Discrete Entanglement Distribution with Squeezed Light

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(Received 22 July 2003; published 8 January 2004)

We show how one can entangle distant atoms by using squeezed light. Entanglement is obtained in steady state, and can be increased by manipulating the atoms locally. We study the effects of imperfections, and show how to scale up the scheme to build a quantum network.

DOI: 10.1103/PhysRevLett.92.013602 PACS numbers: 42.50.Pq, 03.65.Ud, 03.67.Mn

Distributing entanglement among different nodes in a quantum network is one of the most challenging and rewarding tasks in quantum information. This may allow one to extend quantum cryptography over long distances [1], and may lead to some practical applications in the context of secret sharing [2] or distributed quantum computation [3]. From the more fundamental point of view, it may allow us to perform loophole-free tests of Bell inequalities [4].

In a quantum network, photons are used to entangle atoms located at different nodes which store the quantum information. Local manipulation of the atoms using lasers permits then to process this information. In principle, one can construct quantum networks using discrete [5] (qubit) or continuous variable (CV) entanglement [6] (the one contained, e.g., in two-mode squeezed states [7]). However, the fact that Gaussian states cannot be distilled using Gaussian operations [8] may strongly limit the applications of continuous variable entanglement in quantum networks and repeaters.

There have been several proposals to obtain discrete entanglement of distant atoms using high-Q cavities. There are two kind of schemes [5,9–11] (see also [12]): (i) [Fig. 1(a)] An atom A, driven by a laser, emits a photon into the cavity mode. The photon enters the second cavity where it is absorbed by atom B [5,9]. (ii) [Fig. 1(b)] Both atoms are simultaneously driven by a laser; if a photon is detected at halfway between the cavities, the atoms get projected into an entangled state [10,13]. Most of these schemes operate in a transitory regime; i.e., the entanglement is achieved at a specific time and the lasers have to be switched on and off appropriately. Moreover, dissipation may introduce imperfections in the desired entangled state. In this Letter, we propose and analyze a scheme to distribute discrete entanglement which works in steady state. As opposed to these other schemes, dissipation is a necessary ingredient of our scheme which, as we will show, gives it a very robust character. Moreover, it does not create entanglement but rather transforms it from CV (in light) to discrete (atoms). We show how a small amount of CV entanglement can be used to create maximally entangled qubit states. We also show how this scheme can be scaled up by using atoms with several internal levels.

The basic idea is represented in Fig. 1(c). Both cavities are simultaneously driven by squeezed light. The use of squeezed light to drive a single atom was first proposed by Gardiner [14], who studied several phenomena on the atomic steady state. Kimble and colleagues [15], in a remarkable experiment, were able to couple squeezed light in a cavity containing atoms, and confirmed some of the physical phenomena theoretically predicted. Recent experiments in which atoms have been stored in high-Q cavities for relatively long times [16] pave the way for the implementation of several quantum information protocols and, in particular, the one analyzed in the present Letter.

Let us consider two two-level atoms, A and B, confined in two identical cavities. The cavities are driven by an external source of two-mode squeezed light [see Fig. 1(c)]. Assuming that the bandwidth of the squeezed light is larger than the cavity damping rate κ, the evolution of the atoms-plus-cavity modes density operator, ρ, can be described using standard methods [7] by

$$\frac{d\rho}{dt} = -i[H_a + H_b, \rho] + (\mathcal{L}_{\rm cav} + \mathcal{L}_{\rm at}^a + \mathcal{L}_{\rm at}^b)\rho. \quad (1)$$

Here $H_a = g_a(a\sigma_a^+ + a^\dagger\sigma_a^-)$ describes the interaction of

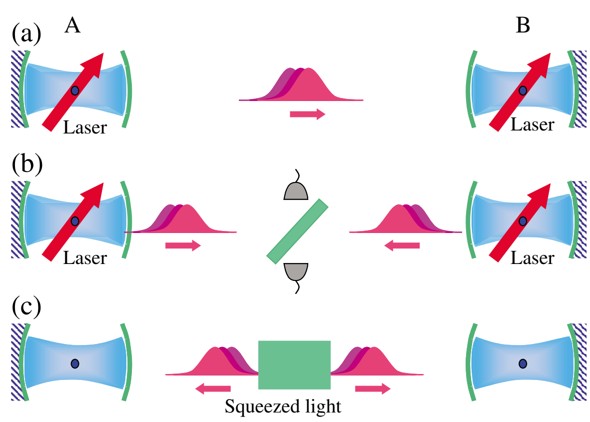

FIG. 1 (color online). Schemes for entanglement creation over long distances. (a) Entanglement is obtained by the emission and subsequent absorption of a photon. (b) A detection of a photon projects the atoms in an entangled state. (c) Both cavities are driven by a common source of two-mode squeezed light. In the steady state, the atoms become entangled.

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atom A with the corresponding cavity mode, where a is the mode annihilation operator and $\sigma_a^+ = (\sigma_a^-)^\dagger = |e\rangle_a\langle g|$, with $|g\rangle$ and $|e\rangle$ denoting the ground and excited atomic states [17]. Spontaneous emission $|e\rangle_a \rightarrow |g\rangle_a$ is described by the usual Liouvillian $\mathcal{L}_{\rm at}^a$ [7], proportional to the spontaneous emission rate Γ. The terms H_b and $\mathcal{L}_{\rm at}^b$ are analogously given. Finally, the interaction between the cavity modes and the squeezed light is given by

$$\begin{aligned}\mathcal{L}_{\rm cav}\rho = \kappa(N+1)\sum_{\alpha=a,b}(\alpha\rho\alpha^\dagger - \alpha^\dagger\alpha\rho) \\ + \kappa N\sum_{\alpha=a,b}(\alpha^\dagger\rho\alpha - \alpha\alpha^\dagger\rho) \\ + \kappa M(2a\rho b + 2b\rho a - 2ba\rho - 2ab\rho) + \text{H.c.},\end{aligned} \tag{2}$$

(H.c. denotes Hermitian conjugate). Here, N and M characterized the two-mode squeezed vacuum and fulfill $M \leq [N(N+1)]^{1/2}$. We will concentrate in the case $g_a = g_b := g$ since the formulas are considerably simplified. The effects for $g_a \neq g_b$ will be analyzed at the end.

We first consider the ideal case $\Gamma = 0$ and perfect squeezing

$$M = [N(N+1)]^{1/2}. \tag{3}$$

We define annihilation operators $\tilde{a} = (N+1)^{1/2}a + N^{1/2}b^\dagger$ and $\tilde{b} = (N+1)^{1/2}b + N^{1/2}a^\dagger$, so that [cf. Eq. (1)]

$$\frac{d\rho}{dt} = -i[\tilde{H}_a + \tilde{H}_b, \rho] + \tilde{\mathcal{L}}_{\rm cav}\rho, \tag{4}$$

where now

$$\tilde{H}_a = g(\tau_a^+\tilde{a} + \tilde{a}^\dagger\tau_a^-), \tag{5a}$$

$$\tilde{H}_b = g(\tau_b^+\tilde{b} + \tilde{b}^\dagger\tau_b^-), \qquad \tilde{\mathcal{L}}_{\rm cav}\rho = \kappa\sum_{\alpha=\tilde{a},\tilde{b}}(\alpha\rho\alpha^\dagger - \alpha^\dagger\alpha\rho), \tag{5b}$$

with $\tau_{a,b}^+ = (\tau_{a,b}^-)^\dagger = (N+1)^{1/2}\sigma_{a,b}^+ - N^{1/2}\sigma_{b,a}^-$. Solving (4) seems to be a difficult task. However, one can easily determine the steady state, which is given by

$$|\Psi\rangle = \left(\sqrt{\frac{N+1}{2N+1}}|g\rangle_a|g\rangle_b + \sqrt{\frac{N}{2N+1}}|e\rangle_a|e\rangle_b\right)|0\rangle_{\tilde{a}}|0\rangle_{\tilde{b}}, \tag{6}$$

where $|0\rangle_{\tilde{a},\tilde{b}}$ are the vacuum states of the new modes $\tilde{a}$ and $\tilde{b}$, respectively. This is a pure state, which for $N \gg 1$ tends to a maximally entangled state. For a realistic value of $N \sim 1$, one still obtains a state with a large entanglement of formation (EoF) $E(\Psi) \sim 0.92$. This quantity is defined as the minimum amount of singlets required to create the state using local operations and classical communication [18].

After the creation of (6), one switches simultaneously off the squeezing source and transfers the excited state $|e\rangle$ of both atoms to some other internal ground state $|g'\rangle$ using a laser, in order to avoid spontaneous emission [Fig. 2(a), (i)]. After that, a maximally entangled state can be obtained as follows [Fig. 2(a), (ii)]. In each of the atoms, a radio frequency (or two-photon Raman) pulse is applied which transforms $|g\rangle \rightarrow \cos\theta|g\rangle + \sin\theta|g''\rangle$, where $|g''\rangle$ is an auxiliary internal ground state, while the state $|g'\rangle$ is not affected. Then, the state $|g''\rangle$ is detected in both atoms using the quantum jump technique. If neither of them is found in $|g''\rangle$, the atomic state will be projected onto one proportional to $|g\rangle_a|g\rangle_b + |g'\rangle_a|g'\rangle_b$ if one chooses $\cos(\theta) = [N/(N+1)]^{1/4}$. Note that this measurement corresponds to a generalized measurement but in which the role of the ancilla is taken by the auxiliary level $|g''\rangle$, i.e., no extra atoms are required. The success probability depends on the value of N, but after a sufficiently large number of trials, a maximally entangled state can be prepared for any $N > 0$.

In practice, there will be several physical phenomena which will distort the atomic entanglement in steady state. In the following, we will evaluate the effect of the most important sources of imperfection.

In order to analyze the nonideal situation $\Gamma \neq 0$ and $M < [N(N+1)]^{1/2}$, we consider $g\sqrt{N+1}, \Gamma \ll \kappa$. Then, we can eliminate the cavity mode by extending the procedure of [19]. We define $\sigma := {\rm tr}_{a,b}(\rho)$, so that

$$\frac{d\sigma}{dt} = \mathcal{L}_1\rho + \mathcal{L}_{\rm at}^a\sigma + \mathcal{L}_{\rm at}^b\sigma, \tag{7}$$

where $\mathcal{L}_1(\rho) = -ig\,{\rm tr}_{a,b}(a[\sigma_a^+, \rho] - \text{H.c.}) + a \leftrightarrow b$. Integrating formally Eq. (1), and substituting the result in (7), one can check that, in the limit $\kappa t \gg 1$, the dominant contribution is given by the term coming from

$$\rho(t) \simeq \int_0^t d\tau e^{\mathcal{L}_{\rm cav}\tau}\mathcal{L}_2[\rho(t-\tau)], \tag{8}$$

where $\mathcal{L}_2(\rho) = -ig([a, \rho\sigma_a^+] - \text{H.c.}) + a \leftrightarrow b$. Using $e^{\mathcal{L}_{\rm cav}\tau}([a,R]) = e^{-\kappa\tau}[a, e^{\mathcal{L}_{\rm cav}\tau}R]$, we see that the integrand will vanish for times $\kappa\tau \gg 1$, so that we can

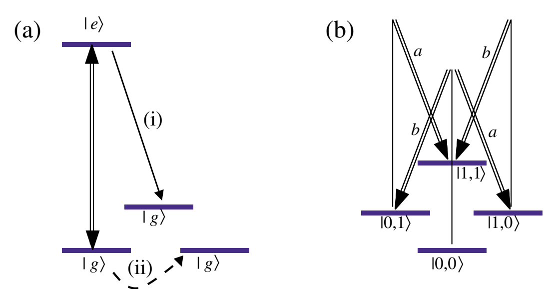

FIG. 2 (color online). Atomic level schemes. Double lines indicate coupling to cavity modes, and single to lasers: (a) $|g\rangle$ and $|e\rangle$ are used for entanglement creation; $|g'\rangle$ to store the qubit once the entanglement has been obtained; $|g''\rangle$ for entanglement concentration. (b) The four internal ground levels are coupled to two cavity modes (a and b) by Raman transitions as indicated.

extend the limit of the integral to infinity. Moreover, since after time t the cavity mode will be driven to its steady state, ρ_{ss}, which fulfills $\mathcal{L}_{\rm cav}(\rho_{ss}) = 0$, we can replace $e^{\mathcal{L}_{\rm cav}\tau}\rho(t-\tau) \to \sigma(t)\otimes\rho_{ss}$. This procedure amounts to performing the Born-Markov approximations [7], but here the bath itself (cavity mode) undergoes a dissipative dynamics. After lengthy algebra, we obtain

$$\dot{\sigma} = \frac{\gamma}{2}(n+1)\sum_{\alpha=a,b}(\sigma_\alpha^-\sigma\sigma_\alpha^+ - \sigma_\alpha^+\sigma_\alpha^-\sigma) + \frac{\gamma}{2}n\sum_{\alpha=a,b}(\sigma_\alpha^+\sigma\sigma_\alpha^- - \sigma_\alpha^-\sigma_\alpha^+\sigma) + \gamma m(\sigma_a^-\sigma\sigma_b^- + \sigma_b^-\sigma\sigma_a^- - \sigma_b^-\sigma_a^-\sigma - \sigma_a^-\sigma_b^-\sigma) + \text{H.c.} \tag{9}$$

Here

$$\gamma = \frac{g^2}{\kappa}(2+\epsilon), \qquad \epsilon := \Gamma\kappa/(g^2), \tag{10a}$$

$$n = N(1+\epsilon/2)^{-1}, \qquad m = -M(1+\epsilon/2)^{-1}. \tag{10b}$$

The interpretation of (9) is straightforward. It describes the interaction of the two atoms with a common squeezed reservoir in which the squeezing parameters are renormalized due to the presence of spontaneous emission. The steady state solution depends only on n and m, and can be easily determined. Instead of analyzing our results in terms of n and m, it is more convenient to analyze them in terms of the physical parameters ϵ and N, choosing (3). Note that it is always possible to find an ϵ, and an N and M fulfilling (3), which give any prescribed values of n and m, so that the effects of imperfect squeezing can be directly read off from our analysis.

In Fig. 3(a), we have plotted the atomic EoF of the steady state as a function of ϵ for various values of N. The most important aspect is that for $\epsilon \neq 0$ increasing the squeezing does not necessarily lead to an increase in the EoF. For each value of ϵ, we have determined the best choice of N, which is shown in the inset. For realistic parameters $\epsilon \lesssim 0.1$, the best choice of N is around 0.6, leading to an EoF of 0.638. In Fig. 3(b), we have plotted the results when the filtering measurement described above is performed. Here we see that the achievable entanglement significantly increases. For $\epsilon = 0.1$, one obtains an EoF of 0.775.

In Fig. 4(a), we have analyzed the effects of the imprecision in the position of the atoms [20]. We have first extended our analysis to the case $g_a = g\cos(\theta_a) \neq g_b = g\cos(\theta_b)$, by deriving a master equation analogous to (9).

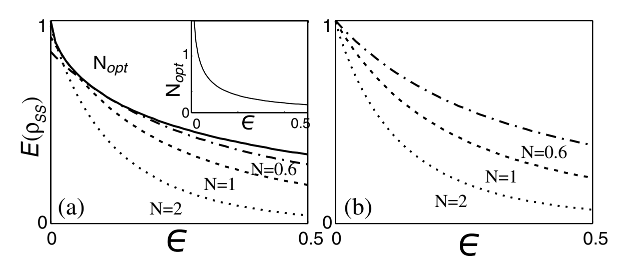

FIG. 3. EoF of the atoms in steady state as a function of ϵ and N: (a) without generalized measurement; (b) with generalized measurement. The solid line indicates the optimal value and the inset gives the N for which the EoF is optimal.

We have then averaged the density operator corresponding to the steady state with respect to θ_a and θ_b, with a weight function $p(\theta) \propto \exp[-\theta^2/(2s^2)]$. We have plotted the resulting EoF vs s, which measures the uncertainty in the atomic position. The figure shows that this uncertainty does not have dramatic effects in the EoF, as long as the position of the particle is not far from the antinode of the cavity mode standing wave.

As mentioned above, we are transforming the CV entanglement contained in the squeezed vacuum state of the incident light into discrete (qubit) entanglement. In Fig. 4(b), we have analyzed the efficiency of this process. We have plotted the achieved EoF vs the EoF contained in the squeezed state for various values of ϵ. The transfer is more efficient for small N, something that can be attributed to the fact that only two Schmidt coefficients are relevant for the two-mode squeezed state.

An important aspect of our scheme is that it can be scaled up to build a quantum communication network or quantum repeaters. The idea is to embed two (or more) atoms in each cavity, and to use two modes in each of them. Atoms $A1$ and $B2$ can interact with modes a_1 and b_2 in their respective cavities, which in turn are driven by two-mode squeezed light. Atoms $B1$ and $C2$ can also become entangled in a similar way by interacting with modes b_1 and c_2, respectively. In the ideal case, after the entanglement is obtained, a measurement in atoms $B1$ and $B2$ will yield an entangled state between atoms $A1$ and $C2$. In the presence of imperfections, the entanglement will be degraded every time we perform one of these operations (i.e., as we try to extend the

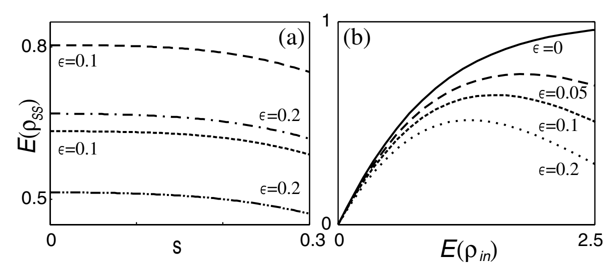

FIG. 4. (a) EoF of the atoms in steady state as a function of the parameter s, for $N = 0.5$. Upper (lower) two curves correspond to the case with (without) measurement after preparation. (b) EoF of the atomic state as a function of the one corresponding to the squeezed state $\rho_{\rm in}$.

entanglement over longer distances). In order to avoid this problem, one can use other auxiliary atoms in each cavity and perform entanglement purification as it is required to build a quantum repeater [1].

For a small number of nodes, it is possible to perform these experiments with a single atom per cavity and without having to perform joint measurements. This is not possible with two-level atoms, since it is known that there is a maximum amount of entanglement that it can share with two neighboring atoms [21]. This problem can be circumvented by using several internal states, since then it is indeed possible that one atom shares two ebits with two other atoms. For example, one may take the scheme of Fig. 2(b), where we have renamed the internal states. Two cavity modes are used, which connect pairs of levels with the help of off-resonant laser beams in Raman configuration. Now, let us consider that we have three atoms A, B, and C, in three different cavities. The atoms in A and C have the same configuration as before, whereas the atom in cavity B has the one indicated in Fig. 2(b). The Hamiltonian, after adiabatically eliminating the excited state of atom B, has the form

$$H = g(\sigma_a^+ a + \sigma_{b_1}^+ b_1 + \sigma_{b_2}^+ b_2 + \sigma_c^+ c) + \text{H.c.} \quad (11)$$

Here, $\sigma_{a,c}^+$ are defined as before, whereas

$$\sigma_{b_1}^+ = |1,0\rangle_B\langle 0,0| + |1,1\rangle_B\langle 0,1|, \quad (12a)$$

$$\sigma_{b_2}^+ = |0,1\rangle_B\langle 0,0| + |1,1\rangle_B\langle 1,0|. \quad (12b)$$

Now, if modes a and b_1 and modes c and b_2 are driven by two independent sources of squeezed light, under ideal conditions ($\Gamma = 0$ and perfect squeezing) the steady state is

$$|\Psi\rangle_{ss} = \frac{N+1}{2N+1}|g\rangle_A|0,0\rangle_B|g\rangle_C + \frac{N}{2N+1}|e\rangle_A|1,1\rangle_B|e\rangle_C + \frac{\sqrt{N(N+1)}}{2N+1}(|g\rangle_A|0,1\rangle_B|e\rangle_C + |e\rangle_A|1,0\rangle_B|g\rangle_C).$$

In the limit $N \gg 1$, this state contains two ebits, one between A and B and another between B and C. Alternatively, an appropriate measurement in B will produce a maximally entangled state between A and C with certain probability. This scheme can be easily generalized to a larger number of nodes. However, as mentioned above, the role of the imperfections will be important and one eventually needs to consider several atoms in each cavity to purify the obtained entanglement.

In conclusion, we have shown that atoms can get entangled by interacting with a common source of squeezed light. The CV entanglement can, in this way, be transformed into a discrete one in steady state. Local measurements result in a more efficient entanglement creation. Given the experimental progress in trapping atoms inside cavities [16] and the successful experiments on coupling squeezed light into a cavity [15], the present scheme may become a very robust alternative to current methods to construct quantum networks for quantum communication.

This work has been supported in part by the *Kompetenznetzwerk Quanteninformationsverarbeitung der Bayerischen Staatsregierung* and by the EU IST projects "RESQ" and "QUPRODIS."

Note added.—After completion of this work, we learned of a related problem, using an atomic Raman configuration [22].

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