_{\text{odd}}$ , is near the predicted switching-current minimum at  $q_{g,B} = 0$  when one quasiparticle occupies the island (odd state) (Fig. 2). If  $I_B > I_{\text{odd}}$  and **B** has not switched, it must be in the even state. The entry of a quasiparticle onto the island effectively changes  $q_{g,B} = e$  to  $q_{g,B} = 0$  which for  $I_B > I_{\text{odd}}$  will cause **B** to switch rapidly to the running state, giving the time of the poisoning and thus the quasiparticle poisoning rate  $\gamma_B$ . Previous studies have shown that  $\gamma_B$  is independent of  $I_B$  in the region between the peaks, giving an exponential decay of the even state. Several of these histograms with increasing  $\gamma_B$  as  $V_E$  and  $I_E$  increase are shown in Fig. 2. The relatively low bandwidth of our filters limits these measurements to  $\gamma_B < 10$  ms $^{-1}$ .

First we determine  $\gamma_B$  when all the electrodes of the electrometer are disconnected from the measurement circuitry and grounded. In this case we still observe a small residual rate  $\gamma_B^0 = 0.06$  ms $^{-1}$  for  $q_{g,B} \approx n_o e$ . This nonzero rate can be caused, e.g., by the presence of impurity levels in the superconducting gap of Al [5]. For the present discussion it is clear that this small residual rate is not related to the backaction of **E**.  $\gamma_B$  is unchanged if **E** is biased on its supercurrent branch or when **E** is biased on its return current branch at low voltage  $V_E < 200$   $\mu$ V  $\approx \Delta/e$  as shown in Fig. 3. For  $V_E > 200$   $\mu$ V,  $\gamma_B$  decreases rapidly, becoming too short to measure for slightly higher voltages. The modulation characteristics of the current  $I_E$  with gate voltage also changes at this point from being

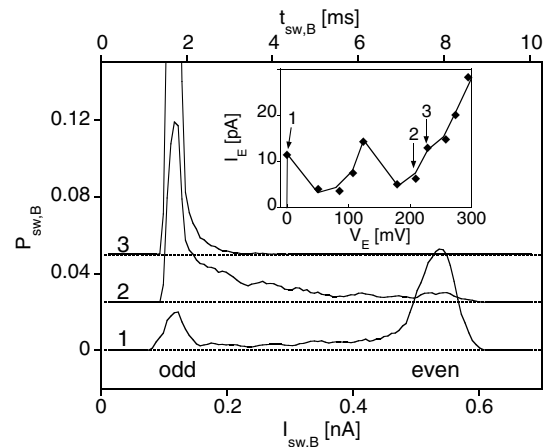


FIG. 2. Switching-current histograms of **B** measured for different bias conditions of **E**. Corresponding time delay from the beginning of the current ramp is shown in the upper horizontal axis. The histograms are shifted on the vertical axis for clarity. The “odd” (“even”) peaks in the histograms correspond to switching from the odd (even) parity states of **B**. Inset:  $I_V$  characteristic of **E**. Arrows indicate bias conditions at which switching-current histograms of the main figure were measured. Diamonds mark positions where  $\gamma_B$  has been measured for data in Fig. 3.

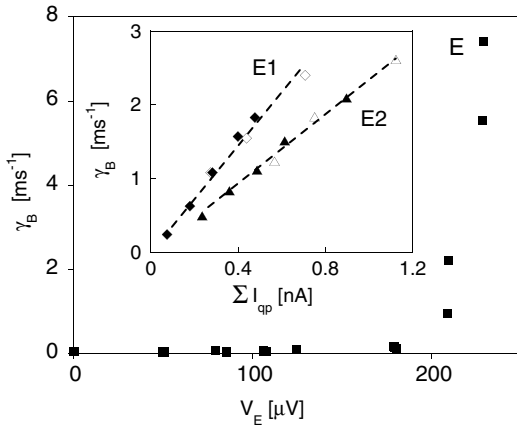


FIG. 3. Quasiparticle poisoning rate  $\gamma_B$  of **B** as a function of the voltage across **E**. Inset:  $\gamma_B$  as a function of the total quasiparticle tunneling current in **E1** (diamonds) and of **E2** (triangles). The open and filled symbols correspond to bias conditions  $V_{E1,E2} > 4\Delta/e$  or  $< 4\Delta/e$ , respectively.  $\sum I_{qp} = I_{E1,2}$  or  $2I_{E1,2}$  for the filled and open symbols, respectively.

$2e$  periodic for  $V_E < 200 \mu\text{V}$  to  $e$  periodic for  $V_E > 200 \mu\text{V}$ . Similar crossovers from  $e$  to  $2e$  periodicity at a voltage  $\Delta/e$  have previously been seen in related systems (see, e.g., [14]).

In order to study how  $\gamma_B$  depends on the voltage and current of **E** through its entire operating range, and, in particular, near  $V_E = 2\Delta/e$  and  $V_E = 4\Delta/e$ , we use two other SCPTs fabricated on the same chip but much more weakly coupled to **B**. Since these two SCPTs, which we call **E1** and **E2**, are much more weakly coupled to **B**, they allow us to measure  $\gamma_B$  for bias currents,  $I_{E1,2}$ , that are several orders of magnitude higher than is possible using **E**. The island of SCPT **E1** is located  $96 \mu\text{m}$  from the island of **B** and has  $R_n = 61.4 \text{ k}\Omega$ , while the corresponding parameters for **E2** are  $143 \mu\text{m}$  and  $R_n = 54.2 \text{ k}\Omega$ . For these devices, we can measure the rate  $\gamma_B$  up to voltages  $4\Delta/e$ . Again, we see a small initial increase of  $\gamma_B$  at  $V_E \approx 200 \mu\text{V}$  and then sharp increases at voltages  $V_E \approx 2\Delta/e$  and  $4\Delta/e$ . These voltages correspond approximately to the Josephson-quasiparticle tunneling and sequential quasiparticle tunneling thresholds in an SCPT and are accompanied by sharp increases in  $I_{E1,2}$ . The inset of Fig. 3 shows the rate  $\gamma_B$  as a function of the currents in **E1** and **E2** above these two thresholds. To test the hypothesis that  $\gamma_B$  is proportional to the total number of quasiparticle tunneling events, the currents for the electrometer voltages  $V_E > 4\Delta/e$  are multiplied by 2. This is done since, for a given current at voltages  $V_E > 4\Delta/e$ , there are twice as many quasiparticle tunneling events through the junctions of **E** as for  $V_E \geq 2\Delta/e$ . As one can see, this scaling collapses all of the data from each electrometer to a common line.

From these data one can conclude that the quasiparticle current of electrometer is the source of backaction noise leading to quasiparticle poisoning and an  $e$ -periodic

Coulomb staircase of **B** when measured by **E**. Further, the quasiparticle generation in **B** is proportional to the total number of quasiparticle tunneling events per second through **E**. This relationship could indicate that the backaction of **E** results from the shot noise of tunneling quasiparticles. On the other hand, this backaction could also be the result of the recombination of quasiparticles in **E** into pairs. This quasiparticle recombination produces phonons and to a smaller extent photons of energy  $\approx 2\Delta$ . These phonons or photons, which propagate from **E** to **B** without energy relaxation, could generate quasiparticles in **B**. Determining the details of the interaction between **E** and **B** will require further work. However, it is interesting to note that this sort of recombination noise would likely be suppressed by having normal metal leads close to the junctions. This may provide an explanation of previous results [15] in which it was possible to observe a  $2e$ -periodic Coulomb staircase using a VM electrometer measuring a superconducting box, which had normal metal “quasiparticle traps” close to the junctions.

Quasiparticle poisoning of **B** due to the measurement of its charge can, in principle, be eliminated by the operation of **E** in the SW mode, where its voltage remains well below  $\Delta/e$  until the measurement is made. The SW mode of operation of the electrometer is illustrated in the inset of Fig. 4, which shows its switching-current vs gate charge dependence. In general,  $q_{g,E}$  contains a component proportional to  $Q_B$ , in addition to the externally applied bias. So, to measure changes in  $Q_B$  as a function of  $q_{g,B}$ , we bias **E** near point 0, where the switching current is

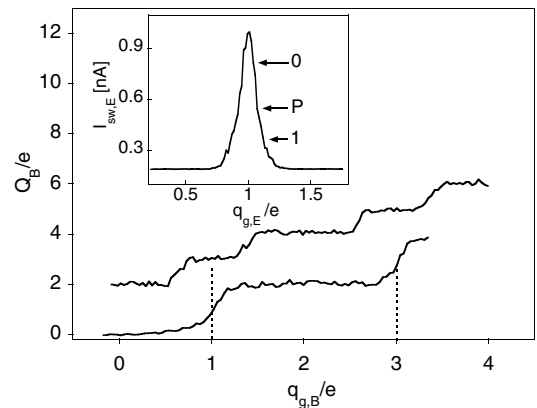


FIG. 4. Average charge of **B** as a function of its gate charge when the electrometer operates in the SW mode: bottom curve, quasiparticle flushing (see text) before each charge measurement; top curve, no flushing. The top curve is shifted in the vertical direction for clarity. The flushing pulse has amplitude  $150 \mu\text{V}$  and duration 25 ms. The dotted lines mark the positions at which the maxima of  $I_{sw,B}(q_{g,B})$  appear. The inset shows the average switching current of **E** as a function of its gate charge. The arrows marked by “0,” “1,” and “P” correspond to switching currents of **E** when the island of **B** has 0 or 1 excess Cooper pairs or when it is in the poisoned state, respectively.

very sensitive to variations of external charge (inset of Fig. 4) and record several hundred switching events of  $\mathbf{E}$  for each value of  $q_{g,B}$ . These switching data are then corrected for the measured nonlinearity in the transfer function of  $\mathbf{E}$  and averaged. The top curve in Fig. 4 shows the result obtained using this procedure.

While these data are no longer strictly  $e$  periodic, as in Fig. 1(b), the Coulomb staircase in Fig. 4 (top line) still shows the split steps around  $q_{g,B} = n_o e$  characteristic of quasiparticle poisoning. We can see that this is consistent with the measured residual rate,  $\gamma_B^0$ , as follows. The edges of these steps should occur for an energy difference between the even and odd states,  $\Delta E(q_{g,B})$ , such that the rates for even to odd ( $\gamma_B^0$ ) and odd to even ( $\bar{\gamma}_B^0$ ) transitions of the island are equal. This ratio is given by  $\gamma_B^0/\bar{\gamma}_B^0 = D_e/D_o \exp[\Delta E(q_{g,B})/kT]$  [16], where  $D_e$  is the sum of the quasiparticle densities on the leads and the island in the even state. For the odd state, this sum is  $D_o = D_e + V_i^{-1}$ , where  $V_i$  is the volume of the island. The second term in  $D_o$  accounts for the extra quasiparticle occupying the island in the odd state, which increases the density by  $V_i^{-1} \approx 100 \mu\text{m}^{-3}$ .  $D_e$  can be estimated from the measured residual poisoning rate  $\gamma_B^0$ , from the normal density of states of the Al film, and from the junction resistances, giving  $D_e \approx 4 \times 10^{-3} \mu\text{m}^{-3}$ . Taking the electron temperature of  $\mathbf{B}$  to be 15 mK, which is reasonable since  $\mathbf{B}$  is completely passive in these measurements, gives a length for the short step of  $0.76e$  in agreement with the data shown in Fig. 4.

The effects which result from the residual rate  $\gamma_B^0$  can be greatly reduced by flushing the quasiparticle from the island of  $\mathbf{B}$  before each measurement. As one possible approach to prepare the even parity state, we apply a voltage pulse  $V_B$  across  $\mathbf{B}$  just prior to each measurement. The amplitude of  $V_B$  is chosen such that  $2E_{c,B} < V_B e < \Delta$ , in order to release the quasiparticle from the electrostatic potential of the island but yet not to generate any new quasiparticles by the pulse [17]. Switching histograms of  $\mathbf{B}$  show that this procedure prepares the even state with a probability of about 85%. Immediately after  $\mathbf{B}$  is flushed, the measurement ramp of  $I_E$  begins. The result of this procedure is shown in the bottom curve in Fig. 4. As one can see, the quasiparticle-induced splitting of the step at  $q_{g,B} = n_o e$  is no longer apparent. However, individual histograms still show about 30% of the switching events in  $\mathbf{E}$  near  $q_{g,B} = n_o e$  are from the poisoned state of  $\mathbf{B}$ . This is consistent with the imperfect preparation of the initial state and additional poisoning with rate  $\gamma_B^0$  in the finite time between flushing and the switching event. Thus we see that, with the effects of residual poisoning greatly reduced by the flushing, the measurement of  $Q_B$  by  $\mathbf{E}$  in the SW mode gives results

consistent with the  $2e$ -periodic switching-current distribution of  $\mathbf{B}$ .

In conclusion, our measurements clearly show that operation of an SCPT electrometer at voltages  $V_E > 200 \mu\text{V}$  ( $\approx \Delta/e$ ) causes a substantial generation of quasiparticles in the circuit of the superconducting box leading to an  $e$ -periodic Coulomb staircase. The rate of quasiparticle poisoning in the box depends linearly on the total number of quasiparticle tunneling events per second through the junctions of the electrometer. To overcome this backaction from the electrometer, we operate it in a mode which uses switching-current modulation for charge detection. Using this mode of operation, we are able to recover the  $2e$ -periodic Coulomb staircase of the SCPT in the box configuration, which is expected both theoretically and from the  $2e$  periodicity of its switching current.

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