**Granger and Greene Reply:** In our Letter [1], the quantum photoabsorption cross section is written in terms of a long- ($\underline{S}^{\rm LR}$) and short- ($\underline{S}^{\rm core}$) range $S$ matrix:

$$\sigma(w) \propto {\rm Re}\omega_0 \vec{d}[\underline{1} - \underline{S}^{\rm eff}(w)]^{-1}[\underline{1} + \underline{S}^{\rm eff}(w)]\vec{d}^{\dagger}, \quad (1)$$

where $\underline{S}^{\rm eff} = \underline{S}^{\rm core}\underline{S}^{\rm LR}(w)$. In closed-orbit theory (COT) [2], on the other hand, the cross section is given as a power series:

$$\sigma^{\rm scl}(w) \propto \sum_{n=0}^{\infty} \sigma_n^{\rm scl}(w). \quad (2)$$

Our work elucidates the convergence properties of the semiclassical result of COT, Eq. (2), and shows how it emerges out of the more accurate quantum expression, Eq. (1).

We claim that it is useful to expand Eq. (1) in a power series using the relationship $[\underline{1} - \underline{S}^{\rm eff}]^{-1} = \sum_{n=0}^{\infty}(\underline{S}^{\rm eff})^n$. Then physical insight can be extracted by studying the Fourier transforms of individual terms in the resulting series expansion of the fully quantal cross section:

$$\sigma(w) \propto {\rm Re}\omega_0 \vec{d}\left[\underline{1} + 2\sum_{n=1}^{\infty}(\underline{S}^{\rm eff})^n\right]\vec{d}^{\dagger}. \quad (3)$$

In his Comment [3], Matzkin asserts that this expansion is "purely formal" and questions whether or not it is physically meaningful to extract physics from its individual terms (through a Fourier transform). We are unclear what is meant by purely formal in this context; this series converges absolutely to the closed form expression, Eq. (1), when $\underline{S}^{\rm eff}$ is evaluated at a complex "energy" $w + i\Gamma/2$. While Matzkin correctly points out that the series form of the cross section, Eq. (3), can and will oscillate wildly about the closed form result, Eq. (1), as more terms are added to the series, we disagree that this invalidates our interpretations.

This is made clear by recognizing that the semiclassical expansion of COT, Eq. (2), has the exact same property of oscillating wildly as more terms are added. In spite of this, researchers have long agreed that each individual term in the semiclassical cross section can be interpreted physically; they are related to classical closed orbits of the system. Moreover, this is assumed even though the convergence properties of the semiclassical cross section have, until now, been on less than solid ground. In some cases (core scattering in high partial waves), the series of Eq. (2) is even known to diverge [4]. Thus, we feel that our interpretation of Eq. (3) stands on the same ground as the interpretation of the semiclassical result.

We have used the phrase "diffractive orbits" to describe paths where the wavelength of the electron is much larger than the distance scale over which the Coulomb potential is varying. We agree that this phrase might be confusing; the term "core-scattered-like" is a better way to describe these recurrences. We also agree with the comment that these orbits are unphysical, if "unphysical" is understood as meaning "not observable in any experiment." Importantly, our theory agrees with experiment on this point.

At the end of his Comment [3], Matzkin questions how we include this cancellation in our semiclassical theory to arrive at our final semiclassical result [Eq. (6) in [1]]. To clarify these issues, we now give a few more details on how we derive our semiclassical result [5]. The first step is to rewrite Eq. (3) as a power series in the matrix $\underline{T} = \underline{S}^{\rm core} - \underline{1}$:

$$\sigma \propto {\rm Re}\omega_0 \vec{d}\left[\underline{1} + 2\underline{S}^{\rm core}\tilde{S}^{\rm LR}\sum_{n=0}^{\infty}(\underline{T}\tilde{S}^{\rm LR})^n\right]\vec{d}^{\dagger}, \quad (4)$$

where $\tilde{S}^{\rm LR}$ is not the original $\underline{S}^{\rm LR}$, but rather a resummed long-range $S$ matrix $\tilde{S}^{\rm LR} = \sum_{n=1}^{\infty}(\underline{S}^{\rm LR})^n$. Equation (4) constitutes an exact rearrangement of Eq. (3). It is at this point, working with $\tilde{S}^{\rm LR}$, that we introduce a semiclassical approximation ($\underline{S}^{\rm LR} \approx \sum_j \underline{A}_j e^{iS_j}$, where $j$ is a sum over the closed orbits with action $S_j$) and include the cancellation [Eq. (5) of [1]] between ghost and core-scattered-like orbits. The final semiclassical result,

$$\tilde{S}^{\rm LR} \approx \underline{S}_{\rm scl}^{\rm LR} = \sum_j \sum_n \underline{A}_{j,n} e^{iS_{j,n}}, \quad (5)$$

is then a sum over all closed orbits $j$ and their repetitions $n$. Semiclassically, the resummed $\tilde{S}^{\rm LR}$ generates repetitions of the closed orbits. Using the $\underline{S}_{\rm scl}^{\rm LR}$ of Eq. (5) in Eq. (4) shows that our final result is a power series in $\hbar$.

B. E. Granger
ITAMP, Harvard-Smithsonian CFA
Cambridge, Massachusetts 02138, USA

C. H. Greene
JILA and the Department of Physics
University of Colorado
Boulder, Colorado 80903-0440, USA

Received 10 April 2003; published 22 December 2003
DOI: 10.1103/PhysRevLett.91.269302
PACS numbers: 32.80.Rm

[1] B. E. Granger and C. H. Greene, Phys. Rev. Lett. **90**, 043002 (2003).
[2] M. L. Du and J. B. Delos, Phys. Rev. A **38**, 1896 (1988).
[3] A. Matzkin, preceding Comment, Phys. Rev. Lett. **91**, 269301 (2003).
[4] J. Shaw (private communication).
[5] B. E. Granger, Ph.D. thesis, University of Colorado, 2001.

  