Entanglement Formation and Violation of Bell's Inequality with a Semiconductor Single Photon Source

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We report the generation of polarization-entangled photons, using a quantum dot single photon source, linear optics, and photodetectors. Two photons created independently are observed to violate Bell's inequality. The density matrix describing the polarization state of the postselected photon pairs is reconstructed and agrees well with a simple model predicting the quality of entanglement from the known parameters of the single photon source. Our scheme provides a method to create no more than one entangled photon pair per cycle after postselection, a feature useful to enhance quantum cryptography protocols based on shared entanglement.

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Entanglement, the nonlocal correlations allowed by quantum mechanics between distinct systems, has recently drawn much attention due to its applications to the manipulation of quantum information [1]. In the past, these nonlocal correlations were often understood as the result of prior interactions between the quantum mechanical systems, as a *memory* of those interactions. In the light of recent progress in the field of quantum information (see, e.g., the Innsbruck teleportation experiment [2]), this is too limited a view. In particular, entanglement can be induced between noninteracting particles, provided they are quantum mechanically indistinguishable. In this type of scheme, an auxiliary degree of freedom such as the particle number is measured, and the measurement result is used to take subsequent action. For instance, the experimental data can be *postselected* based on the "click" of particle detectors. Pionneering work by Shih and Alley [3], followed by Ou and Mandel [4], already used this postselection procedure to induce entanglement between two identical photons produced in a nonlinear crystal. More recently, entanglement swapping experiments [5,6] used two independent entangled photon pairs to induce entanglement between photons of different pairs which never interacted. In this Letter, we use a similar linear-optics technique to induce polarization entanglement between single photons emitted independently in a semiconductor quantum dot source, 2 ns apart. We observed a clear violation of Bell's inequality (BI), which constitutes an experimental proof of nonlocal behavior for the first time with a semiconductor single photon source. The complete density matrix describing the polarization state of the two photons was also reconstructed and satisfies the Peres criterion for entanglement [7]. We show that our results can be quantitatively explained in terms of basic parameters of the single photon source and derive a simple criterion for entanglement generation using those parameters. Eventually, we explain why our technique can be applied to quantum key distribution (QKD) in a straightforward and useful manner.

This experiment relies on two crucial features of our quantum dot single photon source, namely, its ability to suppress multiphoton pulses [8] and its ability to generate consecutively two photons that are quantum mechanically indistinguishable [9]. The idea is to "collide" these photons with orthogonal polarizations at two conjugated input ports of a nonpolarizing beam splitter (NPBS). A quantum interference effect ensures that photons simultaneously detected at different output ports of the NPBS should be entangled in polarization [4]. More precisely, when the two optical modes corresponding to the output ports c and d of the NPBS have a simultaneous single occupation, their joint polarization state is expected to be the EPR-Bell state:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_c|V\rangle_d - |V\rangle_c|H\rangle_d).$$

Denoting a and b the input port modes of the NPBS, they are related to the output modes c and d by the (50-50)% NPBS unitary matrix according to

$$a_{H/V} = \frac{1}{\sqrt{2}}(c_{H/V} + d_{H/V}),$$

$$b_{H/V} = \frac{1}{\sqrt{2}}(c_{H/V} - d_{H/V}),$$

where subscripts H and V specify the polarization (horizontal or vertical) of a given spatial mode. The quantum state corresponding to single-mode photons with orthogonal polarizations at ports a and b can be written as

$$a_H^\dagger b_V^\dagger|\text{vac}\rangle = \tfrac{1}{2}(c_H^\dagger c_V^\dagger - d_H^\dagger d_V^\dagger - c_H^\dagger d_V^\dagger + c_V^\dagger d_H^\dagger)|\text{vac}\rangle.$$

As pointed out in [10], this state already features nonlocal correlations and violates Bell's inequality without the need for postselection by using photodetectors that can distinguish photon numbers 0, 1, and 2. However, since the quantum efficiency of our source is too low (typically 0.1% to 2%) to implement such a "loophole-free" BI test, we implemented a simpler scheme using postselection based on the simultaneous click of two regular photon counter modules. If we discard the events when two photons go the same way (recording only coincidence events between modes c and d), we obtain the postselected state

$$\frac{1}{\sqrt{2}}(c_H^\dagger d_V^\dagger - c_V^\dagger d_H^\dagger)|\mathrm{vac}\rangle = |\Psi^-\rangle$$

with a probability of $\frac{1}{2}$.

The experimental setup is shown in Fig. 1. The single photon source consists of a self-assembled InAs quantum dot (QD) embedded in a GaAs/AlAs distributed Bragg reflector microcavity [9]. It was placed in a helium flow cryostat and cooled down to 4–10 K. Single photon emission was triggered by optical excitation of a single QD, isolated in a micropillar. We used 3 ps Ti:sapphire laser pulses on resonance with an excited state of the QD, ensuring fast creation of an electron-hole pair directly inside the QD. Pulses came by pairs separated by 2 ns, with a repetition rate of 1 pair/13 ns. The emitted photons were collected by a single-mode fiber and sent to a Mach-Zender–type setup with 2 ns delay on the longer arm. A quarter-wave plate followed by a half-wave plate (HWP) were used to set the polarization of the photons after the input fiber to linear and horizontal. An extra half-wave plate was inserted in the longer arm of the interferometer to rotate the polarization to vertical. One time out of four, the first emitted photon takes the long path while the second photon takes the short path, in which case their wave functions overlap at the second nonpolarizing beam splitter (NPBS 2). In all other cases (not of interest), the single photon pulses "miss" each other by at least 2 ns which is greater than their width (100–200 ps). Two single photon counter modules in a start-stop configuration were used to record coincidence counts between the two output ports of NPBS 2, effectively implementing the postselection (if photons exit NPBS 2 by the same port, then no coincidences are recorded by the detectors). Single-mode fibers were used prior to detection to facilitate the spatial mode-matching requirements. They were preceded by quarter-wave and polarizer plates to allow the analysis of all possible polarizations.

The detectors were linked to a time-to-amplitude converter, which allowed us to record histograms of coincidence events versus detection time delay τ. A typical histogram is shown in Fig. 2, with the corresponding postselected events. For given analyzer settings (α, β), we denote by $C(\alpha, \beta)$ the number of postselected events normalized by the total number of coincidences in a time window of 100 ns. This normalization is independent of (α, β) since the input of NPBS 2 is two modes with orthogonal polarizations. $C(\alpha, \beta)$ measures the average rate of coincidences throughout the time of integration.

Two different QD microcavity devices were used to produce single photons. The single count rate for QD 1 at the output of the single-mode fiber was 9400 counts/s, from which we infer a total quantum efficiency of 0.13%

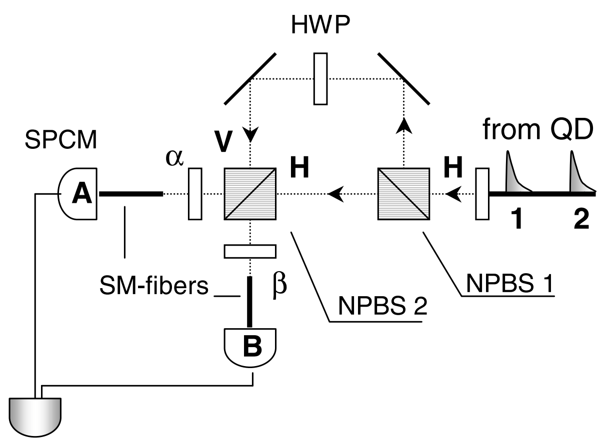

FIG. 1. Experimental setup. Single photons from the QD microcavity device are sent through a single mode fiber and have their polarization rotated to H. They are split by a first NPBS (1). The polarization is changed to V in the longer arm of the Mach-Zender configuration. The two paths of the interferometer merge at a second NPBS (2). The output modes of NPBS 2 are matched to single mode fibers for subsequent detection. The detectors are linked to a time-to-amplitude converter for a record of coincidence counts.

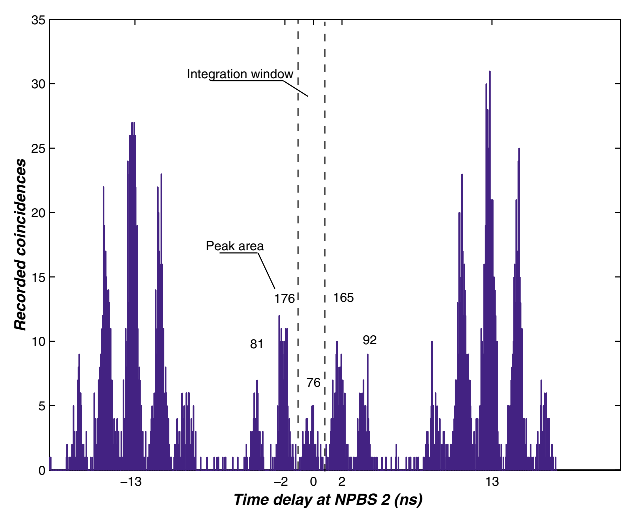

FIG. 2 (color online). Zoom on a typical correlation histogram, taken on QD 1. Coincidences with delay τ between detectors A and B were actually recorded for $-50\ \mathrm{ns} < \tau < 50\ \mathrm{ns}$. The integration time was 2 min, short enough to guarantee that the QD is illuminated by a constant pump power. The central region $-1\ \mathrm{ns} < \tau < 1\ \mathrm{ns}$ corresponds to the postselected events: the corresponding photons overlapped at NPBS 2 where they took different exit ports.

(detection loss included). The total pair production rate for QD 1 was 12/s after fiber, so that useful pairs were generated with a rate of 1.5/s (we loose a factor 8 due to the postselection and by excluding ''bad-timing'' events). Both QD 1 and QD 2 featured a high suppression of two-photon pulses and high overlap (indistinguishability) between consecutive photons. The overlap was measured by the Mandel dip [9], which was estimated by removing the HWP in the long arm, thus colliding completely identical particles at NPBS 2.

A BI test was performed for postselected photon pairs from QD 1. Following Ref. [11], if we define the correlation function $E(\alpha, \beta)$ for analyzer settings α and β as

$$E(\alpha, \beta) = \frac{C(\alpha, \beta) + C(\alpha^\perp, \beta^\perp) - C(\alpha^\perp, \beta) - C(\alpha, \beta^\perp)}{C(\alpha, \beta) + C(\alpha^\perp, \beta^\perp) + C(\alpha^\perp, \beta) + C(\alpha, \beta^\perp)},$$

then local realistic assumptions lead to the inequality

$$S = |E(\alpha, \beta) - E(\alpha', \beta)| + |E(\alpha', \beta') + E(\alpha, \beta')| \leq 2$$

that can be violated by quantum mechanics.

Sixteen measurements were performed for all combination of polarizer settings among $\alpha \in \{0°, 45°, 90°, 135°\}$ and $\beta \in \{22.5°, 67.5°, 112.5°, 157.5°\}$. The corresponding values of the normalized coincidence counts $C(\alpha, \beta)$ are reported in Table I. The statistical error on S is quite large, due to the short integration time used to ensure high stability of the QD device. Bell's inequality is still violated by 2 standard deviations, according to $S \sim 2.38 \pm 0.18$. Hence, nonlocal correlations were created between two single independent photons by linear optics and photon number postselection.

Complete information about the two-photon polarization state can be characterized by a reduced density matrix, where only the polarization degrees of freedom are kept. This density matrix can be reconstructed from a set of 16 measurements with different analyzer settings, including circular [12]. We performed this analysis, known as *quantum state tomography*, on photon pairs emitted by QD 2. The reconstructed density matrix is shown in Fig. 3. It can be shown to be nonseparable, i.e., entangled, using the Peres criterion [7] (negativity ~0.43, where a value of 1 means maximum entanglement).

We next try to account for the observed degree of entanglement from the parameters of the QD single photon source. Because of residual two-photon pulses from the source, giving a nonzero value to its equal time second-order correlation function $g^{(2)}(0)$ [8], a recorded coincidence count can originate from two photons of the same polarization that would have entered NPBS 2 from the same port. A multimode analysis also reveals that an imperfect overlap $V = |\int \psi_1(t)^* \psi_2(t)|^2$ between consecutive photon wave functions washes out the quantum interference responsible for the entanglement generation. Including those imperfections, we could derive a simple model for the joint polarization state of the postselected photons. In the limit of low pump level, this model predicts the following density matrix in the $(H/V) \otimes (H/V)$ basis:

$$\rho_{\text{model}} = \frac{1}{\frac{R}{T} + \frac{T}{R} + 4g^{(2)}} \begin{pmatrix} 2g^{(2)} & & & \\ & \frac{R}{T} & -V & \\ & -V & \frac{T}{R} & \\ & & & 2g^{(2)} \end{pmatrix}.$$

R and T are the reflection and transmission coefficients of NPBS 2 ($\frac{R}{T} \sim 1.1$ in our case). Using the values for $g^{(2)}$ and V measured independently, we obtain an excellent quantitative agreement of our model to the experimental

TABLE I. Normalized coincidences $C(\alpha, \beta) \times 10^3$ for various polarizer angles used in the BI test. They correspond to the coincidences in the integration window (see Fig. 2) divided by the total coincidences recorded for -50 ns $< \tau <$ 50 ns. Note that the quantity $C(\alpha, \beta) + C(\alpha^\perp, \beta^\perp) + C(\alpha^\perp, \beta) + C(\alpha, \beta^\perp)$ is constant for given settings α and β.

$\beta \backslash \alpha$	0°	45°	90°	135°
22.5°	5.6	28.4	28.6	4.7
67.5°	9.0	8.3	25.2	25.1
112.5°	28.9	5.4	4.6	28.4
157.5°	26.0	24.9	8.6	8.8

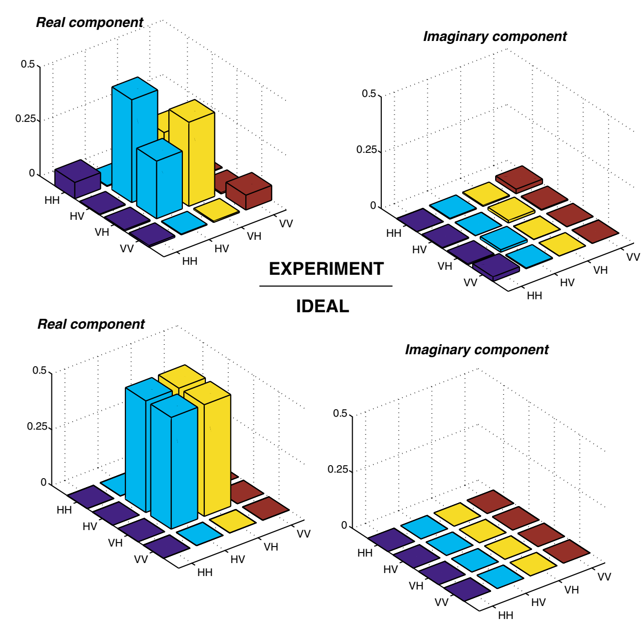

FIG. 3 (color online). Reconstructed polarization density matrix for the postselected photon pairs emitted by QD 2. The small diagonal HH and VV components are caused by finite two-photon pulses suppression ($g^{(2)} > 0$). Additional reduction of the off-diagonal elements originates from the imperfect indistinguishability between consecutively emitted photons.

data, with a fidelity $\mathrm{Tr}(\sqrt{\rho_{\exp}^{1/2}\rho_{\mathrm{model}}\rho_{\exp}^{1/2}})$ as high as 0.997. The negativity of the state ρ_{model} is proportional to $[V - 2g^{(2)}]$, which means that entanglement exists as long as $V > 2g^{(2)}$. This simple criterion indicates whether any given single photon source will be able to generate entangled photons in such a scheme.

Since the present experiment does not distinguish photon numbers 0, 1, 2, only half of the photon pairs colliding at NPBS 2 can be used for a BI test. However, following [10], it would be possible to design a loophole-free BI test by keeping track of photon numbers with existing single photon resolution detectors [13], if, however, the quantum efficiency of the single photon source could be made close to unity. The current scheme also does not allow the creation of an ''event-ready'' entangled photon pair. This is a serious obstacle for many applications to quantum information systems, but not all. The Ekert91 [14] or BBM92 [15] QKD protocols using entangled photons can directly be performed with our postselected technique. The essence of these protocols is to establish a secure key upon local measurement of two distant photons from an entangled pair, which is exactly similar to our scheme. The bit error induced by uncorrelated photon pairs in those protocols is significantly suppressed [16] when single entangled pairs are used, a feature which only our source possesses among the currently demonstrated entangled photon sources. Therefore, those QKD protocols should actually benefit from our method to generate entanglement.

As a final remark, we show how a photon number QND device (discriminating photon numbers 1 and 2) could be used to generate event-ready entangled photons deterministically in an improved version of the current scheme. If two photons are detected in the same output mode c or d of NPBS 2, then we mix those modes in another NPBS to obtain the entangled state $|\psi^+\rangle$, convertible to $|\psi^-\rangle$ with a half-wave plate. If only one photon is detected in a given mode, then we do nothing and obtain $|\psi^-\rangle$.

In summary, we demonstrated the violation of Bell's inequality for the first time with a semiconductor single photon source. Polarization entanglement was induced between two independent but indistinguishable single photons, with linear optics and post selection based on the click of regular photon counters. Our technique naturally produces no more than one entangled pair per cycle, which is a unique feature among previously demonstrated entangled photon sources. Our scheme can be straightforwardly applied to Ekert91/BBM92 QKD and, providing the efficiency of the single photon source can be increased, would perform better than current entangled photon sources for that purpose.

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