Two-Particle Aharonov-Bohm Effect and Entanglement in the Electronic Hanbury Brown–Twiss Setup

P. Samuelsson, E. V. Sukhorukov, and M. Büttiker

Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland
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We analyze a Hanbury Brown–Twiss geometry in which particles are injected from two independent sources into a mesoscopic conductor in the quantum Hall regime. All partial waves end in different reservoirs without generating any single-particle interference; in particular, there is no single-particle Aharonov-Bohm effect. However, exchange effects lead to two-particle Aharonov-Bohm oscillations in the zero-frequency current cross correlations. We demonstrate that this is related to two-particle orbital entanglement, detected via violation of a Bell inequality. The transport is along edge states and only adiabatic quantum point contacts and normal reservoirs are employed.

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Intensity correlations of photons became of interest with the invention by Hanbury Brown and Twiss (HBT) of an interferometer which permitted them to determine the angular diameter of visual stars [1]. The HBT effect contains two important distinct but fundamentally interrelated effects: First, light from different, completely uncorrelated portions of the star gives rise to an interference effect which is visible in intensity correlations but not in the intensities themselves. This is a property of two-particle exchange amplitudes. Exchange amplitudes are a quantum mechanical consequence of the indistinguishability of identical particles. Second, there is a direct statistical effect since photons bunch, whereas fermions antibunch. Fundamentally, both of these effects are related to the symmetry of the multiparticle wave function under exchange of two particles. For photons emitted by a thermal source, a classical wave field explanation of the HBT effect is possible. A quantum theory was put forth by Purcell [2]. For fermions, no classical wave theory is possible.

It has long been a dream to realize the electronic equivalent of the optical HBT experiment. This is difficult to achieve with field emission of electrons into vacuum because the effect is quadratic in the occupation numbers. This difficulty is absent in electrical conductors where at low temperatures a Fermi gas is completely degenerate. Experiments demonstrating fermionic antibunching in electrical conductors were reported by Oliver *et al.* [3], Henny *et al.* [4], and Oberholzer *et al.* [5]. Only very recently was a first experiment with a field emission source successful [6]. In contrast, to date, there is no experimental demonstration of two-electron interference.

In electrical conductors "beams" can be realized in high-magnetic fields in the form of edge states [7]. Edge channels permit the transport of electrons over (electronically) large distances. In the quantized Hall state [8] scattering out of an edge state is suppressed [9]. The second element needed to mimic optical geometries, the half-silvered mirror, is similarly available in the form of quantum point contacts (QPC's) [10,11]. Indeed, in

high-magnetic fields a QPC permits the separate measurement of transmitted and reflected carriers [12]. A Mach-Zehnder interferometer with edge states was recently realized [13]. This shows that it is possible to implement arrangements of linear optics [14] in electrical conductors.

Here we propose an implementation of the HBT experiment in an electrical conductor in the quantum Hall regime. In the setup, there is no single-particle interference; however, two-particle interference is manifested as a magnetic flux dependence of the current correlators, a two-particle Aharanov-Bohm effect. We show that this two-particle effect is closely related to orbital entanglement [15] of electron-hole pairs, recently proposed by Beenakker *et al.* [16], as well as of pairs of electrons. The entanglement is detected via a violation of a Bell inequality (BI). Only normal electronic reservoirs, adiabatic QPC's, and zero-frequency correlators are employed, greatly simplifying an experimental realization.

An optical configuration with two independent sources [17] is shown in Fig. 1. It is a tabletop equivalent to the
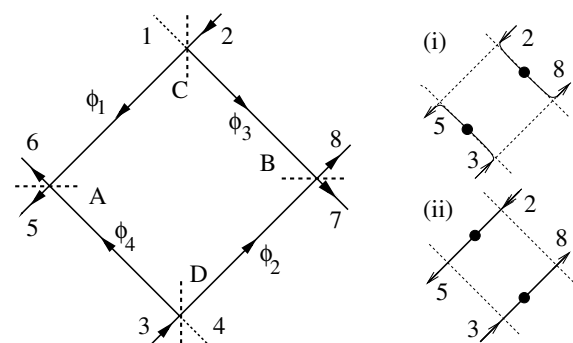

FIG. 1. Left: Two-source, four-detector optical Hanbury Brown–Twiss geometry. Two beams incident from 2 and 3 are split at the mirrors C and D and impinge on the mirrors A and B to reach detectors along the directions 5 to 8. Right: The two scattering processes (i) and (ii) contributing to the phase dependence of the correlators (see text).

stellar interferometer experiment of HBT. Figure 2 shows the implementation of this configuration in an electrical conductor. The geometry has no interfering orbits. Electron waves incident at the ith QPC (with $i = A$ to D) are transmitted with amplitude $\sqrt{T_i}$ and reflected with amplitude $\sqrt{R_i}$ with $T_i + R_i = 1$. Along the edge states electrons accumulate phases ϕ_1 to ϕ_4. The overall scattering behavior is determined by the global scattering matrix $s_{\alpha\gamma}$, which gives the current amplitude at contact α in terms of the current amplitude at the incident contact γ. Since there are no interfering orbits, the global scattering matrix elements depend only in a trivial way on the phases. A particle leaving source contact 2 can after transmission through QPC C reach either contact 5 or 6. For instance, the scattering matrix element $s_{52} = \sqrt{T_A} \times \exp(i\phi_1)\sqrt{T_C}$ and similarly for all other elements of the s matrix. Since the conductance matrix elements are determined by transmission probabilities, it follows immediately that in the setup of Fig. 2 all conductance matrix elements are phase insensitive. For instance, a voltage applied at contact 2 generates a current at contact 5, giving rise to a conductance $G_{52} = -(e^2/h)T_A T_C$. The conductances are thus determined only by products of transmission and reflection probabilities of the QPC's.

Let us now evaluate the current-current correlations for the geometry of Fig. 2. The zero-frequency cross correlations $S_{\alpha\beta}$ of the current fluctuations ΔI_α and ΔI_β are defined through
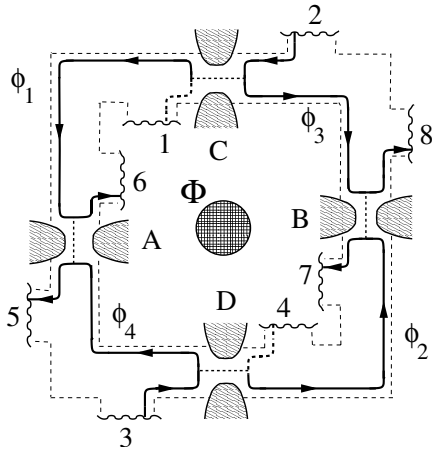

FIG. 2. Two-source, four-detector electrical Hanbury Brown–Twiss geometry: a rectangular Hall bar with inner and outer edges (thin dashed lines) and four quantum point contacts (grey shaded area). Contacts 2 and 3 are sources of electrons (a voltage eV is applied against all other contacts which are at ground). Electrons follow edge states (thick black lines) in the direction indicated by the arrows and pick up phases ϕ_1 to ϕ_4. An Aharanov-Bohm flux Φ penetrates the center of the sample (shaded area) but has no influence on the single particle properties. However, the current correlations are essentially dependent on the phases ϕ_i and the Aharonov-Bohm flux. We note that not all contacts have to be realized experimentally to investigate all physical phenomena discussed.

$$S_{\alpha\beta} = \int dt \langle [\Delta I_\alpha(t)\Delta I_\beta(0) + \Delta I_\beta(0)\Delta I_\alpha(t)] \rangle. \quad (1)$$

Containing two current operators, the correlator provides information about the two-particle properties of the system. Following the scattering approach to noise correlators in Ref. [12], the expression for the cross correlations (at contact $\alpha \in 5, 6$ and $\beta \in 7, 8$) is given in terms of the scattering amplitudes as

$$S_{\alpha\beta} = -(2e^2/h) \int dE |s_{\alpha 2}^* s_{\beta 2} + s_{\alpha 3}^* s_{\beta 3}|^2 (f - f_0)^2, \quad (2)$$

where f is the Fermi distribution function of reservoirs 2 and 3 (at a voltage bias eV) and f_0 is the distribution function of the other reservoirs (grounded). A corresponding calculation of the light-intensity cross correlations in the optical HBT geometry in Fig. 1 with thermal sources would give the same result as in Eq. (2) but with opposite sign, an effect of changing from fermionic to bosonic statistics of the carriers.

The basic scattering processes contributing to the correlator are clearly illustrated by considering the simplest case with transmission and reflection probabilities of all QPC's equal to $1/2$. The correlation function of the currents at, e.g., contact 5 and 8 is then, at zero temperature,

$$S_{58} = -(e^2/4h)|eV|[1 + \cos(\phi_1 + \phi_2 - \phi_3 - \phi_4)], \quad (3)$$

depending in an essential way on the phases ϕ_1 to ϕ_4 in Fig. 2. The phase dependent term results from the process where one particle is emitted from contact 2 and one from contact 3. Detecting one particle in 5 and one in 8, we can quantum mechanically not distinguish which paths the individual particles took, (i) from 2 to 8 and from 3 to 5 or (ii) from 2 to 5 and from 3 to 8 (see Fig. 1). As a consequence, the amplitudes $\exp[i(\phi_1 + \phi_2)]$ and $\exp[i(\phi_3 + \phi_4)]$ of the respective processes (i) and (ii) must be added, giving rise to the interference term $\cos(\phi_1 + \phi_2 - \phi_3 - \phi_4)$ in Eq. (3). We emphasize that it is the fact that both sources 2 and 3 are active that gives rise to this phase dependence. The phase independent term is the sum of the correlations that are obtained if only source 2 is active and if only source 3 is active.

In an experiment, it is possible to modulate the phases with the help of gates which lengthen or shorten the paths of the edge states in Fig. 2. The phases can as well be modulated with the help of an Aharonov-Bohm (AB) flux [18] through the center of the structure. While there are no single particle trajectories which coherently enclose the flux, the paths of two particles, one emitted by 2 and the other emitted by 3, have the possibility to enclose the flux. The flux contributes a positive phase to ϕ_1 and ϕ_2 and a negative phase to ϕ_3 and ϕ_4 to give a total additional phase contribution of $2\pi\Phi/\Phi_0 = \oint d\mathbf{l} \cdot \mathbf{A}$, where $\mathbf{A}$ is the vector potential and $\Phi_0 = h/e$ is the single charge flux quantum. The total phase in Eq. (3) is then $\phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0$. The possibility of such an AB effect due to two-particle exchange was recognized

in early work on noise [19,20] and in the cotunneling current [21]. The geometry of Fig. 2 is unique in that the conductances (second order interference) exhibit no AB effect but current correlators (fourth order interference) are sensitive to the variation of an AB flux.

Apart from this two-particle AB effect, is the opposite sign of the correlator in Eq. (2), resulting from the different statistics of the carriers, the only significant difference between the electronic and the photonic HBT setup? The answer is no. As we now show, due to the degeneracy of the electron sources at low temperatures, for strongly asymmetric source QPC's C and D in Fig. 2, an orbitally entangled [15] electron-hole pair [16] state is emitted from C and D. This has no counterpart in the optical HBT setup with thermal sources [22].

We take the transmission and reflection probabilities at the QPC C to be $T_C = 1 - R_C = T$ and at D to be $T_D = 1 - R_D = R$ (scattering probabilities at A and B are specified below). The many-body transport state generated by the two independent sources is in second quantization (suppressing the spin index) $|\Psi\rangle = \prod_{0 < E < eV} c_2^\dagger(E) c_3^\dagger(E) |0\rangle$, where $|0\rangle$ is the ground state, a filled Fermi sea in all reservoirs at energies $E < 0$. The operator $c_\gamma^\dagger(E)$ creates an injected electron from reservoir γ at energy E. After scattering at C and D, the state $|\Psi\rangle$ consists of two contributions in which the two particles fly off, one to A and one to B, and of two contributions in which the two particles fly off, both towards the same detector QPC.

Consider now the case of strong asymmetry $R \ll 1$, where almost no electrons are passing through the source QPC's towards B. We can write the full state $|\Psi\rangle$ to leading order in $\sqrt{R}$ as $|\Psi\rangle = |\bar{0}\rangle + \sqrt{R}|\tilde{\Psi}\rangle$, with

$$|\tilde{\Psi}\rangle = \int_0^{eV} dE[c_{3B}^\dagger(E)c_{3A}(E) - c_{2B}^\dagger(E)c_{2A}(E)]|\bar{0}\rangle. \quad (4)$$

The second index of the electron operators, A or B, denotes towards which detector the particle is propagating. Here we have redefined the vacuum to be the completely filled stream of electrons, $|\bar{0}\rangle = \prod_{0 < E < eV} c_{2A}^\dagger(E)c_{3A}^\dagger(E)|0\rangle$. The operators $c_{3A}(E)$ and $c_{2A}(E)$ describe hole excitations, i.e., the removal of an electron at energy E from the filled stream. Because of the redefinition of the vacuum [15], we can interpret the resulting state $|\tilde{\Psi}\rangle$ as describing a superposition of "wave packet"-like electron-hole pair excitations out of the new vacuum, i.e., an orbitally entangled pair of electron-hole excitations. This is equivalent to the recent findings by Beenakker *et al.* [16], who discussed the generation of entangled electron-hole pairs at a single nonadiabatic QPC. The state is similar to the two-electron state emitted from a superconductor contacted at two different points in space [15,23]. The new vacuum $|\bar{0}\rangle$ is noiseless and does not contribute to the cross correlators; even so, it carries a current from the sources to the detectors. The cross-correlation measurement is thus sensitive only to $|\tilde{\Psi}\rangle$, the entangled electron-hole pair state.

Following our earlier work [15,24], this two-particle orbital entanglement can be detected via violation of a Bell inequality [25]. In the limit $R \ll 1$, the time between emission of successive electron-hole pairs $h/(eVR)$ is much larger than the coherence time $\tau_C = h/eV$ of each pair. As a consequence, the zero-frequency noise measurement works as a coincidence measurement running over a long time. The BI can then be expressed directly in terms of the zero-frequency cross correlators.

Interestingly, the electron-hole entanglement is not the only feature of the electric HBT setup which has no counterpart in optics [22]. The antibunching of electrons implies that no two electrons can be emitted simultaneously from a single reservoir. As a consequence, electrons emitted from a single source cannot be detected simultaneously in reservoirs α and β. Only the process where one electron is emitted from 2 and one from 3 (shown in Fig. 1) can lead to a joint detection in α and β. As discussed above, since the paths of these two particles cannot be distinguished, the corresponding state is orbitally entangled. The total state emitted from the contacts C and D expressed in terms of electrons is, however, a product state. Thus, the process of jointly detecting one particle in α and one in β means effectively postselecting [26] a pair of orbitally entangled electrons by the measurement.

This postselected entanglement can be detected by a violation of a BI formulated in terms of the joint detection probability of two electrons. In optics, using photodetectors, the joint probability of detecting two photons is given by the theory of Glauber [27]. In close analogy with Ref. [27] we define the probability of simultaneous detection (at energy $0 < E < eV$) of one electron in detector α and one in β as

$$P_{\alpha\beta} \propto \langle c_\beta^\dagger(t) c_\alpha^\dagger(t) c_\alpha(t) c_\beta(t) \rangle, \quad (5)$$

where $c_\alpha^\dagger(t) = \int dE \exp(iEt/\hbar) c_\alpha^\dagger(E)$. The probabilities are normalized such that $\sum P_{\alpha\beta} = 1$. In mesoscopic systems, $P_{\alpha\beta}$ is difficult to measure directly. However, as a nonlocal quantum mechanical correlator, it provides information about the entanglement of two spatially separated particles in a many-particle system.

Since $c_\alpha(t)$ and $c_\beta^\dagger(t)$ anticommute, for the setup in Fig. 2, $P_{\alpha\beta} \propto \langle I_\alpha(t) I_\beta(t) \rangle$ and we find

$$P_{\alpha\beta} \propto S_{\alpha\beta} + 2\tau_C I_\alpha I_\beta, \quad (6)$$

where $I_\alpha = (e^2/h)TV$ and $I_\beta = (e^2/h)RV$ are the currents flowing into reservoirs α and β and $\tau_C = h/eV$ the coherence time. The zero-frequency correlator $S_{\alpha\beta}$ is investigated by varying the transmission through the two QPC's A and B which precede the detector reservoirs. This is similar to schemes in optics where one varies the transmission to the detectors with the help of polarizers. The transmission and reflection probabilities through the detector QPC's are taken to be $T_A = 1 - R_A = \sin^2\theta_A$ for A and with $\theta_A \to \theta_B$ for B. Then Eq. (2) gives

$$S_{58} = -\frac{2e^2}{h}|eV|RT[\sin^2\theta_A \sin^2\theta_B + \cos^2\theta_A \cos^2\theta_B$$
$$+ 2\cos(\phi_0)\cos\theta_A \cos\theta_B \sin\theta_A \sin\theta_B],$$

$$(7)$$

with $S_{67} = S_{58}$ and $S_{57} = S_{68}$ obtained from S_{58} by shifting $\theta_A \rightarrow \theta_A + \pi/2$. Here the phase $\phi_0 = \phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0$.

The BI, following Ref. [28], is expressed in terms of correlation functions

$$E(\theta_A, \theta_B) = P_{58} + P_{67} - P_{57} - P_{68}$$
$$= \cos(2\theta_A)\cos(2\theta_B)$$
$$+ \cos(\phi_0)\sin(2\theta_A)\sin(2\theta_B). \quad (8)$$

The BI is $-2 \le S_B \le 2$, where the Bell parameter $S_B = E(\theta_A, \theta_B) - E(\theta'_A, \theta_B) + E(\theta'_A, \theta_B) + E(\theta'_A, \theta'_B)$, with θ_A, θ'_A, θ_B, and θ'_B four different measurement angles. Optimizing the angles, the maximum Bell parameter is given by [15] $S_B^{max} = 2\sqrt{1 + \cos^2(\phi_0)}$; i.e., the BI can be violated for any $\cos(\phi_0)$. We note that for the electron-hole pair entanglement discussed above, the same maximum Bell parameter is obtained for the BI expressed directly in terms of zero-frequency correlators. Dephasing due to, e.g., a fluctuating AB phase or phases ϕ_i, renormalizes $\cos(\phi_0) \rightarrow \gamma\cos(\phi_0)$, eventually suppressing [29] the entanglement for strong dephasing $\gamma \rightarrow 0$. However, the BI can still be violated for arbitrary strong dephasing. We note that γ is just the visibility of the two-particle AB oscillations. This shows the strong connection between the orbital entanglement and the two-particle AB effect.

We emphasize that in contrast to the electronic HBT setup, the BI cannot be violated in the setup in Fig. 1 with thermal optical sources. The reason for this is that the bunching of photons in the thermal sources allows for two photons to be emitted simultaneously from a single source. These additional two-photon scattering processes, not present in the electronic case, are uncorrelated, i.e., not entangled. Formally, a calculation of the joint detection probability [30] gives the same result as in Eq. (6). However, $S_{\alpha\beta}$ has opposite sign and as a consequence, the correlation function in Eq. (8) should be multiplied with $1/3$, making a violation impossible.

We have treated only the case of integer quantum Hall states. The fractional quantum Hall effect offers a wider, very interesting area for the examination of correlations [31] since in this case fractional statistics is realized.

In conclusion, we have demonstrated the connection between the Hanbury Brown–Twiss effect, the two-particle Aharanov-Bohm effect, and orbital entanglement. The simple adiabatic edge-state geometry described above and the use of zero-frequency correlators brings experimental tests of these effects within reach.

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