Electron Thermal Diffusivity due to the Electron Temperature Gradient Mode

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Charge neutrality breaks down in the short wavelength toroidal electron temperature gradient mode. In contrast to the ion temperature gradient mode, the wave number normalized by the Debye wave number, k/k_{De}, appears as a natural scale parameter, rather than the finite Larmor radius parameter $k_\perp \rho_e$. The growth rate and consequent mixing length estimate yields an electron thermal diffusivity large enough to be relevant to tokamaks.

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It is generally conjectured that the short wavelength electron temperature gradient (ETG) mode [1–3] is dual of the long wavelength ion temperature gradient (ITG) mode, since in the former, ions are adiabatic while in the latter, electrons are (except for the destabilizing roles of trapped electrons on the ITG mode [4]). However, there are some basic differences between the two modes. First, in the ETG mode, charge neutrality does not necessarily hold because of short wavelength nature. Second, while the ITG mode can be stabilized by a modest plasma β factor through the coupling of electron dynamics to the magnetic perturbation [5], the ETG mode is quite resilient against finite β stabilization which can occur through equilibrium modification only at such a large α (the ballooning parameter) as to cause an effective magnetic drift reversal [6]. If isomorphism between the ETG and ITG modes holds, the mixing length estimate for the electron thermal diffusivity would be of the order of [7]

$$\chi_e \simeq \frac{v_{Te}}{L_n}\rho_e^2, \tag{1}$$

which is smaller than the ion thermal diffusivity due to the ITG mode by approximately a factor of $\sqrt{m_i/m_e}$, and thus would not be relevant to the anomalous electron thermal transport commonly observed in magnetic confinement devices. Here, $v_{Te} = \sqrt{T_e/m_e}$, where L_T is the scale length of the temperature gradient, ρ_e is the electron Larmor radius, and m_i/m_e is the ion/electron mass ratio. Such small transport has indeed been observed in a fluid simulation of the ETG mode [8] in which charge neutrality was imposed. In a kinetic simulation without assuming charge neutrality [9,10], thermal transport significantly larger has been observed. The large transport was attributed to the formation of large scale, radially extended streamers.

In this Letter, it is shown that a simple mixing length estimate of the electron thermal diffusivity in tokamaks based on the ETG mode yields an electron thermal diffusivity relevant to those experimentally observed. It is given approximately by

$$\chi_e \simeq \frac{q v_{Te}}{L_n}(\eta_e - \eta_{cr})\left(\frac{c}{\omega_{pe}}\right)^2\sqrt{\beta_e}, \tag{2}$$

where L_n is the density gradient scale length, $\eta_e = L_n/L_T$ is the temperature gradient, η_{cr} is the critical temperature gradient, c/ω_{pe} is the electron skin depth, and $\beta_e = 8\pi n_0 T_e/B^2$ is the electron beta factor. The skin depth appears because the lower cutoff of the ETG mode occurs at $k_\perp \gtrsim \omega_{pe}/c$, which dominate transport. The growth rate of the ETG mode is proportional to $\sqrt{\beta_e}$ through charge non-neutrality and manifests itself in the diffusivity even though the ETG mode is predominantly electrostatic.

The β_e dependence of the growth rate may be shown qualitatively as follows. Substituting adiabatic ions $n_i = -e\phi n_0/T_i$ and approximate electron density perturbation without electron transit effect,

$$n_e \simeq \frac{\eta_e \omega_{*e}\omega_{\mathrm{De}}}{\omega^2} e^{-b_e} I_0(b_e)\frac{e\phi}{T_e} n_0, \tag{3}$$

in the Poisson's equation $\nabla^2\phi = -4\pi e(n_i - n_e)$, we find the growth rate,

$$\gamma(b_e) = \sqrt{\frac{2T_e/m_e}{L_T R}}\sqrt{\frac{b_e e^{-b_e} I_0(b_e)}{\tau + (b_e/\beta_*)}}, \tag{4}$$

where $\eta_e = L_n/L_T$ is the electron temperature gradient parameter, $\omega_{*e} = \frac{v_{Te}}{L_n}\sqrt{b_e}$, $\omega_{\mathrm{De}} = \frac{2v_{Te}}{R}\sqrt{b_e}$, $b_e = (k_\perp\rho_e)^2$, I_0 is the modified Bessel function, $\tau = T_e/T_i$, and

$$\beta_* = \beta_e \frac{mc^2}{2T_e} = \left(\frac{\omega_{pe}}{\Omega_e}\right)^2, \tag{5}$$

with ω_{pe} the electron plasma frequency and Ω_e the cyclotron frequency. The maximum growth rate can be found by scanning the finite Larmor radius (FLR) parameter b_e. When the maximum growth rate is written as $\gamma_{\max} = \sqrt{2T_e/m_e L_T R} f(\beta_*)$, the function $f(\beta_*)$ is approximately proportional to $\sqrt{\beta_*}$ in the regime $\beta_* \lesssim 1$ relevant to tokamaks as shown in Fig. 1 for the case $\tau = 1$. It should be noted that in tokamak stability analysis, the FLR parameter $(k_\perp \rho_e)^2$, the β factor, and the corresponding ballooning parameter $\alpha = q^2(R/L_n)[(1 + \eta_e)\beta_e + (1 + \eta_i)\beta_i]$ are to be specified. Then, the charge non-neutrality factor $(k/k_{\mathrm{De}})^2 = (k_\perp \rho_e)^2 \beta_e \frac{mc^2}{2T_e}$ necessarily involves a normalized temperature.

In order to confirm the predicted β_e dependence of the growth rate, we employ the following fully electromagnetic gyrokinetic dispersion relation to find the mode frequency and growth rate of the ETG mode [11]:

$$\left\{k_\perp^2 + 2\left(\frac{\omega_{pe}}{c}\right)^2 F_{e2} + 2\left(\frac{\omega_{pi}}{c}\right)^2 F_{i2}\right\}\left[F_{e0} - 1 - \left(\frac{k}{k_{\mathrm{De}}}\right)^2 - \tau(1 - F_{i0})\right] = 2\left(\frac{\omega_{pe}}{c}\right)^2 \left(F_{e1} + \sqrt{\frac{\tau m_e}{m_i}} F_{i1}\right)^2, \quad (6)$$

where

$$F_{ej} = \left\langle \left(\frac{v_\parallel}{v_{Te}}\right)^j \frac{\omega - \hat{\omega}_{*e}}{\omega - \hat{\omega}_{\mathrm{De}} - k_\parallel v_\parallel} J_0^2(\Lambda_e) \right\rangle, \quad (7)$$

$$F_{ij} = \left\langle \left(\frac{v_\parallel}{v_{Ti}}\right)^j \frac{\omega + \hat{\omega}_{*i}}{\omega + \hat{\omega}_{Di} - k_\parallel v_\parallel} J_0^2(\Lambda_i) \right\rangle, \quad (8)$$

$\langle \cdots \rangle$ indicates averaging over the velocity with Maxwellian weighting,

$$\hat{\omega}_{*i,e}(v^2) = \frac{cT_{i,e}}{eB^2}\left[1 + \eta_{i,e}\left(\frac{m_{i,e}v^2}{2T_{i,e}} - \frac{3}{2}\right)\right] \times [\nabla(\ln n_0) \times \mathbf{B}] \cdot \mathbf{k}_\perp, \quad (9)$$

$$\hat{\omega}_{Di,e}(\mathbf{v}) = \frac{cm_{i,e}}{eB^3}\left(\frac{1}{2}v_\perp^2 + v_\parallel^2\right)(\nabla B \times \mathbf{B}) \cdot \mathbf{k}_\perp, \quad (10)$$

and J_0 is the zeroth order Bessel function whose argument is $k_\perp v_\perp / \Omega_{i,e}$. The equilibrium ion and electron velocity distributions are assumed to be Maxwellian and trapped electrons and magnetosonic perturbation $\mathbf{A}_\perp$ are ignored. The norms of the differential operators based on a simple trial eigenfunction $\phi(\theta) = 1 + \cos\theta$, $|\theta| \le \pi$, have been given in Ref. [10].

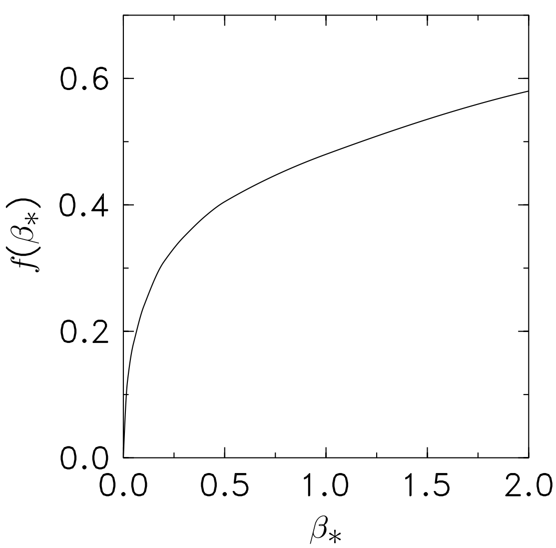

FIG. 1. The function $f(\beta_*)$ in $\gamma_{\max} = (\sqrt{2}v_{Te}/\sqrt{L_T R}) f(\beta_*)$ obtained by scanning b_e in Eq. (4).

Figure 2 shows the dependence of mode frequency and growth rate (both normalized by the electron transit frequency $\omega_{Te} = v_{Te}/qR$) on the normalized perpendicular wave number, $d_e = (k_\theta/k_{\mathrm{De}})^2$ for two values of β, $\beta_e = \beta_i = 0.2\%$ and 0.5% when $T_e = T_i = 5$ keV in (a) and 10 keV in (b). Other parameters assumed are $L_n/R = 0.2$, $s = 1$, $q = 2$, $\eta_e = \eta_i = 2$, and $m_i/m_e = 1836$ (hydrogen). The growth rate peaks at $k^2 \simeq 0.4$ irrespective of variation in the plasma density (β) and temperature. β (actually, the plasma density) destabilizes the ETG mode in two aspects: the maximum growth rate increases with β and also the unstable regime in k^2 broadens toward smaller k^2.

Figure 3 shows the mixing length estimate of the electron thermal diffusivity,

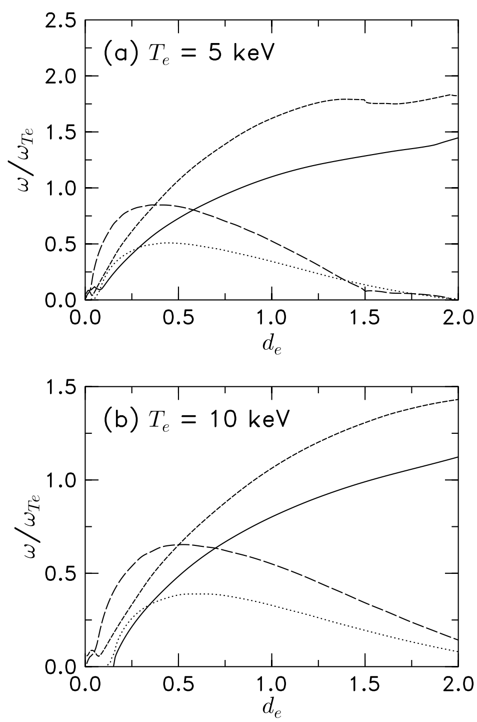

FIG. 2. $(\omega_r + i\gamma)/\omega_{Te}$ vs $d_e = (k_\theta/k_{\mathrm{De}})^2$ when $T_e = T_i = 5$ keV in (a) and 10 keV in (b). Solid and dotted lines show ω_r/ω_{Te} and γ/ω_{Te}, respectively, when $\beta_e = 0.2\%$ and dashed and long-dashed lines show ω_r/ω_{Te} and γ/ω_{Te} when $\beta_e = 0.5\%$. Other parameters are $q = 2$, $s = 1$, $\varepsilon_n = L_n/R = 0.2$, $\eta_e = \eta_i = 2$, and $m_i/m_e = 1840$.

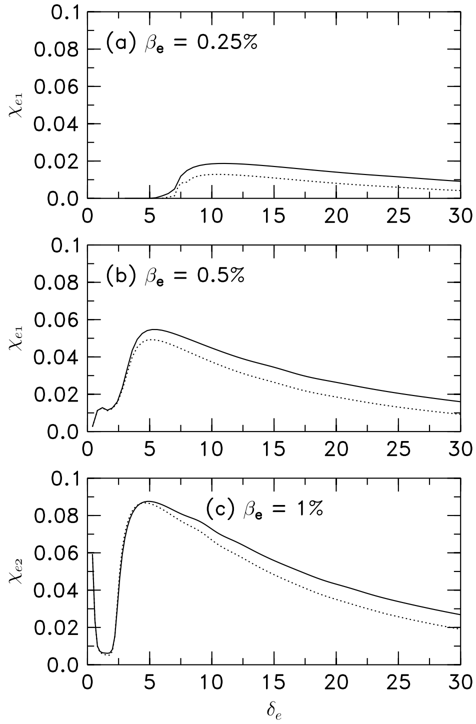

FIG. 3. χ_e in units of Ohkawa diffusivity ($\chi_{e1} = \chi_e/\chi_{\text{Ohkawa}}$) when $T_e = T_i = 5$ keV (solid lines) and 10 keV (dotted lines). $s = 1$, $\varepsilon_n = 0.2$, $\eta_i = \eta_e = 2$, $q = 2$, and $m_i/m_e = 1836$.

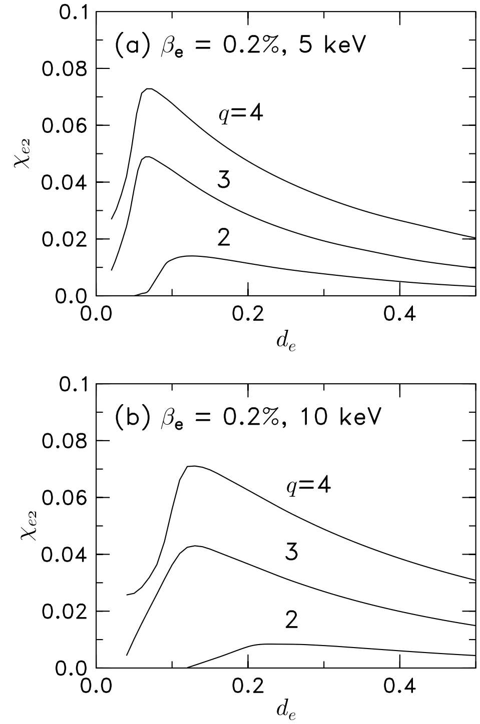

FIG. 4. χ_e in units of $\omega_{Te}/k_{\text{De}}^2$ ($\chi_{e2} = \chi_e k_{\text{De}}^2/\omega_{Te}$) vs $d_e = (k_\theta/k_{\text{De}})^2$ when (a) $T_e = T_i = 5$ keV and (b) 10 keV for different values of q. $\beta_e = 0.2\%$, $s = 1$, $\varepsilon_n = 0.2$, $\eta_i = \eta_e = 2$, and $m_i/m_e = 1836$.

$$\chi_e = \frac{\gamma^3}{\omega_r^2 + \gamma^2}\frac{1}{k_r^2}, \tag{11}$$

in units of $(v_{Te}/qR)(c/\omega_{pe})^2$ (Ohkawa diffusivity [12]) when $T_e = T_i = 5$ keV and 10 keV. [The radial inverse correlation length $1/k_r$ is undetermined in linear analysis and isotropic turbulence with $k_\theta = k_r$ is assumed. The diffusivity would be enhanced by a factor $(k_\theta/k_r)^2$ if fully developed turbulence is anisotropic with $k_r < k_\theta$.] The diffusivity clearly depends on the β factor, and Ohkawa diffusivity is not quite applicable to thermal transport caused by the ETG mode. This is primarily due to the finite β (density) destabilization of the ETG mode.

Scanning the safety factor q has revealed that the diffusivity depends on q strongly as shown in Fig. 4. In Fig. 4, the diffusivity is normalized by $\omega_{Te}/k_{\text{De}}^2$ which is related to the Ohkawa diffusivity through

$$\frac{\omega_{Te}}{k_{\text{De}}^2} = \omega_{Te}\left(\frac{c}{\omega_{pe}}\right)^2 \frac{T_e}{mc^2}.$$

The normalized diffusivity is proportional to q^2 and since $\omega_{Te} \propto 1/q$, we may conclude that the unnormalized diffusivity is proportional to q. (Note that Ohkawa diffusivity is inversely proportional to q.) Such strong dependence on the safety factor q is due to the fact that the mode frequency and growth rate of the toroidal ETG mode are comparable with the electron transit frequency $\omega_{Te} = v_{Te}/qR$. Scanning the temperature gradient η_e and variation of toroidicity $\varepsilon_n = L_n/R$ in the low α regime (ballooning parameter $\alpha \lesssim 1$) has revealed that the diffusivity is proportional to $(\eta_e - \eta_{\text{cr}})/\varepsilon_n$, where η_{cr} is the critical temperature gradient which depends on ε_n: $\eta_{\text{cr}} \simeq 0.2$ when $\varepsilon_n = 0.1$, $\eta_{\text{cr}} \simeq 1$ when $\varepsilon_n = 0.2$, and $\eta_{\text{cr}} \simeq 3$ when $\varepsilon_n = 0.4$. Implementing the dependence on β_e, q, η_e, and ε_n, we may summarize the electron thermal diffusivity due to the ETG mode in the following form:

$$\chi_e \simeq \frac{1}{20}\frac{qv_{Te}}{L_n}(\eta_e - \eta_{\text{cr}})\left(\frac{c}{\omega_{pe}}\right)^2\sqrt{\beta_e}. \tag{12}$$

If $n = 5 \times 10^{13}$ cm^{-3}, $T_e = 10$ keV, $q = 3$, $L_n = 50$ cm, $\eta_e - \eta_{\text{cr}} = 2$, and $\beta_e = 1\%$, this yields $\chi_e \simeq 1.4 \times 10^4$ cm^2/sec which is large enough to be relevant to experiments. Of course, the diffusivity may still be multiplied by factors containing dimensionless parameters. The factor 1/20 should not be taken too seriously, for actual diffusivity may be enhanced by a factor $(k_\theta/k_r)^2$, the spectral anisotropy in fully developed ETG turbulence which is beyond the realm of linear and quasilinear analysis. For example, in fluid simulation in Ref. [8], anisotropy of $k_\theta/k_r \lesssim 3$ has been seen which would enhance the diffusivity to

$$\chi_e \simeq \frac{1}{2}\frac{qv_{Te}}{L_n}(\eta_e - \eta_{cr})\left(\frac{c}{\omega_{pe}}\right)^2\sqrt{\beta_e}. \qquad (13)$$

It is noted that the toroidicity (magnetic curvature) is contained in the safety factor q and χ_e scales as

$$\chi_e \propto \frac{rT_e}{RL_n B_\theta \sqrt{n}}, \qquad (14)$$

where B_θ is the poloidal magnetic field. The diffusivity in Eq. (14) in general increases with the minor radius r which is consistent with the χ_e profiles observed in tokamaks.

In summary, it has been shown that if charge non-neutrality is considered, a natural normalization of the wave number for the ETG mode is $(k/k_{\mathrm{De}})^2$, rather than $(k\rho_e)^2$. The lower cutoff of the ETG mode occurs at $k_\perp \simeq \omega_{pe}/c$, and transport near the cutoff dominates. Consequently, a large electron thermal diffusivity emerges from a simple mixing length estimate despite the fact that the maximum growth rate of the ETG mode occurs at a much shorter wavelength.

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