Effect of Collisional Zonal-Flow Damping on Flux-Driven Turbulent Transport

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The effect of collisional damping of zonal flows (ZFs) on ion-temperature gradient (ITG) driven turbulence in a toroidal plasma is investigated by means of a 3D global fluid model with flux boundary conditions. Results from simulations show an increase of the energy confinement time and a stabilization of turbulence with the inverse of the collisionality ν_*. The stabilization mechanism is identified as an effect of the increased shearing rate of ZFs, which shift upwards the ITG turbulence effective threshold. The shearing rate of ZFs is also seen to depend on the injected power. As a consequence, the effective heat conductivity depends parametrically on the input power.

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A crucial issue for theoretical predictions of tokamak plasma performance is the origin of the empirical dependence of transport on the various dimensionless parameters controlling the confinement. Whereas many studies have been dedicated to the dependence on the normalized gyroradius and on the plasma β, comparably little attention has been paid to the dependence on the collisionality. This was initially motivated by the knowledge that the fundamental instabilities leading to plasma turbulence are essentially collisionless. More recently, the observation of the role of zonal flows (ZFs) as an important regulating factor of the turbulence level [1–5], together with the fact that the poloidal flow is undamped in collisionless plasmas [6], and it is therefore ultimately damped by ion-ion collisions, suggests a possible mechanism by which collisionality can affect the overall transport.

The first studies of the role of the undamped component of the poloidal flow were carried out in Ref. [7], by modifying the closures for the zonal component of the flux-tube gyrofluid model of Ref. [1], and in Ref. [5], with a global gyrokinetic particle simulation code. The latter study has shown a dependence of the ion thermal transport on ion-ion collisions.

In order to analyze the role of the collisional damping of ZFs in machinelike situations, it is important to carry out simulations that emulate steady-state power balance conditions, i.e., characterized by integration times longer than an energy confinement time and by boundary conditions that correspond to a given input power. This allows one to explore separately the effect of the injected energy and of the collisionality on thermal transport. This need is addressed by this work, by means of toroidal ion-temperature gradient (ITG) fluid simulations in global geometry. The main observation is that the energy confinement time increases when the collisionality is reduced. This effect is due primarily to the increase of the zonal flows, at a given input power. The zonal flows act to increase the ITG turbulence effective threshold, which grows with the shearing rate of the ZFs. The ZFs also increase when the input power is augmented at constant collisionality, since they are determined by the balance between the turbulent drive (given by the Reynolds stresses) and the collisional damping [4]. Turbulence also increases with input power, since the fluctuation level must adjust to evacuate the injected energy.

The most important finding of this work is the fact that the input power enters in the effective conductivity through the critical gradient, in a way that partially compensates the degradation of the energy confinement with power, and keeps the transport fairly close to marginality. This effect is not included in the transport models based on an effective conductivity that are routinely used in tokamak transport analysis.

The simulations discussed in this Letter have been performed with an upgraded version of the ETAI3D code [8,9], which solves the minimal fluid model for ITG turbulence in a torus: ion density, parallel velocity, and ion pressure evolution equations to the leading order in the inverse aspect ratio expansion, including curvature effects and collisional damping. Electrons are assumed as adiabatic; use is made of the quasineutrality condition and of a simple parallel Landau damping closure. The equations evolved by the code are

$$\frac{d}{dt}w - 2\varepsilon\omega_d(\Phi + p_i) + A\nabla_{\parallel}v + \frac{\kappa_n}{r}\partial_\theta\Phi = -A\gamma_{pfd}\langle w\rangle + D_w\nabla^2 w, \quad (1)$$

$$\frac{d}{dt}v + A\nabla_{\parallel}(\Phi + p_i) - 4\varepsilon\omega_d v = D_v\nabla^2 v, \quad (2)$$

$$\frac{d}{dt}p_i + \Gamma A\nabla_{\parallel}v - 4\Gamma\varepsilon\omega_d p_i = -\gamma_L|\nabla_{\parallel}|p_i + D_p\nabla^2 p_i, \quad (3)$$

where $w = \Phi - \langle\Phi\rangle - \rho_*^2\nabla^2\Phi$ is the ion guiding center density, Φ being the electrostatic potential and $\rho_* = \rho_s/a$ the ion sound Larmor radius normalized to the minor radius; $d/dt = \partial_t + \vec{v}_E\nabla$, $\vec{v}_E = -(\nabla\Phi \times B)/B^2$, whereas $\omega_d = \sin\theta\partial_r + \frac{1}{r}\cos\theta\partial_\theta$ is the curvature operator. $\varepsilon = a/R$ is the inverse aspect ratio, R is the major radius, $A = \varepsilon/\rho_*$, $\kappa_n = \nabla n_0/n_0$, and $\Gamma = 5/3$. Lengths are normalized to the minor radius, the parallel velocity to $c_s = \sqrt{T_e/m_i}$, the ion temperature to T_e, and the potential to T_e/e; time and conductivity are in Bohm units $t_{\rm Bohm} = a^2/\chi_{\rm Bohm}$, $\chi_{\rm Bohm} = (cT_e/eB)$. The poloidal flow damping is modeled through a damping rate $\gamma_{\rm pfd}$, which is proportional to the collisionality $\nu_* = \nu_{ii}\varepsilon^{-3/2}/(c_s/qR)$, where ν_{ii} is the ion-ion collision frequency and q is the safety factor. The exact relation between the parameter $\gamma_{\rm pfd}$ and ν_* depends on the model used to evaluate the collisional decay of the poloidal flows. The detailed calculation of Ref. [10], well reproduced by gyrokinetic simulations [5], gives $\gamma_{\rm pfd} = 2/3\nu_*\varepsilon^{1/2}/q$. Equations (1)–(3) would have the same property as the gyrofluid model of Ref. [1] that ZF are damped on the ion transit time scale; to avoid this, we switched off the geodesic part of the $m = (0, 1)$ coupling in the curvature operators. Note that this also removes the geodesic acoustic mode oscillations [11]. The equations are solved in a toroidal annulus between two circular magnetic surfaces, $r_a = 0.5$ and $r_b = 1$. The energy is injected in the system at a constant rate by imposing an incoming heat flux at the inner radius $F_{\rm in}(r_a) = -D_p(r_a)\nabla p_i(r_a)$.

Here we present results from three series of simulations carried out for the case of a torus having $\rho_* = 0.02$ with fixed injected flux: respectively, $F_{\rm in} = 0.05, 0.2, 0.5$ [measured in units of $\chi_{\rm Bohm}(T_e n_e/a)$]. The collisional damping is varied in a range typical of the banana regime. The same phenomenology is observed for the case of a cylinder and that of a torus having $\rho_* = 0.01$. A flat density profile and a linear safety factor profile $q = 4r$ are used; $R = 2$ and $\gamma_L = 1$. Starting with high collisional damping of ZFs ($\gamma_{\rm pfd} = 0.5$), the simulations have been run for several confinement times. When the collisionality is decreased, in steps, one observes an increase of the energy confinement time: more precisely, lowering the collisionality by a factor of 5 almost doubles the confinement time. This is evident from Fig. 1, which shows the time evolution of the global confinement time evaluated as the ratio of the instantaneous thermal energy to the constant input power, τ_1 (solid line) and the ratio of thermal energy to the instantaneous losses, τ_2 (dashed line).

The decrease of the collisional damping of ZFs has a strong effect on the turbulence, inducing a rapid drop of the fluctuation amplitudes, uniform over the whole spectrum. Figure 2 shows the time evolution of the root mean squared (rms) amplitudes of pressure, potential, and velocity fluctuations.

Note that since power is injected at a constant rate during all the simulation, after the initial drop that follows the collisionality reduction, the fluctuation amplitudes eventually grow again. Indeed, an important feature of flux driven systems is that the turbulence level is constrained, in a stationary state, by the fact that

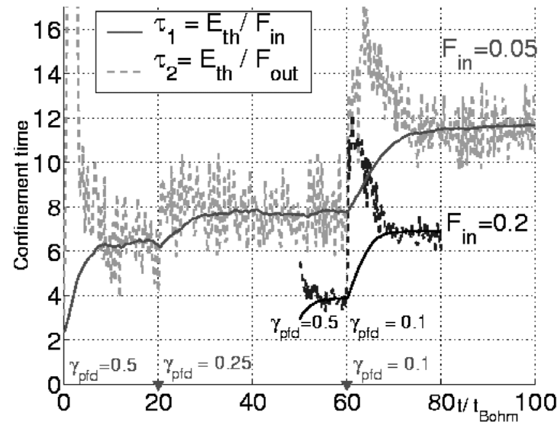

FIG. 1. Energy confinement times for a torus having $\rho_* = 0.02$ and $F_{\rm in} = 0.05$ and 0.2, evaluated as the ratio of the thermal energy to the injected flux, τ_1 (solid line), and the ratio of the thermal energy to the outgoing heat flux, τ_2 (dashed line).

$\Gamma_r = \langle \delta p\, \delta v_r \rangle$, where δp and δv_r are the fluctuating pressure and radial $\mathbf{E} \times \mathbf{B}$ velocity, respectively. This implies that even when the collisionality is reduced, the fluctuation level cannot change too much. A transition to a regime of reduced transport is followed by an increase of the temperature gradient to restore approximately the turbulence level needed to evacuate the injected energy. On the other hand, one finds that the ratio of the pressure to the potential fluctuation amplitude depends on the value of collisionality, as shown in Fig. 3. This occurs indirectly through the temperature gradient, since the pressure fluctuations are more strongly dependent on it than the potential fluctuations. Indeed, simple considerations drawn from linear theory (well above threshold) would give $\delta p/\delta\phi \sim (R/L_T)^{1/2}$, in terms of the temperature scale length L_T.

A first analysis allowed us to identify that the stabilization is caused by the increased ZF shearing rate. Right

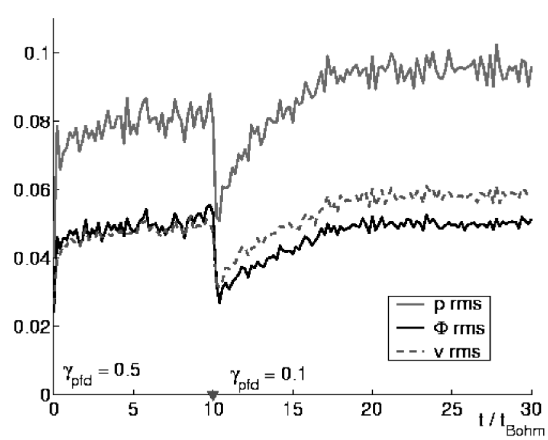

FIG. 2. Time evolution of the rms fluctuation amplitude of pressure (solid gray line), electrostatic potential (black line), and velocity (dashed line) for the case of a torus having $\rho_* = 0.02$ and $F_{\rm in} = 0.2$.

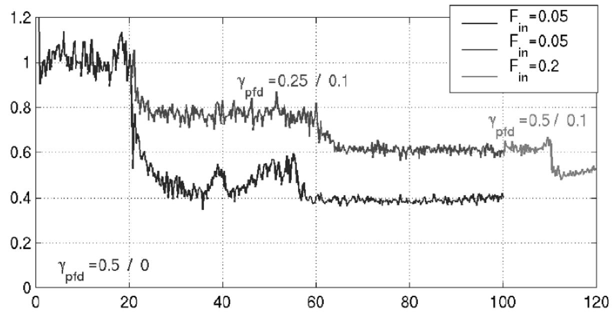

FIG. 3. Time evolution of the ratio between potential and pressure fluctuations for all the simulations with $\rho_* = 0.02$.

after the sudden change of the collisionality, the ZF increases rapidly because of the imbalance between its drive, the Reynolds stresses, and the damping. The large ZF transiently stabilizes the fluctuations. The fluctuations remain completely stable until the ZF shearing rate becomes smaller than γ_0, the average growth rate calculated in the absence of ZFs. This effect is demonstrated by a simulation reinitialized with the ZF at its peak, but with the fluctuations rescaled by a factor 1×10^{-3}, so that they evolve linearly. The ZF is then allowed to decay slowly under a viscous damping. Figure 4 shows that the fluctuation energy growth rate (solid line) becomes positive only when the ZF shearing rate (dash-dotted line) falls below γ_0.

Following Ref. [12], we have also investigated whether the reduction of turbulent transport can be attributed to a change in the cross phase $\phi_{v_r p}$. Defining $\cos\phi_{v_r p} = \langle \delta p \delta v_r \rangle / \langle \delta p^2 \rangle^{1/2} \langle \delta v_r^2 \rangle^{1/2}$, the turbulent flux, averaged on a magnetic surface, can be decomposed as $\langle \Gamma_r \rangle = \langle \delta p^2 \rangle^{1/2} \langle \delta v_r^2 \rangle^{1/2} \cos\phi_{v_r p}$. Evaluating the three terms, we do not observe any relevant variation of the cross phase, not even transiently, when the collisional damping is lowered. A plot of the cross phase evaluated at a fixed radial position, for all the simulations, is shown in Fig. 5.

We now come to the main result of this Letter, namely, the observation that the increase of confinement time is due to an increase of the effective critical gradient for ITG driven turbulence. Linear stability simulations have been performed for each turbulent stationary state, with the actual ZF profile, in order to determine the effective ion-temperature gradient threshold in each case. The resulting critical temperature gradient is roughly proportional to the spatial rms ZF shearing rate, which increases with the inverse of the collisionality, as shown in Fig. 6. The upshift of the threshold was first noticed in Ref. [13] with a gyrokinetic flux-tube code.

Finally, we analyzed the effect of injecting higher power at constant collisionality, observing an increase of turbulence as well as a rise of the ZFs. Figure 7 shows that the shearing rate is approximately a function of the ratio of the injected heat flux to the collisionality, as a consequence of the balance between the Reynolds stresses and the ZF damping. Figure 8 shows the resulting dependence of the effective critical gradient on the ratio of the injected flux to the collisionality.

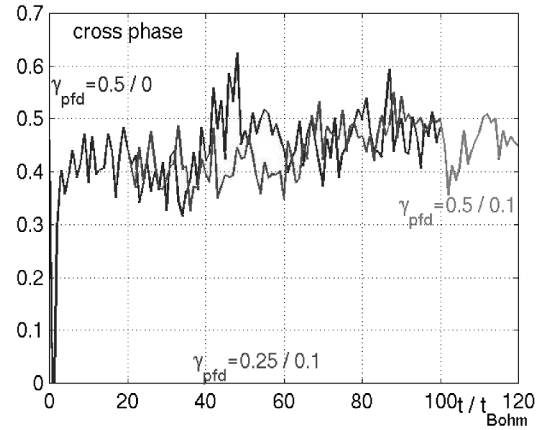

FIG. 5. Cross phase evaluated at the radial position where the ZFs are persistently localized ($r = 0.8$) for all the simulations with $\rho_* = 0.02$ versus time.

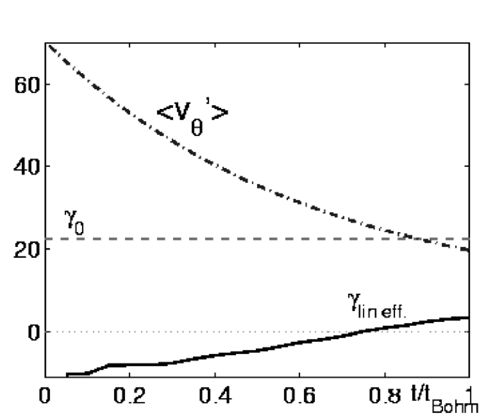

FIG. 4. Fluctuation energy growth rate (solid line) vs shearing rate (dash-dotted line); the linear growth rate γ_0 is plotted (dashed line).

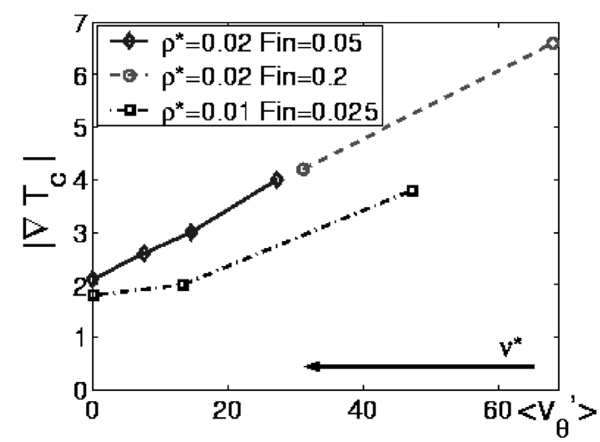

FIG. 6. Effective ion temperature gradient threshold as a function of the spatial rms ZF shearing rate, for the case of a torus with $\rho_* = 0.02$ ($\diamond$, $\bigcirc$) and $\rho_* = 0.01$ ($\square$).

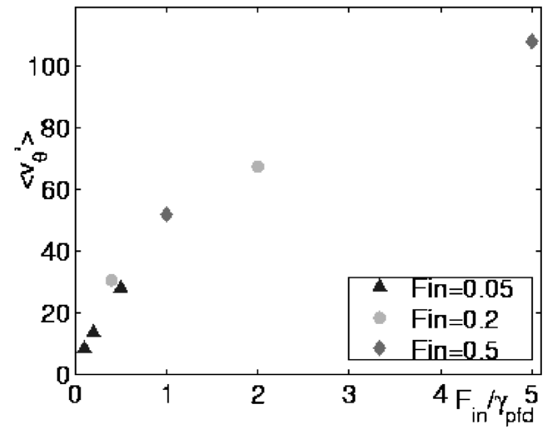

FIG. 7. rms ZF shearing rate as a function of the ratio of the injected heat flux to the collisionality, for the case of a torus with $\rho_* = 0.02$.

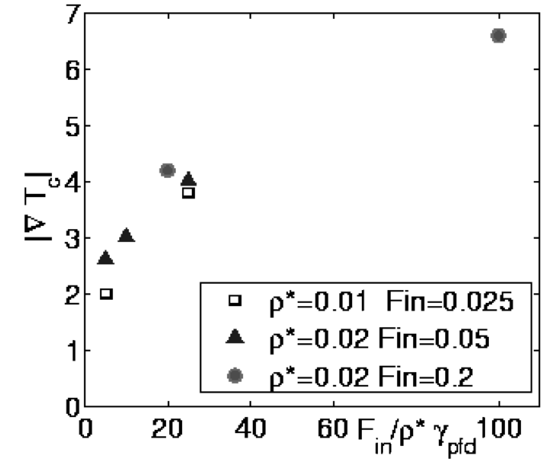

FIG. 8. Effective ion temperature gradient threshold as a function of the ratio of the injected flux to the collisionality, normalized to ρ_*.

Note that as the collisionality is reduced the ZF is progressively dominated by a low frequency component, as recently predicted in Ref. [14]. Little difference is observed between the instantaneous and the time averaged shearing rate.

One of the concepts often advanced is that transport is reduced because of "vortex shredding" by ZFs, which would reduce the radial correlation length. This argument employs the notion that the transport coefficients are proportional to the square of the correlation length and inversely to the correlation time. However, they depend also on the fluctuation level. As has been shown here, the fluctuation level depends primarily on the combination of the temperature gradient and its effective threshold, which is ultimately a function of the ratio of the injected power to the collisionality. The fact that the threshold increases with injected power keeps the system fairly close to threshold. In our simulations, the average gradient does not exceed the actual critical gradient by more than 80%, even at the highest flux, which, for a machine such as Tore Supra, would correspond to an input power in excess of 10 MW.

In conclusion, the main mechanism by which confinement improves at lower collisionality in ITG turbulence is the upshift of the effective critical temperature gradient, mediated by the increased ZF mean shearing rate. The effective critical gradient depends also positively on the injected power, which implies that the effective heat conductivity depends parametrically on the input power. This dependence is not taken into account in existing transport models based on an effective conductivity.

While the effects discussed in this work are quite clear, further studies with a more accurate treatment of the ZF damping, as in Ref. [7], and the inclusion of trapped electrons are required for a quantitative comparison with the experiments.

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