J/ψ and η_c in the Deconfined Plasma from Lattice QCD

M. Asakawa*

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

T. Hatsuda†

Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

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Analyzing correlation functions of charmonia at finite temperature (T) on $32^3 \times (32 - 96)$ anisotropic lattices by the maximum entropy method (MEM), we find that J/ψ and η_c survive as distinct resonances in the plasma even up to $T \simeq 1.6T_c$ and that they eventually dissociate between $1.6T_c$ and $1.9T_c$ (T_c is the critical temperature of deconfinement). This suggests that the deconfined plasma is nonperturbative enough to hold heavy-quark bound states. The importance of having a sufficient number of temporal data points in MEM analyses is also emphasized.

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Whether hadrons survive even in the deconfined quark-gluon plasma is one of the key questions in quantum chromodynamics (QCD). This problem was first examined in [1] and [2] in different contexts. In the former, it was shown that collective q-$\bar{q}$ excitations with a low mass and a narrow width in the π-σ channels exist even above T_c (the critical temperature) from analyses of the spectral functions in the Nambu–Jona-Lasinio model. The fate of heavy mesons such as J/ψ in the deconfined plasma was also investigated in a phenomenological potential picture taking into account the Debye screening [3]. In general, there is no *a priori* reason to believe that the dissociation of bound states should take place exactly at the phase transition point [4].

Experimentally, measurements of dileptons (diphotons) in heavy ion collisions may provide a clue to the properties of vector (pseudoscalar) mesons in hot/dense matter. Indeed, data from CERN Super Proton Synchrotron indicate anomalies in the dilepton spectra relevant to ρ and J/ψ. Also, BNL Relativistic Heavy Ion Collider is going to produce ample data of dileptons in a few years [5].

From the theoretical point of view, the spectral function (SPF) at finite temperature T, which has all the information of in-medium hadron properties, is a key quantity to be studied. Recently, the present authors have shown [6] that the first-principle lattice QCD simulation of SPFs is possible by utilizing the maximum entropy method (MEM). We have also formulated the basic concepts and applications of MEM on the lattice at $T = 0$ and $T \neq 0$ in [7]. The purpose of this Letter is to report our latest results of SPFs of low lying charmonia (J/ψ and η_c) above T_c extracted from quenched QCD simulations on anisotropic lattices. We find that J/ψ and η_c survive even up to $T \simeq 1.6T_c$ and disappear between $1.6T_c$ and $1.9T_c$. This together with our previous results for $s\bar{s}$ mesons on the same lattice [8] suggests that the system is still strongly interacting above T_c.

To draw the above conclusion with a firm ground, we put special emphasis on (i) the MEM error analysis of the resultant SPFs and (ii) the sensitivity of the SPFs to $N_{\rm data}$ (the number of the temporal data points adopted in MEM). These tests are crucial to prevent fake generation and/or smearing of the peaks and must be always carried out as emphasized in [7,8].

Let us first summarize the basic formulation of MEM applied to lattice data at finite T [7]. We consider the Euclidean correlation function of the local interpolating operator $J_l(\tau, \mathbf{x}) = \bar{c} i \gamma_l c$ with $l = 1, 2, 3$ for J/ψ and $l = 5$ for η_c. The spectral decomposition of the correlator in the imaginary time $0 < \tau < 1/T$ reads

$$D(\tau) = \int \langle J_l(\tau, \mathbf{x}) J_l^\dagger(0) \rangle d^3x = \int_0^\infty K(\tau, \omega) A(\omega) d\omega, \tag{1}$$

where ω is a real frequency and $A(\omega)$ is the spectral function. The sum over $l = 1, 2, 3$ is taken for J/ψ. $K(\tau, \omega)$ is the integral kernel, $K(\tau, \omega) = (e^{-\tau\omega} + e^{-(1/T - \tau)\omega})/(1 - e^{-\omega/T})$. For simplicity, we take the three momentum of the correlation function to be zero.

Monte Carlo simulations provide $D(\tau_i)$ with statistical error at a discrete set of temporal points τ_i. Although there exist infinitely many $A(\omega)$ which give the same $D(\tau_i)$ through the integral Eq. (1), MEM provides a way to select a *unique* $A(\omega)$ without introducing parametrization of SPF and to give its statistical significance on the basis of Bayes' theorem. The most probable $A(\omega)$ given lattice data D is obtained by maximizing the conditional probability $P[A|D] \propto e^{\alpha S - L}$, where L is the standard likelihood function and S is the Shannon-Jaynes entropy:

$$S = \int_0^\infty \left[A(\omega) - m(\omega) - A(\omega) \log\left(\frac{A(\omega)}{m(\omega)} \right) \right] d\omega. \tag{2}$$

Here α is a parameter dictating the relative weight of

S and L. The statistical significance (error) of the resultant $A(\omega)$ is estimated by the second variation, $(\delta/\delta A)^2 P[A|D]$. The default model m in Eq. (2), the first estimate of which is obtained from the perturbative QCD calculation of $A(\omega \gg 1~\text{GeV})$, may be chosen so that the MEM errors become minimum. The final result $A(\omega)$ is given by the weighted average over α: $A(\omega) = \int A_\alpha(\omega) P[\alpha|Dm] d\alpha$, where $A_\alpha(\omega)$ is obtained by minimizing $P[A|D]$ for a fixed α. The conditional probability $P[\alpha|Dm]$ can be calculated by using Bayes' theorem and the lattice data. Thus α is eventually integrated out and does not appear in the final result.

MEM has been successfully applied to the lattice QCD data at $T = 0$ to extract the parameters of the ground and excited state hadrons [6,9,10]. On the other hand, applications to $T \neq 0$ system have been known to be a big challenge [7], although there are some studies at high T [11]. The difficulty originates from the fact that the temporal lattice size L_τ is restricted as $L_\tau = 1/T = N_\tau a_\tau$, where a_τ (N_τ) is the temporal lattice spacing (the number of the temporal lattice sites). Because of this, it becomes more difficult to keep enough N_{data} to obtain reliable SPFs as T increases. In other words, simulations up to a few times T_c with N_{data} as large as 30 [12] require fine lattices at least in the temporal direction (anisotropic lattices).

On the basis of the above observation, we have carried out quenched simulations with $\beta = 7.0$ on a $32^3 \times N_\tau$ anisotropic lattice. The renormalized anisotropy is $\xi = a_\sigma/a_\tau = 4.0$ with a_σ being the spatial lattice spacing. We take the naive plaquette gauge action and the standard Wilson quark action. The corresponding bare anisotropy $\xi_0 = 3.5$ is determined from the data given in [13,14]. The fermion anisotropy $\gamma_F \equiv \kappa_\tau/\kappa_\sigma$ with κ_σ (κ_τ) being the spatial (temporal) hopping parameter is determined by comparing the temporal and spatial effective masses of the pseudoscalar and vector mesons on a $32^2 \times 48 \times 128$ lattice. $a_\tau = a_\sigma/4 = 9.75 \times 10^{-3}$ fm is determined from the ρ meson mass in the chiral limit [15]. The physical lattice size in the spatial direction $L_\sigma = 1.25$ fm can well accommodate J/ψ or η_c, whose root mean square radius is about 0.5 fm. N_τ, the corresponding T/T_c, and the number of gauge configurations N_{gauge} are summarized in Table I. Calculations close to T_c ($T/T_c = 1.04$ and 0.93) are also currently undertaken. Gauge configurations are generated by the pseudo heatbath and overrelaxation algorithms with a ratio 1:4. Initially, the gauge field is thermalized with 10 000 sweeps, and then each configuration is separated by 1000 sweeps.

TABLE I. Temporal lattice size, the corresponding temperature, and the number of gauge configurations in our simulation.

N_τ	32	40	46	54	96
T/T_c	2.33	1.87	1.62	1.38	0.78
N_{gauge}	141	181	182	150	194

We have measured the point-point $q\bar{q}$ correlation functions in the scalar, pseudoscalar (PS), vector (V), and axial-vector channels for the spatial hopping parameters $\kappa_\sigma = 0.082\,85$, 0.0850, 0.0853, and 0.08545. In this Letter, we focus our attention on the heaviest quark mass ($\kappa_\sigma = 0.082\,85$ with $\gamma_F = 3.476$) and study the V (J/ψ) and PS (η_c) channels. The masses determined on the $T = 0$ lattice ($32^2 \times 48 \times 128$) are $m_{J/\psi} = 3.10$ GeV and $m_{\eta_c} = 3.03$ GeV.

In all the figures below, we adopt the continuum kernel, $K(\tau, \omega)$, defined after Eq. (1). Use of the lattice kernel [7,11] does not lead to any appreciable difference. This is because our lattice spacing is quite small, $a_\tau \sim 0.01$ fm [16]. Following [7], we define dimensionless SPFs: $A(\omega) = \omega^2 \rho(\omega)$ for η_c and $A(\omega) = 3\omega^2 \rho(\omega)$ for J/ψ. If the temporal distance between the source and the sink is closer than ξa_τ, lattice artifacts due to anisotropy would appear in the SPFs for $\omega \geq \pi/\xi a_\tau$. To avoid this, we exclude the six points near the edge ($\tau_i = 1, 2, 3$ and $N_\tau - 3, N_\tau - 2, N_\tau - 1$) and adopt the points $\tau_i = 4, 5, \cdots$ and $N_\tau - 4, N_\tau - 5, \cdots$ until we reach the total number of points $N_{\text{data}} (< N_\tau - 7)$.

The current operator on anisotropic lattice $J_l^{\text{LAT}}(x)$ and that in the continuum $J_l^{\text{CON}}(x)$ are related by $J_l^{\text{LAT}} = \sqrt{a_\tau a_\sigma^5} J_l^{\text{CON}} / (2 Z_l \sqrt{\kappa_\tau \kappa_\sigma})$, where Z_l is a nonperturbative renormalization constant. The default model $m(\omega)$ extracted from the perturbative evaluation of $A(\omega \gg 1~\text{GeV})$ also depends on Z_l [7]. We vary Z_l from 1 (the value in the weak coupling limit) and the known value at $\beta = 6.0$ on the isotropic lattice [7,17]. The central values of m thus obtained are $m(\omega)/(3\omega^2) = 0.40$ and $m(\omega)/\omega^2 = 1.15$ for the V and PS channels, respectively. We use these values throughout the following figures. We have checked that our conclusions are not modified within the variation of Z_l mentioned above.

Shown in Fig. 1 are $\rho(\omega)$s for J/ψ at $T/T_c = 0.78, 1.38$, and 1.62 [Fig. 1(a)] and those at $T/T_c = 1.87$ and 2.33 [Fig. 1(b)]. (The corresponding N_{data} used in these figures are $N_{\text{data}} = 89$, 40, 34, 33, and 25 from low T to high T.) If the deconfined plasma were composed of almost free quarks and gluons, SPFs would show a smooth structure with no pronounced peaks above the $q\bar{q}$ threshold. To the contrary, we find a sharp peak near the zero temperature mass even up to $T \simeq 1.6 T_c$ as shown in Fig. 1(a), while the peak disappears at $T \simeq 1.9 T_c$ as shown in Fig. 1(b). The width of the first peak in Fig. 1(a) partly reflects the unphysical broadening due to the statistics of the lattice data and partly reflects possible physical broadening at finite T. At the moment, the former width of a few hundred MeV seems to dominate and we are not able to draw definite conclusions on the thermal mass shift and broadening.

The second and third peaks in Fig. 1(a) may be related to the fermion doublers as first pointed out for light mesons at $T = 0$ [10]. This must be checked by studying whether the peak position scales as $1/a_\sigma$ by varying a_σ,

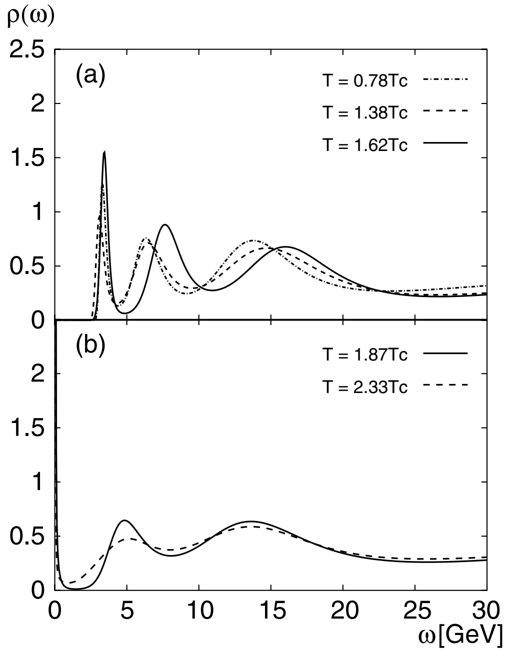

FIG. 1. Spectral functions for J/ψ (a) for $T/T_c = 0.78$, 1.38, and 1.62 and (b) for $T/T_c = 1.87$ and 2.33.

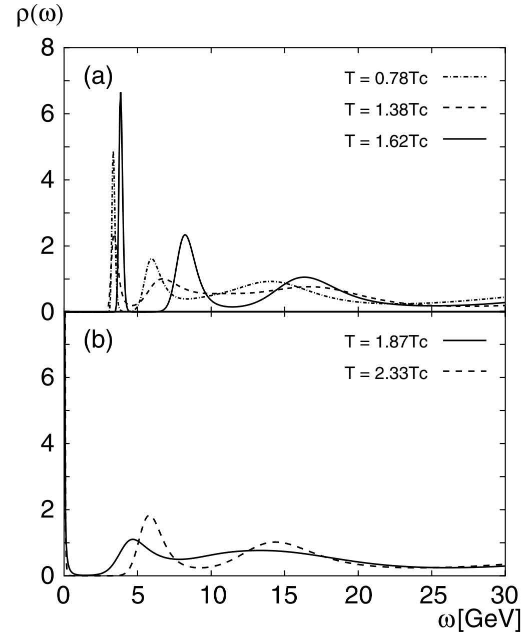

FIG. 2. Spectral functions for η_c (a) for $T/T_c = 0.78$, 1.38, and 1.62 and (b) for $T/T_c = 1.87$ and 2.33.

which remains to be a future problem. Our error analysis shows that the peak around $\omega = 0$ in Fig. 1(b) is not significant in the present statistics. The qualitative change of the spectral structure between $T/T_c = 1.62$ and 1.87, and other features seen in the J/ψ channel are also observed in the η_c channel as shown in Fig. 2.

Let us now evaluate the reliability of the existence (absence) of the sharp peak at $T/T_c = 1.62(1.87)$ by the two tests (i) and (ii) mentioned before. The first test is the error analysis of the peak. Shown in Fig. 3 are the SPFs for J/ψ at $T/T_c = 1.62$ and 1.87 with MEM error bars. (The frequency interval over which the SPF is averaged is characterized by the horizontal position and extension of the bars, while the mean value and the 1σ uncertainty of the integrated strength within the interval are characterized by the heights of the bars.) The sharp peak at $T = 1.62T_c$ is statistically significant, and the absence of the peak at the same position at $T = 1.87T_c$ is also statistically significant. The same features are also observed for η_c.

The second test is the $N_{\rm data}$ dependence of the SPFs. Shown in Fig. 4(a) is a comparison of the SPF obtained with $N_{\rm data} = 34$ and that with $N_{\rm data} = 39$ for the same temperature $T = 1.62T_c$ ($N_\tau = 46$). The two curves are almost identical with each other. (Note that the maximum number of temporal data available in this case is $46 - 7 = 39$.) Figure 4(b) shows SPFs obtained with $N_{\rm data} = 26$ and 33 for a higher temperature $T = 1.87T_c$ ($N_\tau = 40$). The maximum number of data points available is $40 - 7 = 33$ in this case. Again the two curves are almost identical. Therefore, the qualitative change of the SPF between $T = 1.62T_c$ and $1.87T_c$ is the real thermal effect and is not caused by the artifact of the insufficient number of data points.

Here we make a brief comment on related works aiming at studying the charmonia above T_c using MEM [18,19]. In contrast to our large temporal grids ($N_\tau = 46$

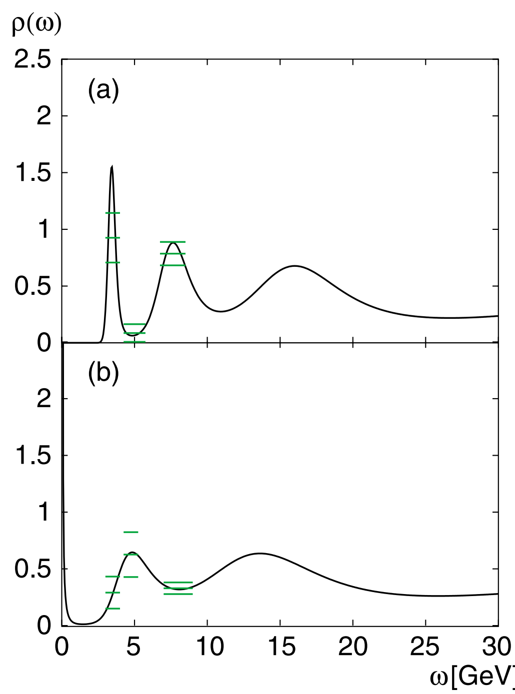

FIG. 3 (color online). Spectral functions for J/ψ with MEM errors (a) for $T/T_c = 1.62$ and (b) for $T/T_c = 1.87$.

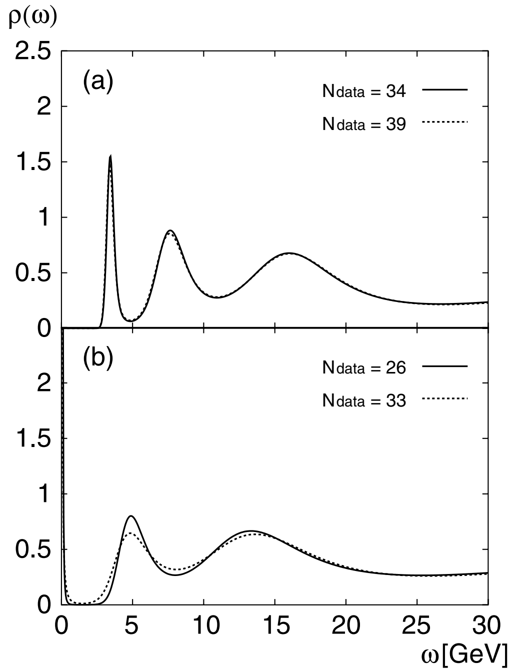

FIG. 4. Comparison of the SPF for J/ψ (a) for $N_{\rm data} = 34$ and 39 with $N_\tau = 46$ ($T/T_c = 1.62$) and (b) for $N_{\rm data} = 26$ and 33 with $N_\tau = 40$ ($T/T_c = 1.87$).

for $T = 1.62T_c$), N_τ in these papers are 3–4 times smaller ($N_\tau = 17$ in [18] and $N_\tau = 12$ in [19] at $T = 1.62T_c$). For such small N_τ with point-point correlation, the reliability tests (i) and (ii) need to be done before drawing physics conclusions. There is indeed evidence that $N_\tau = 17$ fails test (ii) as shown in [18].

In summary, we have extracted the spectral functions of J/ψ and η_c in the deconfined plasma using lattice Monte Carlo data and the maximum entropy method. In the quenched approximation, the number of temporal sites N_τ is taken as large as 46 and 40 for $T/T_c = 1.62$ and 1.87, respectively. Careful analyses of the MEM errors and the $N_{\rm data}$ dependence of the results are carried out. It is found that there are distinct resonances up to $T \simeq 1.6T_c$ and they disappear between $1.6T_c$ and $1.9T_c$. This together with our previous results on $s\bar{s}$ mesons [8] on the same lattice indicates that the quark-gluon plasma is still strongly interacting above T_c so that it can develop hadronic resonances. Whether these resonances found in quenched simulation survive in full simulation with dynamical quarks is an interesting future problem. It is also important to unravel the nature of such resonances by studying their spatial structure on the lattice [20] or by using nonrelativistic potential approaches [21].

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*Electronic address: yuki@ruby.scphys.kyoto-u.ac.jp
†Electronic address: hatsuda@phys.s.u-tokyo.ac.jp

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