Interferometric Methods to Measure Orbital and Spin, or the Total Angular Momentum of a Single Photon

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We propose interferometric methods capable of measuring either the total angular momentum, or simultaneously measuring the spin and orbital angular momentum of single photons. This development enables the measurement of any angular momentum eigenstate of a single photon. The work allows the investigation of single-photon two-qubit entangled states and has implications for high density information transfer.

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Polarization is associated with the orientation of the electric field vector. An electric field that oscillates in one plane with respect to the direction of propagation is linearly polarized. When the electric field rotates with respect to the direction of propagation, the beam is circularly polarized (rotation is circularly symmetric). When a light beam is circularly polarized, each photon carries a spin angular momentum (SAM) given by $\sigma\hbar$, where $\sigma = \pm 1$ corresponds to left- and right-handed circular polarization, respectively. These two states are the eigenstates of the quantum-mechanical spin operator. The polarization of a photon can therefore be described by a point in a two-dimensional state space.

As well as spin angular momentum, photons can also carry orbital angular momentum (OAM) [1,2]. This form of angular momentum arises from phase fronts that are inclined with respect to the beam's propagation axis. OAM eigenstates have an azimuthal phase dependence of the form $\exp(il\phi)$, which gives rise to l intertwined helical phase fronts. The value l is an integer and corresponds to the number of times the phase changes by 2π in a closed loop around the beam. The photons of such a beam carry an OAM of $l\hbar$ [3,4]. As l can take any integer value, there are an infinite number of possible eigenstates of OAM.

The total angular momentum (TAM) of a photon is given by the sum of SAM and OAM. The eigenvalues of TAM are therefore given by $j\hbar$, where $j = \sigma + l$ and the eigenvectors are described by a point in an infinite dimensional state space. It is possible for two photons to possess the same TAM, while having different values of OAM and SAM. For example, $j = 2$ could correspond to either $l = 1, \sigma = +1$ or $l = 3, \sigma = -1$, or indeed any superposition of these. We can infer the TAM from simultaneous measurements of the SAM and OAM, but we cannot infer either the SAM or the OAM from a measurement of the TAM.

The measurement of the SAM of a single-photon is straightforward. A quarter-wave plate is used to convert a circularly polarized state into one of two orthogonal linear polarization states which can be separated using a polarizing beam splitter, for example, a Wollaston prism. Until recently, all previous methods for measuring OAM could not be used to measure the OAM of single photons: Either they required many photons in the same state [5–7] or they were binary devices that test only for one selected state [8,9]. A method for distinguishing between modes of two different orders using the difference in Gouy phase was used within an interferometer [10]. However, the mode order does not depend on the total angular momentum. In contrast to previous schemes, we recently described an interferometric method to select the optical path of a single-photon based on its OAM state [11]. The extension of this method to sorting TAM states was not possible as in the OAM sorting process the polarization state was not maintained. This was due to Dove prisms, which were used to rotate the phase, but also introduced a change to the polarization state [12].

The problem that is addressed in this Letter is the sorting of single photons on the basis of TAM while leaving the SAM and OAM unchanged. The method we report does not collapse the wave function associated with SAM or OAM, and they both remain undefined. We also discuss a method to simultaneously measure the SAM and the OAM of a single photon.

The OAM may be sorted by utilizing the rotational symmetry of the phase that characterizes photons in OAM eigenstates [11]. The symmetry arises from an azimuthal dependence which is given by $\exp(il\phi)$, where l is the OAM per photon in units of $\hbar$. On rotation of the beam through an angle α, this phase dependence becomes $\exp[il(\phi + \alpha)]$, corresponding to a phase shift of [13]

$$\Delta\Phi = l\alpha.$$

In our earlier work, this rotation was accomplished with two Dove prisms, rotated with respect to each other through an angle $\alpha/2$ (this rotates the phase and intensity of a passing beam through an angle α). If the Dove prisms

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are inserted into one arm of a Mach-Zehnder interferometer with $\alpha/2 = \pi/2$, the relative phase difference between the two arms is $\Delta\psi = l\pi$ [Fig. 1(a)]. Adjusting the path length of the interferometer such that beams with even l interfere constructively to give an output at one particular port means that beams with odd l interfere constructively at the other. This forms the first stage of OAM sorting: A cascade of interferometers with different rotation angles allows further sorting of OAM states [11,14].

In principle, an analogous system can sort photons with SAM. Rather than using Dove prisms to rotate the phase and intensity, the polarization vector is rotated using half-wave plates. Two half-wave plates, with their optical axes set at an angle $\alpha/2$ with respect to each other, rotate the polarization through an angle α. For circularly polarized light, this results in a phase shift of

$$\Delta\Phi = \sigma\alpha$$

(where $\sigma = \pm 1$) [15]. We can thus create another Mach-Zehnder interferometer with the wave plates inserted into one arm such that left- and right-hand circularly polarized light are directed to different output ports [Fig. 1(b)]. This provides an alternative mechanism to sort individual photons in the two circular polarization states.

The methods for sorting photons with SAM and OAM are versions of a general sorting technique that can be applied to photons with angular momentum (Fig. 1). To sort photons on the basis of a particular angular momentum (SAM or OAM), we require to rotate the appropriate variable that is associated with that angular momentum (polarization or phase and intensity). It follows then that, to sort photons with TAM (the sum of the SAM and OAM), we require to rotate both the polarization and the phase and intensity, i.e., the whole beam.

Combining the two schemes for sorting OAM and SAM provides a mechanism to sort photons on the basis of TAM. The phase shift given to a beam passing through two half-wave plates and two Dove prisms with appropriate orientation is

$$\Delta\Phi = l\alpha + \sigma\alpha = j\alpha. \quad (1)$$

Incorporating this into a Mach-Zehnder interferometer [Fig. 1(c)] introduces a phase shift that is sensitive to the TAM quantum number, j. To enable the sorting of a greater number of these states, the general principle can be extended further (Fig. 2). The sorting is achieved by cascading additional Mach-Zehnder interferometers with different rotation angles set between the prisms and wave plates.

Unfortunately, in general a Dove prism slightly changes the polarization state of a passing beam and therefore cannot be used for TAM sorting. Instead we designed a prism that rotates both phase and intensity but acts as a quarter-wave plate on the polarization vector (Fig. 3). The three internal reflections give a phase shift between horizontal and vertical polarization components of 90°. They therefore act exactly as achromatic quarter-wave plates. The addition of a conventional quarter-wave plate can be used either to cancel the prism's phase shift or to complement it, giving a half-wave retardation (Fig. 3).

We performed three experiments which incorporated our new prisms: one to demonstrate sorting photons on the basis of the TAM, one to sort superpositions of TAM

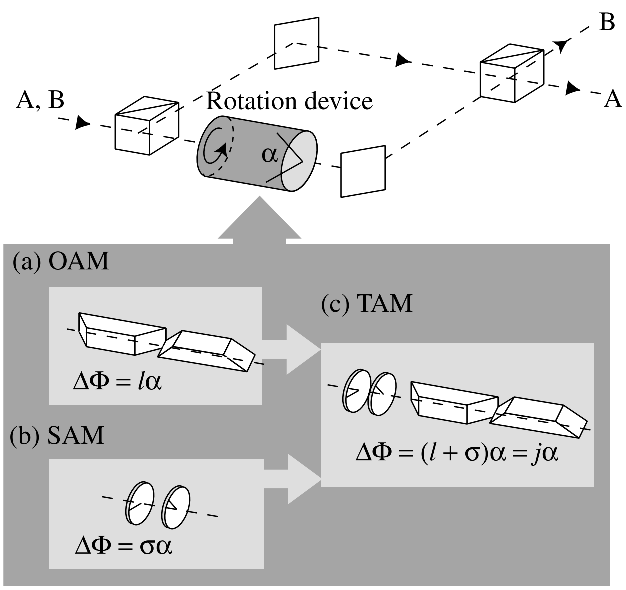

FIG. 1. A single stage of the general sorting scheme. The optical path of a photon can be selected using a Mach Zehnder interferometer with a rotation device in one arm. Rotation of the phase and intensity (Dove prisms) will sort photons containing OAM (a); rotation of the polarization (half-wave plates) will sort photons containing SAM (b). Combining these two systems so that the whole beam is rotated will sort photons containing TAM (c). Note that we show second stage sorting of OAM and TAM, and we show idealized Dove prisms that do not change the polarization state.

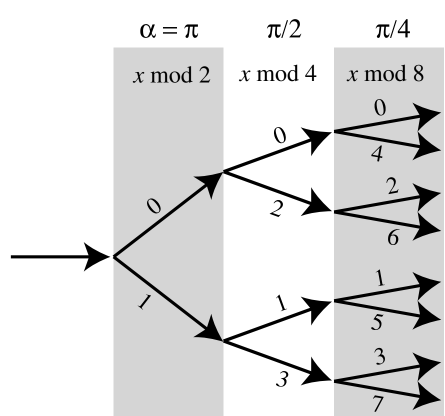

FIG. 2. The general sorting network for angular momentum. This cascade of interferometers can be used to sort photons on the basis of SAM, OAM, or TAM, where x represents σ, l, or j, respectively.

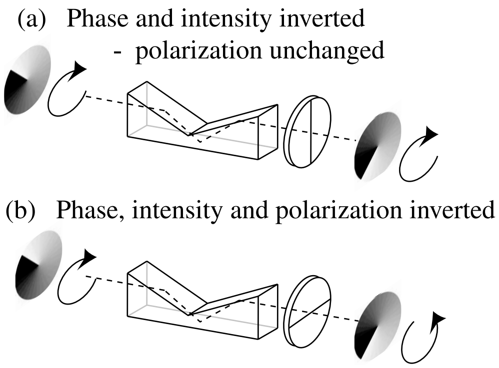

FIG. 3. The image and phase rotating prism acting in opposition with a conventional quarter-wave plate to leave the polarization state unchanged (a) or, in addition, to act as a half-wave plate (b).

states, and one to show a simultaneous measurement of the SAM and OAM. The interferometers were built from standard optical components and the length of each arm of the interferometer was approximately 10 cm long. The light source used in this experiment was a helium-neon laser with an output at 633 nm and a power of <1 mW. The specific j states were created by using the first diffraction order of a computer addressable hologram and then passing the beam through appropriately aligned wave plates. The output ports of the sorter were directed into a camera so that an intensity pattern could be recorded.

Figure 4(a) shows the output from TAM sorting. It shows the second stage of sorting where we select the optical path of photons where the TAM is even. An increment/decrement in the TAM by $2\hbar$ results in the mode's path changing to the other output port. This can be achieved either by changing the SAM state or by incrementing/decrementing the OAM state by $2\hbar$. The first stage would sort modes where the TAM was even from when it was odd. We were successful in sorting different TAM states into different ports.

Clearly, it is also possible to infer the TAM from measurements of the SAM and OAM. A simple extension to the OAM sorting experiment allows us to simultaneously measure the SAM and OAM state of a photon. This is achieved by first sorting on the basis of OAM [using new prisms within the general sorting technique, Fig. 1(a)]. If the beams of the output ports of the interferometer are then passed through a Wollaston prism, then the resulting path of the photon also becomes dependent on the SAM state. This enables us now to sort first on the basis of OAM, using an OAM sorter, and then, second, on the basis of SAM [see Fig. 4(b)]. As can be seen, we succeeded in sorting even and odd OAM states while simultaneously sorting different polarization states into different ports.

It is important to note that, in the case of Fig. 4(a), we measure the TAM directly, not through obtaining information about the SAM and OAM and then inferring the TAM. To illustrate this point explicitly, we created superpositions of TAM states and passed them through the TAM sorter. The superposition states were again created using the computer generated hologram. Whereas the first order diffracted beam has the desired value of l, the zero order is always $l = 0$. The angular deviation induced by the hologram allows the two beams to be separated, orthogonally polarized, and then combined on a beam splitter. Subsequent passage through a quarter-wave plate means the two component beams have different values of l and opposite values of σ. Modifying the hologram design results in the l and, hence, j values of one of the beams to be changed. The results of sorting the superposition states and their constituents are shown in Fig. 5. This demonstrates that the optical path of the superposition states is selected purely on the basis of the TAM.

Note that a single photon in a superposition of pure eigenstates will register in only one of the ports with a probability given by the statistical weight of that component. In other words, the measurement collapses the

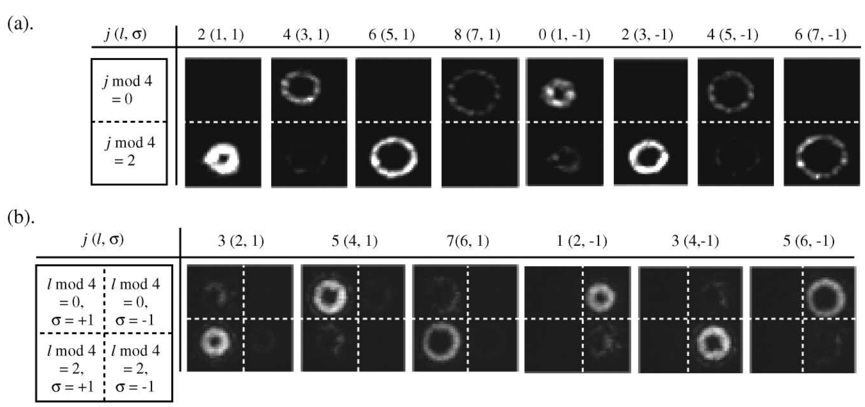

FIG. 4. Intensity outputs from various sorting schemes based on the total angular momentum. (a) Output from second stage sorting where the optical path of the photon is selected on the basis of the TAM. (b) Output from sorting first on the basis of the OAM state and then on the basis of the SAM state.

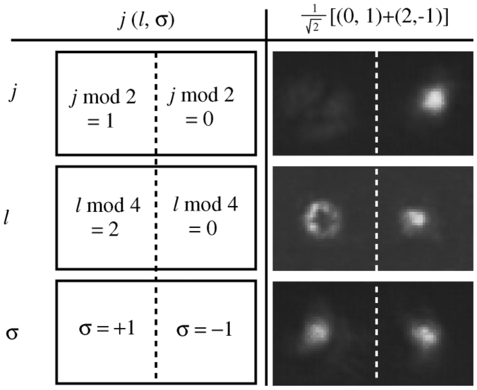

FIG. 5. Results for sorting superpositions of TAM states. The input mode was a superposition of $l = 0$, $\sigma = +1$, and $l = 2$, $\sigma = -1$ so that $j = +1$.

photon into a pure eigenstate (such as all quantum-mechanical measurements).

In the above experiments, the contrast of each channel was measured as >10:1 and the absolute efficiency of the devices was 50%. However, it should be noted that it would be possible to obtain a higher contrast with a more rigidly designed interferometer and a higher absolute efficiency with better quality and specifically designed optical components.

It has recently been demonstrated that single-photon two-qubit entangled states can be prepared in which the two qubits correspond to photon polarization and optical path [16]. Note that this type of entanglement is different from two-photon entanglement, which is of significant importance to quantum computation. Our TAM sorter could similarly be employed to sort single-photon Bell states of the form

$$|\Psi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|l = +1, \sigma = -1\rangle \pm |l = -1, \sigma = +1\rangle),$$
$$|\Phi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|l = +1, \sigma = +1\rangle \pm |l = -1, \sigma = -1\rangle). \quad (2)$$

The states $|\Psi^{(\pm)}\rangle$ have TAM 0, while the states $|\Phi^{(\pm)}\rangle$ have TAM 2 mod 2 and our TAM sorter can separate the two Ψ states from the Φ states. The Bell measurement could be completed by a combination of measuring the linear polarization and Hermite Gaussian mode [14]. The ability to control SAM, OAM, and TAM at the single-photon level will also be useful in manipulating photon pairs entangled in SAM and OAM [4]. In particular, we envisage the possibility of preparing and analyzing embedded Bell states, entangled in both SAM and OAM following the ideas suggested in [17].

We have demonstrated experimentally that the TAM of a single photon can be measured. We have also shown that the OAM and SAM of a single photon can be measured simultaneously. An appropriate sorting device can be used to the optical path of a photon on the basis of either SAM, OAM, or TAM. These approaches are in principle 100% efficient, limited only by the efficiency of the components.

The ability to measure a single photon to be in any one of an arbitrarily large number of orthogonal states has a number of potential implications for quantum information processing. The efficient measurement of the OAM or TAM of a single photon allows us access to a larger state space than that associated with optical polarization. This provides the possibility of a greater density of information transfer along with the generation and analysis of entanglement involving large numbers of states.

The implications of this work for entanglement based applications such as superdense coding, teleportation, and quantum computation [18] remain to be explored.

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[1] L. Allen, M. J. Padgett, and M. Babiker, in *Progress in Optics XXXIX*, edited by E. Wolf (Elsevier Science, New York, 1999), pp. 291–372.
[2] L. Allen, S. M. Barnett, and M. J. Padget, *Optical Angular Momentum* (Institute of Physics, Bristol, 2003).
[3] L. Allen, M.W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A **45**, 8185 (1992).
[4] A. Mair, A.Vaziri, G. Weihs, and A. Zeilinger, Nature (London) **412**, 313 (2001).
[5] M. Harris, C. A. Hill, P. R. Tapster, and J. M. Vaughan, Phys. Rev. A **49**, 3119 (1994).
[6] M. J. Padgett, J. Arlt, N. B. Simpson, and L. Allen, Am. J. Phys. **64**, 77 (1996).
[7] V. V. Kotlyar, V. A. Soifer, and S. N. Khonina, J. Mod. Opt. **44**, 1409 (1997).
[8] V. Y. Bazhenov, M.V. Vasnetsov, and M. S. Soskin, JETP Lett. **52**, 429 (1990).
[9] N. R. Heckenberg, R. McDuff, C. P. Smith, and A. G. White, Opt. Lett. **17**, 221 (1992).
[10] M.V. Vasnetsov, V.V. Slyusar, and M. S. Soskin, Quantum Electron. **31**, 464 (2001).
[11] J. Leach, M. J. Padgett, S. M. Barnett, S. Franke-Arnold, and J.Courtial, Phys. Rev. Lett. **88**, 257901 (2002).
[12] M. J. Padgett and J. P. Lesso, J. Mod. Opt. **46**, 175 (1999).
[13] J. Courtial, D. A. Robertson, K. Dholakia, L. Allen, and M. J. Padgett, Phys. Rev. Lett. **81**, 4828 (1998).
[14] H. Wei and X. Xue, quant-ph/0208146.
[15] B. A. Garetz and S. Arnold, Opt. Commun. **31**, 1 (1979).
[16] Y.-H. Kim, Phys. Rev. A **67**, 040301 (2003).
[17] P. G. Kwiat and H. Weinfurter, Phys. Rev. A **58**, R2623 (1998).
[18] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University, Cambridge, England, 2000).