Dynamical Coulomb Blockade and Spin-Entangled Electrons

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We consider the creation of mobile and nonlocal spin-entangled electrons from tunneling of a BCS-superconductor (SC) to two normal leads of finite resistivity. The resulting dynamical Coulomb blockade effect, which we describe phenomenologically in terms of an electromagnetic environment, is shown to be enhanced for tunneling of two electrons from a Cooper pair into the same lead compared to the desired pair-split process where each electron enters a different lead. Conversely, this latter process is suppressed by a finite separation between the tunneling points on the SC.

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Introduction.—The controlled creation of nonlocal entanglement is crucial in quantum communication as well as in quantum computation tasks [1]. Several solid state entanglers, a device that creates mobile and nonlocal pairwise entangled electrons, were proposed recently [2–6]. A particularly interesting quantity is the spin of the electron which was shown to be a promising realization of a quantum bit [7]. A natural source of spin entanglement is provided by Cooper pairs in an *s*-wave superconductor (SC), since the Cooper pairs are in a spin-singlet state. Weakly coupling the SC to a normal region allows for (pair-)tunneling of Cooper pairs from the SC to normal leads and single-particle tunneling is suppressed at low energies below the SC gap. Coulomb interaction between the two electrons of a pair can then be used to separate them spatially leading to nonlocality. To mediate the necessary interaction entangler setups containing quantum dots [3] or that exhibit Luttinger liquid correlations [4,5] (e.g., nanotubes in the metallic regime) were proposed recently.

In this Letter we show that a considerably simpler experimental realization can be used to generate the necessary Coulomb interaction between the electrons of a pair. Indeed, if the normal leads are resistive a dynamical Coulomb blockade (CB) effect is generated with the consequence that in a pair tunneling process into the same lead the second electron still experiences the Coulomb repulsion of the first one, which has not yet diffused away. Natural existing candidates with long spin decoherence lengths ($\sim$100 μm [8]) for such a setup are, e.g., semiconductor systems tunnel coupled to a SC, as experimentally implemented in InAs [9], InGaAs [10], or GaAs/AlGaAs [11]. Recently, a two-dimensional electron gas (2DEG) with a resistance per square approaching the quantum resistance $R_Q = h/e^2 \sim 25.8\ \text{k}\Omega$ could be achieved by depleting the 2DEG with a voltage applied between a back gate and the 2DEG [12]. In metallic normal NiCr leads of width $\sim$100 nm and length $\sim$10 μm, resistances of $R = 22$–$24\ \text{k}\Omega$, have been produced at low temperatures. Even larger resistances $R = 200$–$250\ \text{k}\Omega$ have been measured in Cr leads [13].

We use a phenomenological approach to describe charge dynamics in the electromagnetic circuit which is described in terms of normal-lead impedances and junction capacitances; see Fig. 1. The subgap transport of a single SN junction under the influence of an electromagnetic environment has been studied in detail [14,15]. In order to create nonlocal entangled states in the leads we have to go beyond previous work to investigate the physics of two tunnel junctions in parallel with two distinct transport channels for singlets. A Cooper pair can tunnel as a whole into one lead, or the pair can split and the two electrons enter separate leads, leading to a nonlocal spin singlet in the leads. Subsequently, the degree of spin entanglement can be detected via an enhanced shot noise in a beam splitter setup [16] or, alternatively, by measuring Bell's inequality [17]. By using spin filters, e.g., quantum dots in the CB regime [18], one can then use current-current correlation measurements for testing Bell's inequality. In the case where the pair splits we

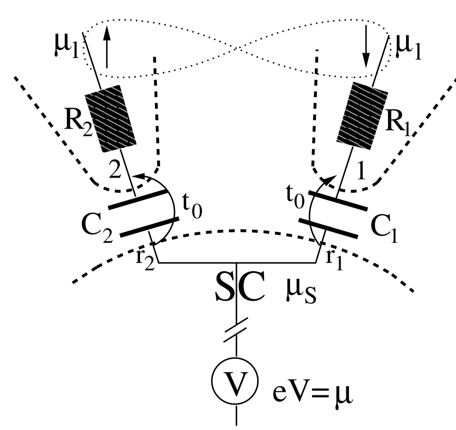

FIG. 1. Entangler setup: A BCS bulk superconductor (SC) with chemical potential μ_S is tunnel coupled (amplitude t_0) via points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the SC to two Fermi liquid leads 1, 2 with resistance $R_{1,2}$. The leads are held at the same chemical potential μ_l such that a bias voltage $\mu = \mu_S - \mu_l$ is applied between the SC and the two leads via the voltage source V. The tunnel junctions 1, 2 have capacitances $C_{1,2}$.

find that the dynamical CB effect is uncorrelated for the two electron charges. In contrast, if the two electrons tunnel into the same lead we find a dynamical CB consistent with a charge $q = 2e$, where e is the elementary charge. Thus the CB effect is twice as large for the unsplit process which enhances the probability for a nonlocal (pair-split) process. On the other hand, we show that the spatial correlations of a Cooper pair results in a suppression factor for tunneling via different junctions which is weaker for lower dimensional SCs.

Setup and formalism.—The setup is sketched in Fig. 1. The SC is held at the (electro-)chemical potential μ_S by a voltage source V. The two electrons of a Cooper pair can tunnel via two junctions placed at points r_1 and r_2 on the SC to two separate normal leads 1 and 2 which have resistances R_1 and R_2, respectively. They are kept at the same chemical potential μ_l so that a bias voltage $\mu \equiv \mu_S - \mu_l$ is applied between SC and leads [19]. The system Hamiltonian decomposes into three parts $H = H^e + H_{\rm env} + H_T$. Here $H^e = H_S + \sum_{n=1,2} H_{ln}$ describes the electronic parts of the isolated subsystems consisting of the SC and Fermi liquid leads $n = 1, 2$, with $H_{ln} = \sum_{\mathbf{p}\sigma} \varepsilon_p c^\dagger_{n\mathbf{p}\sigma} c_{n\mathbf{p}\sigma}$, where $\sigma = (\uparrow, \downarrow)$. The s-wave bulk SC is described by the BCS Hamiltonian $H_S - \mu_S N_S = \sum_{\mathbf{k}\sigma} E_k \gamma^\dagger_{\mathbf{k}\sigma}\gamma_{\mathbf{k}\sigma}$ with the quasiparticle spectrum $E_k = (\xi_k^2 + \Delta^2)^{1/2}$ where $\xi_k = k^2/2m - \mu_S$. The electron creation ($c^\dagger_{\mathbf{k}\sigma}$) and annihilation ($c_{\mathbf{k}\sigma}$) operators are related to the quasiparticle operators by the Bogoliubov transformation $c_{\mathbf{k}(\uparrow/\downarrow)} = u_k \gamma_{\mathbf{k}(\uparrow/\downarrow)} \pm v_k \gamma^\dagger_{-\mathbf{k}(\downarrow/\uparrow)}$, where u_k and v_k are the usual BCS coherence factors. To describe resistance and dissipation in the normal leads we use a phenomenological approach [20], where the electromagnetic fluctuations in the circuit (being bosonic excitations) due to electron-electron interaction and the lead resistances are modeled by a bath of harmonic oscillators which is linearly coupled to the charge fluctuation Q_n of the junction capacitor n (induced by the tunneling electron). This physics is described by [20,21]

$$H_{\mathrm{env},n} = \frac{Q_n^2}{2C_n} + \sum_{j=1}^{N}\left[\frac{q_{nj}^2}{2C_{nj}} + \frac{(\phi_n - \varphi_{nj})^2}{2e^2 L_{nj}}\right]. \tag{1}$$

The phase ϕ_n of junction n is the conjugate variable to the charge satisfying $[\phi_n, Q_m] = ie\delta_{n,m}$. As a consequence $e^{-i\phi_n}$ reduces Q_n by one elementary charge e. We remark that the SC is held at constant chemical potential μ_S by the voltage source; see Fig. 1. Therefore the charge relaxation of a nonequilibrium charge on one of the capacitors described by (1) does not influence the charge dynamics of the other junction and, as a consequence, $H_{\rm env} = \sum_{n=1,2} H_{\mathrm{env},n}$ [22]. Electron tunneling through junctions 1, 2 located at points $\mathbf{r}_1$, $\mathbf{r}_2$ of the SC nearest to the leads 1, 2 is described by the tunneling Hamiltonian $H_T = \sum_{n=1,2} H_{Tn}$ + H.c., where

$$H_{Tn} = t_0 \sum_\sigma \psi^\dagger_{n\sigma}\Psi_\sigma(\mathbf{r}_n)\, e^{-i\phi_n}. \tag{2}$$

Here t_0 is the bare electron tunneling amplitude which we assume to be spin independent and the same for both leads. Since H_T conserves spin we have $[H, \mathbf{S}^2_{\rm tot}] = 0$, and thus the two electrons from a given Cooper pair singlet which have tunneled to the lead(s) remain in the singlet state.

Current of two electrons tunneling into different leads.—We use a T-matrix approach [23] to calculate tunneling currents. At zero temperature the current I_1 for tunneling of two electrons coming from the same Cooper pair into different leads is given to lowest order in t_0 by [4]

$$I_1 = 2e \sum_{\substack{n\neq n'\\ m\neq m'}} \int_{-\infty}^{\infty} dt \int_0^\infty dt' \int_0^\infty dt'' e^{-\eta(t'+t'')+i(2t-t'-t'')\mu} \langle H^\dagger_{Tm}(t-t'') H^\dagger_{Tm'}(t) H_{Tn}(t') H_{Tn'}(0)\rangle, \tag{3}$$

where $\eta \to 0^+$, and the expectation value is to be taken in the ground state of the unperturbed system. The physical interpretation of Eq. (3) is a hopping process of two electrons with opposite spins from two spatial points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the SC to the two leads 1, 2, thereby removing a Cooper pair in the SC, and back again. The delay times between the two tunneling processes of the electrons within a pair are t' and t'', respectively, whereas the time between destroying and creating a Cooper pair is given by t. This process is contained in the correlation function

$$\begin{aligned}\sum_{\substack{n\neq n'\\ m\neq m'}} \langle H^\dagger_{Tm}(t-t'')\, H^\dagger_{Tm'}(t)\, H_{Tn}(t')\, H_{Tn'}(0)\rangle = |t_0|^4 \sum_{\sigma,\, n\neq m} \{&G_{n\sigma}(t-t'')\, G_{m,-\sigma}(t-t')\, \mathcal{F}_{nm\sigma}(t')\, \mathcal{F}^*_{nm\sigma}(t'')\langle e^{i\phi_n(t-t'')} e^{-i\phi_n(0)}\rangle \\ &\times \langle e^{i\phi_m(t-t')}\, e^{-i\phi_m(0)}\rangle - G_{m,-\sigma}(t-t'-t'')\, G_{n\sigma}(t)\, \mathcal{F}_{nm\sigma}(t') \\ &\times \mathcal{F}^*_{mn,-\sigma}(t'')\langle e^{i\phi_m(t-t'-t'')} e^{-i\phi_m(0)}\rangle\langle e^{i\phi_n(t)}\, e^{-i\phi_n(0)}\rangle\}. \end{aligned} \tag{4}$$

The lead Green's functions are $G_{n\sigma}(t) \equiv \langle \psi_{n\sigma}(t)\psi^\dagger_{n\sigma}(0)\rangle \simeq (\nu_l/2)/it$, with ν_l being the density of states (DOS) per volume at the Fermi level μ_l of the leads. The anomalous Green's function of the SC is $\mathcal{F}_{nm\sigma}(t) \equiv \langle \Psi_{-\sigma}(\mathbf{r}_m, t)\Psi_\sigma(\mathbf{r}_n, 0)\rangle = [\mathrm{sgn}(\sigma)/V_S]\sum_{\mathbf{k}} u_k v_k \exp(-iE_k t + i\mathbf{k}\cdot\delta\mathbf{r})$ with $\delta\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, and V_S is the volume of the SC. The bath correlator can be expressed as $\langle \exp(i\phi_n(t))\exp(-i\phi_n(0))\rangle = \exp[J(t)]$ with $J(t) = 2\int_0^\infty (d\omega/\omega)[\mathrm{Re}Z_T(\omega)/R_Q][\exp(-i\omega t) - 1]$. Here we introduced the total impedance $Z_T = (i\omega C + R^{-1})^{-1}$, with a purely Ohmic lead impedance $Z_n(\omega) = R$, which we assume to be the same for both tunnel junctions and leads. For small times, $\omega_R|t| \ll 1$, we can approximate $J(t) \sim -iE_c t$ where $E_c = e^2/2C$ is the charging energy and $\omega_R = 1/RC$ is the bath frequency cutoff which is the inverse classical charge relaxation time τ_{cl} of an RC circuit. For the long-time behavior, $\omega_R|t| \gg 1$, we get $J(t) \sim -(2/g)[\ln(i\omega_R t) + \gamma]$ with $\gamma = 0.5772$ the Euler number and $g = R_Q/R$ is the dimensionless lead conductance.

We first consider the low bias regime $\mu \ll \Delta, \omega_R$. In this limit the delay times t' and $t'' \lesssim 1/\Delta$ can be neglected compared to $t \lesssim 1/\mu$ in all correlators in (4) and the bath correlators are dominated by the long-time behavior of $J(t)$. We then obtain for the current

$$I_1 = e\pi\mu\Gamma^2 F_d^2(\delta r)\frac{e^{-4\gamma/g}}{\Gamma(4/g+2)}\left(\frac{2\mu}{\omega_R}\right)^{4/g}. \quad (5)$$

The spatial correlation of a Cooper pair results in the suppression factor $F_{d=3}(\delta r) = [\sin(k_F\delta r)/k_F\delta r] \times \exp(-\delta r/\pi\xi)$ with $\delta r = |\delta \mathbf{r}|$. The exponential decay sets in on the length scale of the coherence length ξ. It is on the order of micrometers for usual s-wave materials and therefore $\delta r \ll \xi$ for δr in the range of nanometers. More severe is the power-law decay $\propto 1/(k_F\delta r)^2$ with k_F the Fermi wave number in the SC. This power law is sensitive to the effective dimensionality d of the SC with weaker decay in lower dimensions. Indeed, in two dimensions [24] and for $k_F\delta r \gg 1$, but still $\delta r < \xi$, we get $F_{d=2}^2 \propto 1/(k_F\delta r)$, and in one dimension there is no power-law decay as a function of $k_F\delta r$. In (5) we introduced the gamma function $\Gamma(x)$ and the dimensionless tunnel conductance $\Gamma = \pi\nu_S\nu_l|t_0|^2$ with ν_S being the DOS per volume of the SC at the Fermi level μ_S. The exponent $4/g$ in (5) is twice the value for single electron tunneling [20] via one junction since the tunneling events into different leads are uncorrelated.

We consider now the large bias regime $\Delta, \mu \gg \omega_R$. For $\Delta \gg |\mu - E_c| \gg \omega_R$ we can use the short time expansion for $J(t)$ in (4) and can again neglect the delay times t' and t'' compared to t in all correlators in (4). We then obtain for I_1 in the large bias limit and up to small contributions $\sim e\pi\Gamma^2 F_d^2(\delta r)\omega_R[\mathcal{O}(\omega_R/\mu) + \mathcal{O}(\omega_R/|\mu - E_c|)]$

$$I_1 = e\pi\Gamma^2 F_d^2(\delta r)\Theta(\mu - E_c)(\mu - E_c). \quad (6)$$

This shows a gap in I_1 for $\mu < E_c$ and $R \to \infty$ which is the hallmark of dynamical CB.

Current of two electrons tunneling into the same lead.—The current I_2 for tunneling of two electrons into the same lead, 1 or 2, is given by (3) but with $n = n'$ and $m = m' = n$. We assume that the two electrons tunnel off the SC from the same point and therefore $\delta r = 0$ here. Since both electrons tunnel into the same lead the bath correlation functions do not separate anymore as was the case in (4). Instead we have to look at the full 4-point correlator

$$\langle e^{i\phi_n(t-t'')}e^{i\phi_n(t)}e^{-i\phi_n(t')}e^{-i\phi_n(0)}\rangle = e^{J(t-t'-t'')+J(t-t')+J(t-t'')+J(t)-J(t')-J(-t'')}. \quad (7)$$

The lead correlators again factorize into a product of two single-particle Green's functions since they are assumed to be Fermi liquids and in addition there appear no spin correlations due to tunneling of two electrons with opposite spins.

We first consider the low bias regime $\mu \ll \omega_R, \Delta$. Here again we can assume that $|t|$ is large compared to the delay times t' and t'', but it turns out to be crucial to distinguish carefully between $\Delta \gg \omega_R$ and $\Delta \ll \omega_R$. We first treat the case $\Delta \gg \omega_R$ and approximate $\exp[-(J(t') + J(-t''))] \simeq \exp[-iE_c(t'' - t')]$ in (7). In this limit and for $\Delta > E_c$ the current I_2 becomes

$$I_2 = e\pi\mu\Gamma^2\frac{(4\Delta/\pi)^2}{\Delta^2 - E_c^2} \times \arctan^2\left\{\sqrt{\frac{\Delta + E_c}{\Delta - E_c}}\right\}\frac{e^{-8\gamma/g}}{\Gamma(8/g+2)}\left(\frac{2\mu}{\omega_R}\right)^{8/g}. \quad (8)$$

The tunneling of a charge $q = 2e$ clearly shows up in the exponent $8/g$ in (8). In addition to this double charging effect we see from (8) that increasing E_c has two opposite effects. On the one hand the factor $(2\mu/\omega_R)^{8/g}$ decreases, while the Δ-dependent terms increase. The latter terms result from a charge relaxation in the virtual state with one electron of the entangled pair still being on the SC. The formula (8) is valid if $\sqrt{(\Delta - E_c)/(\Delta + E_c)} \gg \sqrt{\omega_R/\Delta}$.

In the other limit where $\Delta \ll \omega_R$, e.g., for small R, we can assume that $\omega_R t'$ and $\omega_R t'' \gg 1$ and therefore approximate $\exp[-(J(t') + J(-t''))] \simeq \exp(4\gamma/g) \times \omega_R^{4/g}(t't'')^{2/g}$. In this limit we obtain

$$I_2 = e\pi\mu\Gamma^2 A(g)\left(\frac{2\mu}{\omega_R}\right)^{4/g}\left(\frac{2\mu}{\Delta}\right)^{4/g}, \quad (9)$$

with $A(g) = (2e^{-\gamma})^{4/g}\Gamma^4(1/g + 1/2)/\pi^2\Gamma(8/g + 2)$. Here the relative suppression of the current I_2 compared to I_1 is proportional to $(2\mu/\Delta)^{4/g}$ and not to $(2\mu/\omega_R)^{4/g}$ as in the case of an infinite Δ. This is because the virtual state with a quasiparticle in the SC can last much longer than τ_{cl}, and, as a consequence, the power-law suppression of the current is weakened since $\Delta \ll \omega_R$ here. To our knowledge, the result (9) was not discussed in the literature so far [27], but similar results are obtained when SCs are coupled to Luttinger liquids [4]. A large gap Δ is therefore crucial to suppress I_2.

In the large voltage regime $\Delta, \mu \gg \omega_R$ we expect a Coulomb gap due to a charge $q = 2e$. Indeed, in the parameter range $|\mu - 2E_c| \gg \omega_R$ and $\Delta \gg |\mu - E_c|$ we obtain for I_2 again up to small contributions $\sim e\pi\Gamma^2\omega_R[\mathcal{O}(\omega_R/\mu) + \mathcal{O}(\omega_R/|\mu - 2E_c|)]$,

$$I_2 = e\pi\Gamma^2\Theta(\mu - 2E_c)(\mu - 2E_c). \quad (10)$$

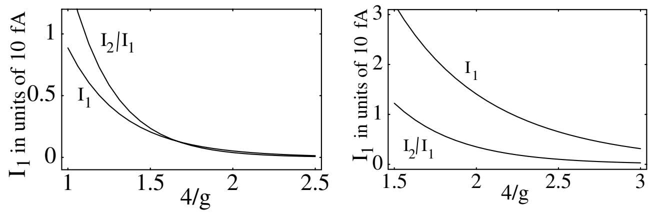

FIG. 2. Current ratio I_2/I_1 (entangler efficiency) and I_1 in the regime $\mu \ll \Delta, \omega_R$ and $\Delta \gg E_c, \omega_R$ as a function of $4/g = 4R/R_Q$. Chosen parameters: $E_c = 0.1$ meV, $k_F \delta r = 10$, $\Gamma = 0.1$, and $\mu = 5$ μeV (left plot), $\mu = 15$ μeV (right plot). For a 2D SC, I_1 and I_1/I_2 can be multiplied by 10.

This shows that I_2 is small $(\propto \omega_R^2/|\mu - 2E_c|)$ in the regime $E_c < \mu < 2E_c$, whereas I_1 is finite $[\propto F_d^2(\delta r)(\mu - E_c)]$.

Discussion and conclusions.—We now give numerical values for the current magnitudes and efficiencies of our entangler. We first discuss the low bias regime $\mu \ll \Delta, \omega_R$. In Fig. 2 we show the ratio I_2/I_1(efficiency of entangler) and I_1 for $\Delta \gg E_c, \omega_R$ as a function of $4/g$ for realistic system parameters (see caption of Fig. 2). The plots show that a very efficient entangler can be expected for lead resistances $R \lesssim R_Q$. The total current is then on the order of $I_1 \gtrsim 10$ fA. In the large bias regime $\mu \gg \omega_R$ and for $E_c < \mu < 2E_c$ we obtain $I_2/I_1 \propto (k_F\delta r)^{d-1}\omega_R^2/(2E_c - \mu)(\mu - E_c)$, where we assume that $2E_c - \mu$ and $\mu - E_c \gg \omega_R$. For $\mu \simeq 1.5E_c$ and using $\omega_R = gE_c/\pi$ we obtain approximately $I_2/I_1 \propto (k_F\delta r)^{d-1}g^2$. To have $I_2/I_1 < 1$ we require $g^2 < 0.01$ for $d = 3$, and $g^2 < 0.1$ for $d = 2$. Such small values of g have been produced approximately in Cr leads [13]. For I_1 we obtain $I_1 \simeq e(k_F\delta r)^{1-d}(\mu - E_c)\Gamma^2 \simeq e(k_F\delta r)^{1-d}E_c\Gamma^2 \simeq 2.5$ pA for $d = 3$ and for the same parameters as used in Fig. 2. This shows that I_1 is much larger than for low bias voltages, but an efficient entangler requires high lead resistances $R \gtrsim 10R_Q$. Our discussion shows that the proposed device should be realizable within state-of-the-art techniques. Finally, we note that the delay time $\sim 1/\Delta$ within a pair is much shorter than the time separation between subsequent pairs $\sim 2e/I_1$, so that different pairs do not overlap in time. This is crucial for detection of entanglement via correlation measurements described in the Introduction.

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[1] M. Nielsen and I. Chuang, *Quantum Computation & Quantum Information* (Cambridge University, Cambridge, England, 2000).

[2] M.-S. Choi, C. Bruder, and D. Loss, Phys. Rev. B **62**, 13 569 (2000).

[3] P. Recher, E.V. Sukhorukov, and D. Loss, Phys. Rev. B **63**, 165314 (2001); D.S. Saraga and D. Loss, Phys. Rev. Lett. **90**, 166803 (2003).

[4] P. Recher and D. Loss, Phys. Rev. B **65**, 165327 (2002).

[5] C. Bena *et al.*, Phys. Rev. Lett. **89**, 037901 (2002).

[6] G.B. Lesovik, T. Martin, and G. Blatter, Eur. Phys. J. B **24**, 287 (2001); S. Bose and D. Home, Phys. Rev. Lett. **88**, 050401 (2002); W.D. Oliver, F. Yamaguchi, and Y. Yamamoto, Phys. Rev. Lett. **88**, 037901 (2002); P. Samuelsson, E.V. Sukhorukov, and M. Büttiker, Phys. Rev. Lett. **91**, 157002 (2003); C.W.J. Beenakker *et al.*, Phys. Rev. Lett. **91**, 147901 (2003); D.S. Saraga *et al.*, cond-mat/0310421.

[7] D. Loss and D.P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).

[8] J.M. Kikkawa and D.D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998).

[9] J. Nitta *et al.*, Phys. Rev. B **46**, 14 286 (1992); C. Nguyen, H. Kroemer, and E.L. Hu, Phys. Rev. Lett. **69**, 2847 (1992).

[10] S. De Franceschi *et al.*, Appl. Phys. Lett. **73**, 3890 (1998).

[11] A.M. Marsh, D.A. Williams, and H. Ahmed, Phys. Rev. B **50**, 8118 (1994).

[12] A.J. Rimberg *et al.*, Phys. Rev. Lett. **78**, 2632 (1997).

[13] L.S. Kuzmin *et al.*, Phys. Rev. Lett. **67**, 1161 (1991).

[14] J.J. Hesse and G. Diener, Physica (Amsterdam) **203B**, 393 (1994).

[15] A. Huck, F.W.J. Hekking, and B. Kramer, Europhys. Lett. **41**, 201 (1998).

[16] G. Burkard, D. Loss, and E.V. Sukhorukov, Phys. Rev. B **61**, R16 303 (2000); X. Hu and S. Das Sarma, cond-mat/0307024.

[17] S. Kawabata, J. Phys. Soc. Jpn. **70**, 1210 (2001).

[18] P. Recher, E.V. Sukhorukov, and D. Loss, Phys. Rev. Lett. **85**, 1962 (2000).

[19] This prevents a current to flow from one lead to the other via the SC.

[20] See, e.g., G.-L. Ingold and Y.V. Nazarov, in *Single Charge Tunneling*, editedy by H. Grabert and M.H. Devoret (Plenum, New York, 1992), Chap 2.

[21] Any lead impedance $Z_n(\omega)$ can be modeled with Eq. (1) via $Z_n^{-1} = \int_{-\infty}^{+\infty} dt \exp(-i\omega t) Y_n(t)$ with admittance $Y_n(t) = \sum_{j=1}^{N}[\Theta(t)/L_{nj}]\cos(t/\sqrt{L_{nj}C_{nj}})$.

[22] A correction to this decoupling assumption is determined by the cross capacitance $C_{12} \ll C_n$, $n = 1, 2$ between the leads 1, 2 and is therefore small.

[23] E. Merzbacher, *Quantum Mechanics* (Wiley, New York, 1998), 3rd ed., Chap. 20.

[24] It was predicted [25] and shown experimentally [26] that a SC on top of a 2DEG can induce superconductivity in the 2DEG via the proximity effect.

[25] A.F. Volkov *et al.*, Physica (Amsterdam) **242C**, 261 (1995).

[26] J. Eroms *et al.*, Europhys. Lett. **58**, 569 (2002).

[27] The result (9) is in contrast to predictions made in [14].