Fragment-Asperity Interaction Model for Earthquakes

Oscar Sotolongo-Costa[1,2,4] and A. Posadas[2,3,4]

[1]*Department of Theoretical Physics, Havana University, Habana 10400, Cuba*
[2]*Department of Applied Physics, Almería University, Almería, Spain*
[3]*Andalusian Institute of Geophysics and Seismic Disasters Prevention, Almería University, Almería, Spain*
[4]*Henri Poincaré Division of Complex System, Havana University, Havana 10400, Cuba*

(Received 7 June 2003; published 28 January 2004)

A model for earthquake dynamics consisting of two rough profiles interacting via fragments filling the gap is introduced, the fragments being produced by the local breakage due to the interaction of the local plates. The irregularities of the fault planes can interact with the fragments between them to develop a mechanism for triggering earthquakes. The fragment size distribution function comes from a nonextensive formulation, starting from first principles. An energy distribution function, which gives the Gutenberg-Richter law as a particular case, is analytically deduced.

DOI: 10.1103/PhysRevLett.92.048501 PACS numbers: 91.30.Bi, 89.75.Da, 91.30.Dk

The Gutenberg-Richter (GR) law has been considered as a paradigm of manifestation of self-organized criticality since the dependence of the cumulative number of earthquakes with energy, i.e., the number of earthquakes with energy greater than E, $N(E)$, behaves as a power law:

$$N(E) \sim E^{-b}, \tag{1}$$

where b is a critical exponent. A great number of studies have been originated as a result of this law, where the importance of the knowledge of the energy distribution of the earthquakes and the physical and practical implications are emphasized [1].

Some famous models such as those of Burridge and Knopoff [2] or Olami *et al.* [3] have focused on the mechanical phenomenology of earthquakes through simple images which capture essential aspects of the nature and genesis of a seism: the relative displacement of tectonic plates or the relative motion of the hanging wall and footwall on a fault and also the existence of a threshold for a catastrophic release of energy in the system.

Today it is widely accepted that most earthquakes are originated by relative motion of fault planes, whereas the images to model this energy release are diverse.

The standard picture usually assigns the cause of an earthquake to some kind of rupture or some stick-slip mechanism in which the friction properties of the fault play the determinant role. A review of these viewpoints and some generated paradoxes can be found in [4].

The GR law accounts for the seismicity generated in large geographic areas, usually identified as "seismic regions," covering many geological faults. This is a statistical law—it does not say anything about a specific earthquake, but it stems from direct measurements and has not previously been connected with general principles in physics.

In [2,3,5] the GR law was computationally reproduced, showing that a stick-slip mechanism in a single fault representation is adequate to model the seismicity generated in such large areas characterized by a diversity of fault sizes and depths, changes in mineral composition, volcanic activity, etc.

The influence of the fault profiles in the characteristics of earthquakes have been highlighted in [5] using a geometric viewpoint to simulate the power law dependence of the earthquake energy distribution (GR law) in what represents one of the most important contributions to the consideration of the role of the geometry of the fault profiles in earthquake dynamics.

The energy release in the formation of the faults due to the breakage of the earth crust can be manifested in seismicity in the moment of fault formation, so it makes sense to consider that the size distribution of faults in a seismic region (which has been studied in [6,7]) must play a role in the regional distribution of seismicity, but here, following the same idea as [2,3,5], we focus on the mechanism of relative displacement of fault plates, which is recognized as a main cause of earthquakes.

In our picture we incorporate the fact that the space between faults is filled with the residues of the breakage of the tectonic plates, from where the faults originated. In this respect, the influence of the size distribution of these fragments in the energy distribution of earthquakes has been predicted. In [8] this fact has been highlighted.

The great pressure existent between the two fault plates is the main factor conditioning the complexity of the fragment-asperity interaction, where eventually the fragments may act as roll bearings as suggested in [9] and also as hindering entities of the relative motion of the plates until the growing stresses produce their liberation with the subsequent triggering of the earthquake. To get a realistic image of earthquakes, the role of the fragments in the phenomenon must be recognized.

Nevertheless, its quantitative treatment, up to now has not been considered, with the exception of [9] where the fragments filling the gap are considered as circular disk-shaped pieces which act like bearings filling the space between two planes. Though that image does not deal with the GR law it is useful to explain the curious fact that over very extended areas called "seismic gaps," two tectonic plates can creep on each other without producing either earthquakes or the amount of heat expected from the usual friction forces.

We present a more realistic approximation, considering that the surfaces of the tectonic plates are irregular and the space between them contains fragments of very diverse and irregular shapes. We present a "geometric" image involving the fragments and irregularities between the two fault plates with a fragment size distribution deduced from a nonextensive entropy formulation.

The irregularities of the fault planes can be combined with the distribution of fragments between them to develop a mechanism of triggering earthquakes. In this case, it is tempting to relate fragment size distribution function with the energy distribution of the earthquakes.

In geophysics, the GR law is more frequently expressed as a log-linear dependence between the number of earthquakes of a magnitude greater than a given one and the value of this magnitude. However, the graphical representation of this law for different catalogs reflects that for the smallest magnitudes the dependence is not fulfilled. It is usual to consider that this misalignment is due to the threshold of sensitivity of the instruments, and, therefore, the catalog is complete from a minimum value on, clearly detectable by the instruments, so that only those values larger than a given threshold are considered here.

On the other hand, for large magnitudes, the GR law also fails, which reveals the limitations of this empirical formula. The model we present here describes very well the energy distribution in all the clearly detectable range of magnitudes.

To start, let us consider the situation illustrated in Fig. 1: two irregular profiles are able to slip as shown in the figure. Their motion can be hindered not only by the overlapping of two irregularities of the profiles, but also by the eventual relative position of several fragments as illustrated in the figure between the points "a" and "b." Stress in the resulting structure accumulates until a displacement of one of the asperities, due to the displacement of the hindering fragment, or even its breakage in the point of contact with the fragment leads to a relative displacement of the fault planes of the order of the size of the hindering fragment "r."

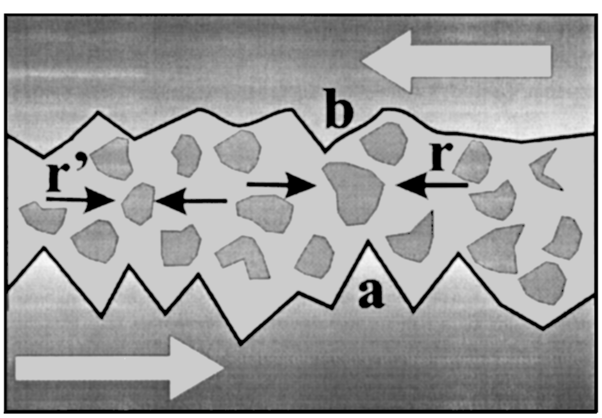

FIG. 1. An illustration of the relative motion of two irregular faults in the presence of material filling the space between them. Observe that this material may play the role of bearings or also of particles that hinder the relative motion of the plates as seen in the figure between points a and b.

It is natural to consider that the displacement of the fragments is more abundant than the breakage of asperities, so most of the earthquakes (though not all of them) may have their origin in that mechanism. Then, the eventual release of stress, whatever is the cause, leads to such a displacement with the subsequent liberation of energy. As large fragments are more difficult to release than small ones, we assume this energy "ε" to be proportional to r, so that the energy distribution of earthquakes generated by this mechanism can reflect the size distribution of the fragments between plates.

We can assume that the constant interaction and local breakage of the fault planes produce the fragments. The process of fault slip can be considered to occur in a homogeneous fashion in the depth of the fault so that in any plane transverse to the depth of the fault the situation is the same. Then, to deduce the size distribution function of the fragments, we consider a two-dimensional frame such as the one illustrated in Fig. 1. Our problem is, then, to find the distribution of fragments by area.

Here, we deduce the distribution starting from first principles, i.e., the maximum entropy formalism, from where we deduce the probability of finding a fragment of a given size.

The Boltzmann-Gibbs formulation in the maximum entropy principle proved to be useful in the study of the fragmentation phenomena [10], though in that work an important feature of fragmentation, i.e., the eventual presence of scaling in the size distribution of fragments was not obtained and the size distribution function there obtained does not fit well with all the experimental results.

The process of violent fractioning leads to the existence of long-range interactions among all parts of the object being fragmented. Then, fractioning is a paradigm of nonextensivity. This suggests that it may be necessary to use nonextensive statistics instead of the Boltzmann-Gibbs one to describe the size distribution function of the fragments.

We apply the maximum entropy principle for the Tsallis entropy [11], which has been proposed and successfully employed in a wide variety of situations where nonextensivity is the hallmark in all of them [12].

The Tsallis entropy for our problem has the form

$$S_q = k\frac{1 - \int p^q(\sigma)d\sigma}{q - 1}, \tag{2}$$

where $p(\sigma)$ is the probability of finding a fragment of relative surface σ referred to as a characteristic surface of the system, q is a real number, and k is Boltzmann's constant. It is easy to see that this entropy reduces to Boltzmann's when $q \rightarrow 1$. The sum by all states in the entropy is here expressed through the integration in all sizes of the fragments.

The maximum entropy formulation for Tsallis entropy involves the introduction of at least two constraints. The first one is the normalization of $p(\sigma)$:

$$\int_0^\infty p(\sigma)d\sigma = 1, \tag{3}$$

and the other is the *ad hoc* condition about the q-mean value, which in our case can be expressed as

$$\int_0^\infty \sigma p^q(\sigma)d\sigma = \langle\langle\sigma\rangle\rangle_q. \tag{4}$$

This condition reduces to the definition of the mean value when $q \rightarrow 1$. More information concerning the constraints that can be imposed in the formulation can be seen in [12,13]. The formulation can also be performed in terms of the escort probabilities, but the results in this case are not essentially different.

As we already said, fracture is a paradigm of such long-range interaction phenomenon, and recently our group gave a formulation in terms of Tsallis statistics with results that explain the experimental behavior of fragmentation phenomena [14]. Then the problem is to find the extreme of s_q/k subject to the conditions given by formulas (2) and (3). To simplify, we assume $\langle\langle\sigma\rangle\rangle_q = 1$ (we see that this has no effect on the final result).

The standard method of conditional extremization of the entropy functional leads to the expression of the fragment size distribution function:

$$p(\sigma)d\sigma = \frac{(2-q)^{1/(2-q)}d\sigma}{[1+(q-1)(2-q)^{(q-1)/(2-q)}\sigma]^{1/(q-1)}} \tag{5}$$

for the area distribution of the fragments of the fault plates.

If we now introduce the proportionality of the released relative energy ε with the linear dimension r of the fragments, as σ scales with r^2, the resulting expression for the energy distribution function of the earthquakes due to this mechanism is

$$p(\varepsilon)d\varepsilon = \frac{C_1\varepsilon d\varepsilon}{[1+C_2\varepsilon^2]^{1/(q-1)}}, \tag{6}$$

The probability of the energy $p(\varepsilon) = n(\varepsilon)/N$, where $n(\varepsilon)$ is the number of earthquakes of energy ε and N the total number of earthquakes. C_1 and C_2 are constants involving q and the proportionality constant between ε and r.

Hence, starting from first principles, in this case the extremization of the entropy functional, we have obtained an analytic expression for the energy distribution of earthquakes. No *ad hoc* hypothesis was introduced except the proportionality of ε and r, which seems justified. (Of course, a similar treatment can be performed with Boltzmann's entropy, and the inadequacy of the formulation in that case will become evident for the reasons we already explained.)

To use the common frequency-magnitude distribution, the cumulative number was calculated as the integral from ε and "∞" of the formula (6); then

$$\frac{N(\varepsilon >)}{N} = \int_\varepsilon^\infty p(\varepsilon)d\varepsilon, \tag{7}$$

where N is the total number of earthquakes and $N(\varepsilon >)$ the number of earthquakes with energy larger than ε. This rate defines the "excedence," i.e., the relative cumulative number, in this case applied to the earthquakes with energy larger than ε.

On the other hand, $m \propto \log(\varepsilon)$ where m is the magnitude, so we substitute formula (6) in (7) and change from the variable ε to m, so we get

$$\begin{aligned}\log(N(> m)) = \log N + \left(\frac{(2-q)}{1-q}\right) \\ \times \log[1 + a(q-1) \\ \times (2-q)^{(1-q)/(q-2)} \times 10^{2m}]. \end{aligned} \tag{8}$$

This is not a trivial result, and incorporates the characteristics of nonextensivity into the distribution of earthquakes by magnitude. a is the constant of proportionality between ε and r.

This is the simpler model we can make, since a proportionality of the energy with another power of r can be introduced in this representation, or the deduction of fragment sizes by volume instead of area can be calculated, but this leads only to a new constant to be adjusted or a new value of the nonextensivity constant as can be seen from the analysis of the process of deduction of formulas (6) and (7).

Figure 2 shows the application of this formula to the catalogs of the Iberian Peninsula (IGN, Spain, http://www.geo.ign.es/), Andalucía (IAG, Spain, http://www.ugr.es/iag/), and California earthquakes (NEIC, USA, http://neic.usgs.gov/neis/epic/). We are considering earthquakes with magnitude $m > 3$, i.e., detectables, and this is unbiased due to the sensitivity threshold of instruments. We compare the observed data with $G(> m) = N(> m)/N$, i.e., the excedence, defined as the relative cumulative number, which can be obtained from formula (8), to have a measure of the relative occurrence of earthquakes. As the total number of earthquakes, it was chosen the total number with magnitude greater than the threshold. There is a nice agreement of formula (8) with the data.

For the first time, a functional dependence was obtained for the distribution of earthquakes produced by interactions in the space between the fault planes, starting from first principles, i.e., a nonextensive formulation of the maximum entropy principle (the Tsallis formulation).

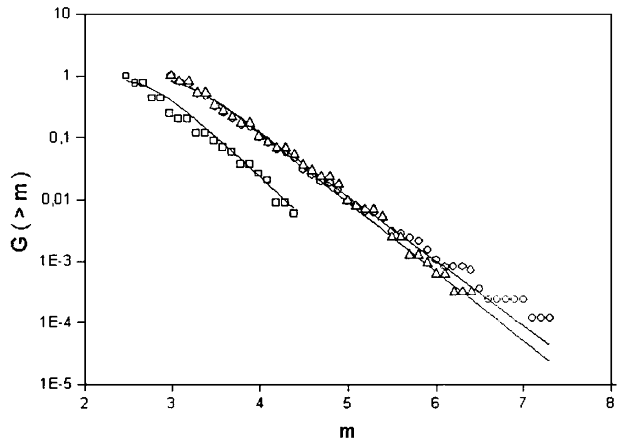

FIG. 2. We use formula (8) to calculate the relative cumulative number of earthquakes (excedence) to different catalogs: California (circles: over 10 000 earthquakes, $q = 1.65$, and $a = 5.73 \times 10^{-6}$), Iberian Peninsula (triangles: 3000 earthquakes, $q = 1.64$, and $a = 3.37 \times 10^{-6}$), and Andalusian region (squares: 300 earthquakes, $q = 1.60$, and $a = 3 \times 10^{-5}$).

The active role of the material between the fault planes was revealed with this model. Nonextensivity is, as can be seen, determinant to obtain the energy distribution of earthquakes in a wide energy range. No *a priori* assumption about the fault profile or shape of the fragments was needed.

The nice agreement with the data also expresses an advantage of this model. Other representations as that of [5] are computationals, reproduce only the region of power law behavior, and then cannot explain the observed data of so varied catalogs in a wide range of magnitudes. This model provides the same physics for all the scales. No particular explanations for different ranges of magnitudes are needed.

It is very interesting to observe the similarity in the value of the nonextensivity parameter q for all the catalogs used. Though intriguing to some extent, this reveals that the obtained formula is not a mere fitting artifact, and we think that a more exhaustive study of the nonextensive statistics and its relation with earthquakes is needed to give a deeper interpretation of this result. But, even if it were not so and the obtained expression could be interpreted by someone as a fitting tool, let us note that it is based on a physical image that recovers the main characteristics of earthquake dynamics, i.e., the interactions between plates and fragments in a more realistic representation than those proposed up to now.

This work was partially supported by the MCYT Projects No. REN2001-2418-c04-02/RIES and No. REN2002-04198-C02-02/RIES (University of Almería, Spain), by the European Community with FEDER, and by the research team of Geofisica Aplicada (RNM194) of Junta de Andalucía, Spain. One of us (O. S.) sincerely thanks Almería University for kind hospitality.

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