# Experimental Nonlinear Sign Shift for Linear Optics Quantum Computation

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We have realized the nonlinear sign shift operation for photonic qubits. This operation shifts the phase of two photons reflected by a beam splitter using an extra single photon and measurement. We show that the conditional phase shift is $(1.05 \pm 0.06)\pi$ in clear agreement with theory. Our results show that, by using an ancilla photon and conditional detection, nonlinear optical effects can be implemented using only linear optical elements. This experiment represents an essential step for linear optical implementations of scalable quantum computation.

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A promising system for quantum computation is to use single photons to encode quantum information [1,2]. This is due to the photon's robustness against decoherence and the availability of single-qubit operations. However, it has been very difficult to achieve the necessary two-qubit operations since the physical interaction between photons is much too small. Surprisingly, Knill, Laflamme, and Milburn (KLM) showed that effective nonlinear interactions can be implemented using only linear optical elements in conjunction with single-photon sources and conditional dynamics in such a way that scalable quantum computation can be achieved [3–10]. The fundamental element of the KLM scheme is the nonlinear sign-shift (NS) operation from which the two-qubit conditional sign flip gate can be constructed. Universal quantum computation is then possible with this two-qubit gate together with all single-qubit rotations [11,12] Here, we experimentally demonstrate the NS operation using photons produced via parametric down-conversion. In contrast with the KLM scheme, our method to observe the NS operates in the polarization basis and therefore does not require interferometric phase stability.

A simplified version of NS operation is shown in Fig. 1(a). An input state, $|\Psi_{\mathrm{IN}}\rangle = |n\rangle$, impinges on a beam splitter (BS) with reflection probability $R$; a single ancilla photon, $|1\rangle$, impinges from the other side of the beam splitter. The two input modes, 1 and 2, undergo a unitary transformation into two output modes, 3 and 4, described by $\hat{a}_1 \rightarrow \sqrt{R}\hat{a}_3 + \sqrt{1-R}\hat{a}_4$ and $\hat{a}_2 \rightarrow -\sqrt{1-R}\hat{a}_3 + \sqrt{R}\hat{a}_4$. The NS operation is successful when one and only one photon reaches the detector in mode 4. Provided the photons are indistinguishable, the two paths leading to exactly one photon in mode 4 will interfere. The two interfering processes are depicted in Fig. 1(b) for $n$ input photons. Either all $n+1$ photons are reflected or $n-1$ of the photons in mode 1 are reflected and one photon in each of modes 1 and 2 are transmitted. When a single photon ends up in mode 4, the photon-number state undergoes the following transformation:

$$|\Psi_{\mathrm{IN}}\rangle = |n\rangle \rightarrow |\Psi_{\mathrm{OUT}}\rangle = (\sqrt{R})^{n-1}[R - n(1-R)]|n\rangle, \tag{1}$$

where the unusual normalization of the output state reflects the probability amplitude of success. The sign of the phase shift depends on the number of incident photons and the reflection probability of the BS. For $n < R/(1-R)$, the sign of the amplitude is unchanged and for $n > R/(1-R)$ it picks up a negative sign. For the critical case, where $n = R/(1-R)$, the output probability amplitude becomes zero [13].

In the original KLM proposal, the NS gate is achieved using a phase sensitive interferometer. In our experiment, we induce the phase shift between two polarizations in the same spatial mode and therefore have much less stringent stability requirements. The extension of the NS operation to include a second polarization mode is straightforward. We inject a horizontally polarized

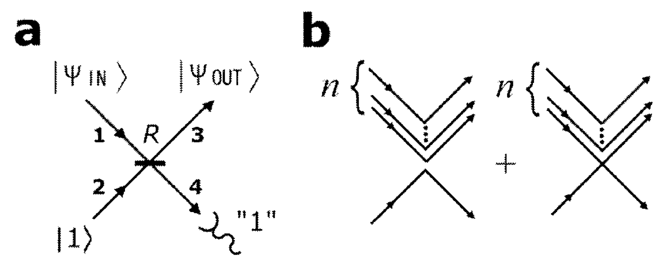


FIG. 1. (a) Schematic of a simplified version of NS operation constructed by a nonpolarizing beam splitter of reflectivity $R$. $|\Psi_{\mathrm{IN}}\rangle$ and $|\Psi_{\mathrm{OUT}}\rangle$ are the quantum states of input and output photons. The operation is successful when the single-photon detector in ancilla mode 4 counts a single photon. (b) The two paths that lead to the detection of exactly one photon in output mode 4. As long as all of the photons are indistinguishable, these two paths can interfere.

  

ancilla photon into the BS in Fig. 1(a) and consider only the cases where the single-photon detected in mode 4 is horizontally polarized. The transformation for the horizontal polarization is the same as in Eq. (1). There is only one possible path which leads to no vertically polarized photons in mode 4; that is for all vertically polarized photons to be reflected. This operation for the input state with $m$ vertically polarized photons and $n$ horizontally polarized photons is given by

$$\begin{aligned}|\Psi_{\mathrm{IN}}\rangle &= |m_{\mathrm{V}}; n_{\mathrm{H}}\rangle \to |\Psi_{\mathrm{OUT}}\rangle \\ &= (\sqrt{R_{\mathrm{V}}})^m (\sqrt{R_{\mathrm{H}}})^{n-1}[R_{\mathrm{H}} - n_{\mathrm{H}}(1 - R_{\mathrm{H}})]|m_{\mathrm{V}}; n_{\mathrm{H}}\rangle,\end{aligned} \tag{2}$$

where $R_{\mathrm{V}}$ and $R_{\mathrm{H}}$ are the reflection probabilities for vertical and horizontal polarization, respectively. As expected, the vertical photon number, $m$, does not appear in the square bracketed nonlinear sign term. The only change the vertically polarized photons contribute is the reflection amplitude raised to the power of $m$.

A quantum phase gate for the KLM scheme can be implemented using two such NS gates when the BS has reflection probabilities, $R_{\mathrm{V}} = 5 - 3\sqrt{2} \approx 0.76$ and $R_{\mathrm{H}} = (3 - \sqrt{2})/7 \approx 0.23$ [14]. For the experiment, we use input states where $m + n = 2$ and the typical 50/50 BS, where $R_{\mathrm{V}} = R_{\mathrm{H}} = 1/2$. The three possible input states are transformed by the NS operation according to

$$|2_{\mathrm{V}}; 0_{\mathrm{H}}\rangle \to \frac{1}{2\sqrt{2}}|2_{\mathrm{V}}; 0_{\mathrm{H}}\rangle, \tag{3a}$$

$$|1_{\mathrm{V}}; 1_{\mathrm{H}}\rangle \to 0, \tag{3b}$$

$$|0_{\mathrm{V}}; 2_{\mathrm{H}}\rangle \to -\frac{1}{2\sqrt{2}}|0_{\mathrm{V}}; 2_{\mathrm{H}}\rangle. \tag{3c}$$

The operation with this set of input parameters serves to change only the phase of the input state $|0_{\mathrm{V}}; 2_{\mathrm{H}}\rangle$. The input state $|1_{\mathrm{V}}; 1_{\mathrm{H}}\rangle$ is "annihilated" by this operation [13]. This means that for that input state the condition of having exactly one horizontally polarized photon in mode 4 never occurs. The NS operation using this particular BS reflectivity is important to a related protocol for a "quantum filter" [15].

In the experiment (Fig. 2), frequency-doubled pulses from a mode-locked Ti:sapphire laser (center wavelength 394.5 nm, 200 fs pulse duration, 76 MHz repetition rate) make two passes through a type-II phase-matched 2-mm beta-barium-borate (BBO) crystal (BBO1) [16–19]. Through spontaneous parametric down-conversion, there is some probability for one pair of entangled photons to be created on the first pass and another pair on the second [20]. Additional 1-mm crystals (BBO2, BBO3) and a half-wave plate (HWP1) are used for the compensation of the birefringence effect inside BBO1 and also for the selection of appropriate polarization-entangled photons. The first pair (right going modes 1 and 2 in Fig. 2) serves as the input to the NS operation in mode 3. The second pair (left going modes 5 and 6) is used to produce the ancilla photon. Upon detection of a "trigger" photon in mode 6, a single-photon state will be present in mode 5 with high probability. BS2 is a normal 50/50 BS and its reflectivity determines the NS operation. Four-photon events are postselected by single-photon counting detectors $D_1$, $D_2$, $D_A$, and $D_B$. We first verified the operations (3a) and (3c). The state of the polarization-entangled photons in modes 1 and 2 was prepared as $|\Phi_\theta\rangle = 1/\sqrt{2}(|1_{\mathrm{V}}\rangle_1|1_{\mathrm{V}}\rangle_2 + e^{i\theta}|1_{\mathrm{H}}\rangle_1|1_{\mathrm{H}}\rangle_2)$, where the relative phase, $\theta$, was controlled by tilting the compensation crystal BBO2. BS1 (also 50/50) transforms this polarization-entangled state to a photon-number entangled state. The photons in mode 3 are in the state $|\Psi_{\mathrm{IN}}\rangle = 1/\sqrt{2}(|2_{\mathrm{V}}; 0_{\mathrm{H}}\rangle_3 + e^{i\theta}|0_{\mathrm{V}}; 2_{\mathrm{H}}\rangle_3)$. Pairs created from the second pass are used as a source of triggered single photons. A translatable mirror on the pump allows for the relative creation time of the two pairs to be varied. Down-converted photons are coupled into single-mode fibers (SMF) for mode filtering. The photons come out of the fibers to free space again for interference in BS2 (also 50/50 BS). The polarization of the ancilla photon $|1_{\mathrm{H}}\rangle$ in mode 5 is set to horizontal using a polarizer (POL). The entangled photons are sent to BS2 and combined with the horizontally polarized ancilla photon in mode 5. A successful operation occurs when the single-photon detector, $D_2$, in output mode 8 counts a single photon in

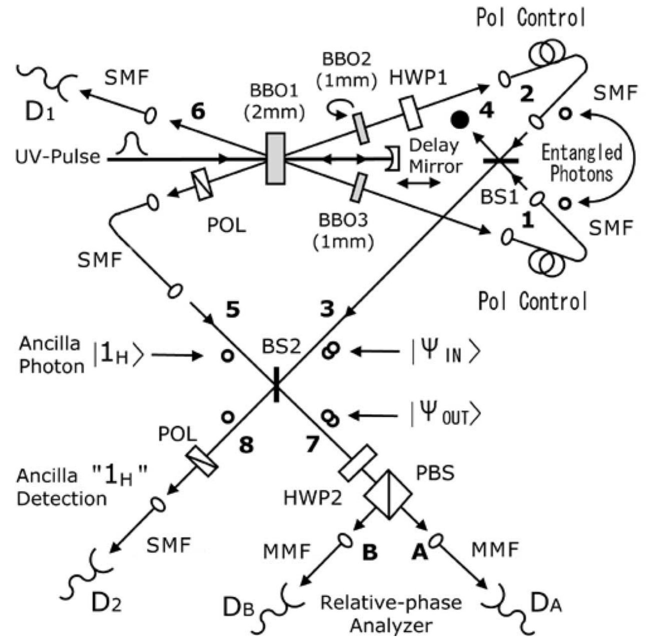


FIG. 2. Experimental setup for the demonstration of NS operation using double-pass parametric down-conversion. Photon pairs created from first pass are used for the input of NS operation and pairs from the second pass are used for the triggered single photon source as ancilla. Successful operation is identified through fourfold coincidence counts between all four detectors.

horizontal polarization state. The output state in mode 7 is analyzed using HWP2, a polarizing beam splitter (PBS), multimode fibers (MMF), and single-photon counters placed in modes $A$ and $B$ ($D_A$and $D_B$); together these form a relative-phase analyzer. Successful operation is identified through the fourfold coincidence counts between all four detectors ($D_1$, $D_2$, $D_A$, and $D_B$).

In general, $D_2$ would need to be able to distinguish one photon from multiple photons. However, in a multiphoton coincidence experiment events with five or more photons are of much lower probability and contribute negligibly to the signal. The production probably of down-converted photons to the measurement is estimated to be about $10^{-11}$ with four-photon events per UV pulse of pump beam, and less than $10^{-16}$ with five or more photon events. This postselection process allows for the NS operation signal to be observed using current generation photodetectors and probabilistic signal photon sources. Equations (3a) and (3c) show that the maximum successful probability of the NS operation is 1/8; however, the experimental results become much lower (about $10^{-5}$) because we use pulsed down-converted photons for the operation instead of ideal single-photon sources. Further developments of novel single-photon sources and photodetectors will allow for subsequent refinements of this experiment, such as eliminating the need for postselection and pushing the experimental probability of success towards its theoretical maximum [21–26].

For our first measurement, we act on the photon-number entangled states in mode 3. Since the NS operation is an interference effect, it proceeds only when the entangled photons and the ancilla photon arrive at BS2 within their coherence time, $\tau_{\mathrm{coh}}$. In this case, the operation performs the following:

$$|\Psi_{\mathrm{IN}}\rangle = \frac{1}{\sqrt{2}}(|2_{\mathrm{V}};0_{\mathrm{H}}\rangle_3 + e^{i\theta}|0_{\mathrm{V}};2_{\mathrm{H}}\rangle_3) \rightarrow |\Psi_{\mathrm{OUT}}\rangle$$
$$= \frac{1}{4}(|2_{\mathrm{V}};0_{\mathrm{H}}\rangle_7 - e^{i\theta}|0_{\mathrm{V}};2_{\mathrm{H}}\rangle_7). \tag{4}$$

To analyze this effect, we use HWP2 to rotate the polarization states by 45°. Under such an operation, $1/\sqrt{2}(|2_{\mathrm{V}};0_{\mathrm{H}}\rangle + |0_{\mathrm{V}};2_{\mathrm{H}}\rangle) \rightarrow 1/\sqrt{2}(|2_{\mathrm{V}};0_{\mathrm{H}}\rangle + |0_{\mathrm{V}};2_{\mathrm{H}}\rangle)$ (i.e., it is invariant), however $1/\sqrt{2}(|2_{\mathrm{V}};0_{\mathrm{H}}\rangle - |0_{\mathrm{V}};2_{\mathrm{H}}\rangle) \rightarrow |1_{\mathrm{V}};1_{\mathrm{H}}\rangle$ (i.e., it can produce coincidences between $D_A$ and $D_B$). We can verify the transformation (4) by measuring one vertically and one horizontally polarized photon using the PBS and photodetectors $D_A$ and $D_B$. Figure 3(a) shows the observed variations of the count rate as functions of the pump mirror position which varies the relative arrival times of the entangled photons and the ancilla photons at BS2. Solid squares (circles) show the fourfold coincidence counts for $\theta = 0(\pi)$ in (4) as a function of the pump delay. At zero delay, the coincidences for $D_1D_2D_AD_B$ are enhanced (suppressed) by the NS operation (4). When the arrival time difference

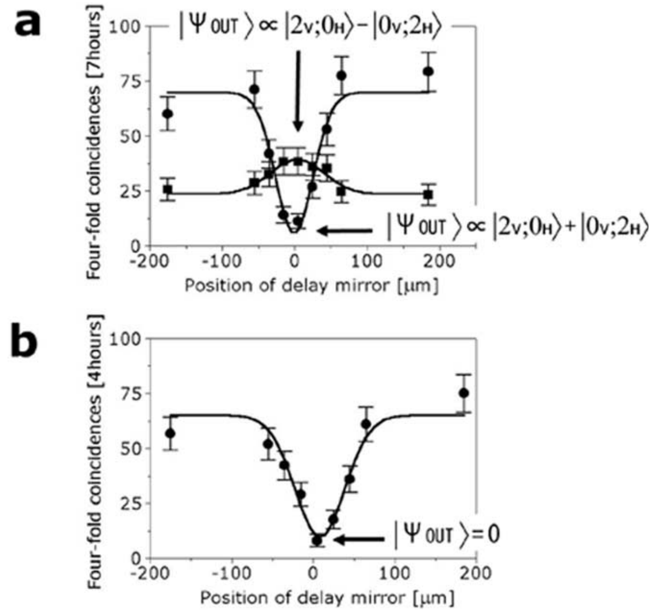


FIG. 3. (a) The fourfold coincidences as a function of the pump delay mirror position for the photon-number entangled photons. HWP2 is set to rotate the polarization state by 45°. Solid squares (circles) show the fourfold coincidence counts for the input state where $\theta = 0(\pi)$. The peak and dip of the two curves are different in size only because of accidental coincidences occurring when the delay is much larger than the photons' coherence length. (b) The fourfold coincidences as functions of the pump delay mirror position for the input photons in the state $|1_{\mathrm{V}};1_{\mathrm{H}}\rangle_3$. The HWP2 is set to leave the input polarization states unchanged. At zero delay, the coincidences are suppressed nearly to zero by the HOM effect.

is larger than $\tau_{\mathrm{coh}}$, we obtain coincidence counts from the state $|\Psi_{\mathrm{IN}}\rangle$ in (4) and also accidental coincidence counts between the ancilla photon and one entangled photon. If one could resolve the subpicosecond level time differences, the size of the dip of one curve would be equal to the size of the peak in the other. The fidelity of the sign-shifted entangled photons can be estimated from the data to be $77 \pm 6\%$. When taking the fidelity $92.9 \pm 0.5\%$ of the initial state into account, this confirms the high quality of our NS operation.

We then fix the pump delay at zero so that the NS operation can proceed with maximum efficiency. We analyze the output mode 7 by tilting the compensation crystal BBO2—this allows for the variation of the correlations with $\theta$ to be directly observed. For the state $|2_{\mathrm{V}};0_{\mathrm{H}}\rangle + e^{i\theta}|0_{\mathrm{V}};2_{\mathrm{H}}\rangle$ in mode 7, the coincidence rate between detectors $D_A$ and $D_B$ is proportional to $\sin^2(\theta/2)$. We input the state $|2_{\mathrm{V}};0_{\mathrm{H}}\rangle + e^{i\theta}|0_{\mathrm{V}};2_{\mathrm{H}}\rangle$ into BS2 and record both the twofold $D_AD_B$ and the fourfold $D_1D_2D_AD_B$ coincidences with the ancilla path open. The twofold coincidence rate reflects the initial correlations for the input entangled-photon pair, whereas the fourfold coincidence rate shows the correlations after a successful NS operation. Figure 4 shows the observed coincidence

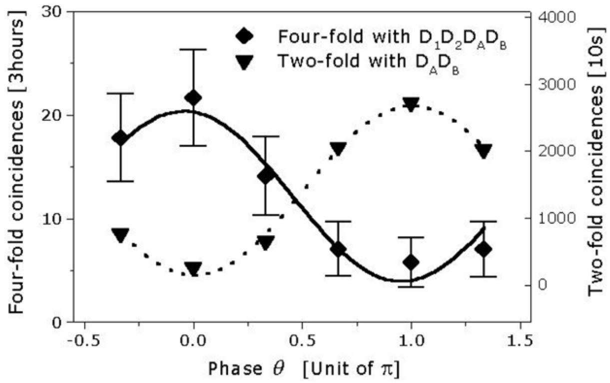


FIG. 4. The observed variation of the coincidence count rates as functions of the phase of entangled photons $\theta$ at zero delay for the photon-number entangled photons. Solid triangles represent the twofold coincidences with $D_A D_B$ and show the phase of input photons. Solid diamonds represent the fourfold coincidences with $D_1 D_2 D_A D_B$ which show the phase of the output photons demonstrating a successful NS operation. Error bars are based on the usual Poisson fluctuation in the number of counts on the uncorrected data (error bars of twofold coincidences are too small to display). The phase of fourfold coincidence is shifted $(1.05 \pm 0.06)\pi$ against twofold coincidence in agreement with the expected $\pi$ phase shift.

rates (twofold are solid triangles and fourfold are solid diamonds) at zero delay for different phase angles set by BBO2. Clearly, the phase of the correlations has been changed by the NS operation; the relative phase between the two curves is $(1.05 \pm 0.06)\pi$ in excellent agreement with the expected shift of $\pi$.

To complete the experimental confirmation of the NS operation, we also verified its action on the input state $|1_V; 1_H\rangle$ [Eq. (3b)]. We prepare the input state $|\Psi_{IN}\rangle = |1_V; 1_H\rangle_3$ from the polarization-entangled photons that was prepared as $|\Psi^+\rangle = 1/\sqrt{2}(|1_V\rangle_1|1_H\rangle_2 + |1_H\rangle_1|1_V\rangle_2)$. The HWP2 is set such that it does not rotate the polarization. Figure 3(b) shows the fourfold coincidences as a function of the pump delay. At zero delay, the fourfold coincidences $D_1 D_2 D_A D_B$ are suppressed nearly to zero because of the effect at BS2 [13]. The visibility of the fringe is about $89 \pm 4\%$. The results shown in Figs. 3 and 4 confirm all of the important features of the NS operation for input states with two photons as described theoretically in Eq. (3a)–(3c).

We have experimentally demonstrated the nonlinear sign shift operation using linear optical elements and the best available technologies for single-photon generation and detection. This includes using a triggered single-photon source from parametric down-conversion and single-photon counting by avalanche photodiodes. This experiment is a proof-in-principle demonstration of the operation of the nonlinear sign shift, which is the critical element in the KLM scheme. These experimental results are of utmost importance for the realization of scalable quantum computer with linear optics.

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