Spin Symmetry in the Antinucleon Spectrum

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We discuss spin and pseudospin symmetry in the spectrum of single nucleons and single antinucleons in a nucleus. As an example we use relativistic mean field theory to investigate single antinucleon spectra. We find a very well developed spin symmetry in single antineutron and single antiproton spectra. The dominant components of the wave functions of the spin doublet are almost identical. This spin symmetry in antiparticle spectra and the pseudospin symmetry in particle spectra have the same origin. However, it turns out that the spin symmetry in antinucleon spectra is much better developed than the pseudospin symmetry in normal nuclear single particle spectra.

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Symmetries in the single particle spectra of atomic nuclei have been extensively discussed in the literature as, e.g., violation of spin symmetry by the spin-orbit term or approximate pseudospin symmetry in nuclear single particle spectra. Atomic nuclei are characterized by a very large spin-orbit splitting; i.e., pairs of single particle states with opposite spin ($j = l \pm \frac{1}{2}$) have very different energies. This fact has allowed the understanding of magic numbers in nuclei and forms the basis of nuclear shell structure. More than 30 years ago [1,2] pseudospin quantum numbers were introduced by $\tilde{l} = l \pm 1$ and $\tilde{j} = j$ for $j = l \pm \frac{1}{2}$, and it has been observed that the splitting between pseudospin doublets in nuclear single particle spectra is by an order of magnitude smaller than the normal spin-orbit splitting.

After the observation that relativistic mean field models yield spectra with nearly degenerate pseudospin-orbit partners [3], Ginocchio showed clearly that the origin of pseudospin symmetry in nuclei is given by a relativistic symmetry in the Dirac Hamiltonian ([4,5] and references therein). He found that pseudospin symmetry becomes exact in the limiting case, where the strong scalar and vector potentials have the same size but opposite sign. However, this condition is never fulfilled exactly in real nuclei, because in this limit the average nuclear potential vanishes and nuclei are no longer bound. It has been found that the quality of pseudospin symmetry is related to the competition between the centrifugal barrier and the pseudospin orbital potential [6].

In relativistic investigations a Dirac Hamiltonian is used. In its spectrum one finds single particle levels with positive energies as well as those with negative energies. The latter are interpreted as antiparticles under charge conjugation. This has led to many efforts to explore configurations with antiparticles and their interaction with nuclei. The possibility of producing a new kind of nuclear system by putting one or more antibaryons inside ordinary nuclei has recently gained renewed interest [7]. For future studies of antiparticles in nuclei it is therefore of great importance to investigate the symmetries of such configurations.

In a relativistic description nuclei are characterized by two strong potentials, an attactive scalar field $-S(\boldsymbol{r})$ and a repulsive vector field $V(\boldsymbol{r})$ in the Dirac equation which for nucleons (labeled by a subscript "N") reads

$$\{\boldsymbol{\alpha} \cdot \boldsymbol{p} + V_N(\boldsymbol{r}) + \beta[M - S_N(\boldsymbol{r})]\}\psi_N(\boldsymbol{r}, s) = \epsilon_N \psi_N(\boldsymbol{r}, s), \tag{1}$$

where $V_N(\boldsymbol{r}) = V(\boldsymbol{r})$ and $S_N(\boldsymbol{r}) = S(\boldsymbol{r})$. For a spherical system, the Dirac spinor ψ_N has the form

$$\psi_N(\boldsymbol{r}, s) = \frac{1}{r}\begin{pmatrix} iG_{n\kappa}(r)Y^l_{jm}(\theta, \phi, s) \\ -F_{\tilde{n}\kappa}(r)Y^{\tilde{l}}_{jm}(\theta, \phi, s)\end{pmatrix}, \qquad j = l \pm \frac{1}{2}, \tag{2}$$

where $Y^l_{jm}(\theta, \phi)$ are the spin spherical harmonics. $G_{n\kappa}(r)/r$ and $F_{\tilde{n}\kappa}(r)/r$ form the radial wave functions for the upper and lower components with n and $\tilde{n}$ radial nodes. $\kappa = \langle 1 + \boldsymbol{\sigma} \cdot \mathbf{l}\rangle = (-1)^{j+l+1/2}(j + 1/2)$ characterizes the spin-orbit operator and the quantum numbers l and j. $\tilde{l} = l - \mathrm{sign}(\kappa)$ is the orbital angular momentum of the lower component. It is therefore well accepted that the pseudospin quantum numbers of a particle state with positive energy are nothing but the quantum numbers of the lower component [4,5].

Charge conjugation leaves the scalar potential $S_N(\boldsymbol{r})$ invariant while it changes the sign of the vector potential $V_N(\boldsymbol{r})$. That is, for antinucleons (labeled by "A"), $V_A(\boldsymbol{r}) = -V_N(\boldsymbol{r}) = -V(\boldsymbol{r})$ and $S_A(\boldsymbol{r}) = S_N(\boldsymbol{r}) = S(\boldsymbol{r})$. Charge conjugation of Eq. (2) gives the Dirac spinor for an antinucleon,

$$\psi_A(\boldsymbol{r}, s) = \frac{1}{r}\begin{pmatrix} -F_{\tilde{n}\tilde{\kappa}}(r)Y^{\tilde{l}}_{jm}(\theta, \phi, s) \\ iG_{n\tilde{\kappa}}(r)Y^{l}_{jm}(\theta, \phi, s) \end{pmatrix}, \qquad j = l \pm \frac{1}{2}, \quad (3)$$

with $\tilde{\kappa} = -\kappa$.

We are interested only in positive energy states of the Dirac equations. Therefore normal quantum numbers follow the upper component which is dominant. A particle state is labeled by $\{nl\kappa m\}$, while its pseudoquantum numbers are $\{\tilde{n}\,\tilde{l}\,\tilde{\kappa}\,m\}$. Following Ref. [8], $\tilde{n} = n + 1$ for $\kappa > 0$; $\tilde{n} = n$ for $\kappa < 0$. An antiparticle state is labeled by $\{\tilde{n}\,\tilde{l}\,\tilde{\kappa}\,m\}$, and its pseudoquantum numbers are $\{nl\kappa m\}$. In analogy to Ref. [8], we deduce the relation

$$n = \tilde{n} + 1, \quad \text{for } \tilde{\kappa} > 0; \qquad n = \tilde{n}, \quad \text{for } \tilde{\kappa} < 0. \quad (4)$$

With $\kappa(1 - \kappa) = \tilde{l}(\tilde{l} + 1)$ and $\kappa(1 + \kappa) = l(l + 1)$ in mind, one derives Schrödinger-like equations for the upper and the lower components

$$\left[-\frac{1}{2M_+}\left(\frac{d^2}{dr^2} + \frac{1}{2M_+}\frac{dV_+}{dr}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right) - \frac{1}{4M_+^2}\frac{\kappa}{r}\frac{dV_+}{dr} + M - V_-\right]G(r) = \begin{cases} +\epsilon_N G(r), \\ -\epsilon_A G(r), \end{cases} \quad (5)$$

$$\left[-\frac{1}{2M_-}\left(\frac{d^2}{dr^2} - \frac{1}{2M_-}\frac{dV_-}{dr}\frac{d}{dr} + \frac{\tilde{l}(\tilde{l}+1)}{r^2}\right) + \frac{1}{4M_-^2}\frac{\tilde{\kappa}}{r}\frac{dV_-}{dr} + M - V_+\right]F(r) = \begin{cases} -\epsilon_N F(r), \\ +\epsilon_A F(r). \end{cases} \quad (6)$$

where $V_\pm(r) = V(r) \pm S(r)$ and $M_\pm = M_\pm(\epsilon) = M \pm \epsilon \mp V_\pm$ with $\epsilon = +\epsilon_N$ (for particle states) or $-\epsilon_A$ (for antiparticle states). Both equations are fully equivalent to the exact Dirac equation with the full spectrum of particle and antiparticle states.

We give the relation between spin or pseudospin symmetry and the external fields in Table I. If $dV_+/dr = 0$, we have exact spin symmetry in the particle spectrum and exact pseudospin symmetry in the antiparticle spectrum because states with the same l (but different κ) are degenerate in Eq. (5). l is the orbital angular momentum of particle states and pseudo-orbital angular momentum of antiparticle states. When $dV_+/dr \neq 0$, the symmetries are broken. But if dV_+/dr is so small that the spin-orbit term (the term $\sim\kappa$) in Eq. (5) is much smaller than the centrifugal term, there will be approximate symmetries.

Similarly, when $dV_-/dr = 0$ in Eq. (6), there is an exact pseudospin symmetry in the particle spectra [4,5]. On the other hand, if we focus on antiparticle states, we have in this case exact spin symmetry because now $\tilde{l}$ is the orbital angular momentum. If $dV_-/dr \neq 0$ but small, we have approximate pseudospin symmetry in particle spectra and approximate spin symmetry in antiparticle spectra. This implies that the spin symmetry in the antiparticle spectrum has the same origin as the pseudospin symmetry in particle spectrum as realized in Ref. [5]. However, there is an essential difference in the degree to which the symmetry is broken in both cases: the factor $1/M_-^2 = 1/(M - \epsilon + V_-)^2$ is much smaller for antinucleon states than that for nucleon states. The bound antiparticle energies ϵ_A are in the region between $M - V_+(0) \lesssim \epsilon_A \lesssim M$. For realistic nuclei roughly we therefore have 0.3 GeV $\lesssim \epsilon_A \lesssim$ 1 GeV. On the other hand, the bound particle states are in the region of $M - |V_-(0)| \lesssim \epsilon_N \lesssim M$, i.e., for realistic nuclei close to 1 GeV. We therefore have $|M_-(\epsilon_A)| > 2|M - S(0)|$ and $|M_-(\epsilon_N)| < |V_-(0)|$. Thus the factor in front of the $\tilde{\kappa}$ term is for antiparticle states by more than a factor $(2|M - S(0)|/|V_-(0)|)^2 \approx 400$ smaller than for particle states. Spin symmetry for antiparticle states is therefore much less broken than pseudospin symmetry for particle states.

TABLE I. Relation between symmetry and external fields.

	Particle	Antiparticle
$dV_+/dr = 0$	Spin symmetry	Pseudospin symmetry
$dV_-/dr = 0$	Pseudospin symmetry	Spin symmetry

Since the spin-orbit term in Eq. (6) is so small for antinucleon states, we expect, in addition, that the radial wave functions of the spin doublets are nearly identical; i.e., the dominant components of spin partners for antiparticle solutions are much more similar than the small components of pseudospin partners for particles.

Although the present discussion is meant for single particle spectra in atomic nuclei, the idea is very general. It has first been discovered that the equality of the vector and scalar potentials results in spin symmetry in Refs. [9,10] where the authors suggested applications to meson spectra. However, this symmetry was only recently found to be valid for mesons with one heavy quark [11]. In the present Letter, we illustrate for the first time in realistic nuclei nearly exact spin symmetry in the single particle spectra for antinucleons. We use for that purpose nonlinear relativistic mean field (RMF) theory [12] with modern parameter set NL3. Relativistic Hartree calculations are carried out in coordinate space for the doubly magic nuclei ^{16}O and ^{208}Pb. With $V_N(\boldsymbol{r})$ and $S_N(\boldsymbol{r})$ replaced by $V_A(\boldsymbol{r})$ and $S_A(\boldsymbol{r})$, respectively, and $\psi_N(\boldsymbol{r}, s)$ replaced by $\psi_A(\boldsymbol{r}, s)$, Eq. (1) is solved for the antinucleon states by the same way it is solved for the nucleon states.

For ^{16}O, pseudospin symmetry cannot be studied successfully because there are only a few bound nucleon states. However, as seen in Fig. 1, there are many more antiparticle states. We find excellent spin symmetry for them. Since there are too many levels in antiparticle spectra of ^{208}Pb (around 400 for either antineutrons or antiprotons), we will not give a similar figure in this case.

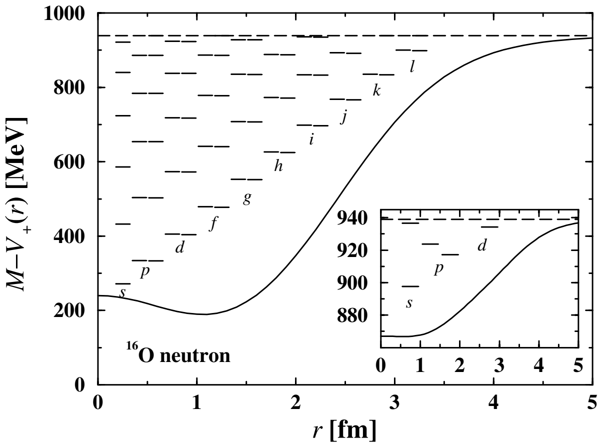

FIG. 1. Antineutron potential and spectrum of ^{16}O. For each pair of the spin doublets, the left level is with $\tilde{\kappa} < 0$ and the right one with $\tilde{\kappa} > 0$. The inset gives neutron potential $M + V_-(r)$ and spectrum.

In Fig. 2 we present the spin-orbit splitting in antineutron spectra of ^{16}O and ^{208}Pb. For ^{16}O, the spin-orbit splittings are around 0.2–0.5 MeV for p states ($l = 1$). With increasing particle number A the spin symmetry in the antiparticle spectra becomes even more exact. For ^{208}Pb, the spin-orbit splittings are $\sim$0.1 MeV for p states and less than 0.2 MeV even for h states ($l = 5$) as seen in the lower panel of Fig. 2. We show in Table II the pseudospin orbit splitting of the neutron spectrum of ^{208}Pb to compare them with the spin-orbit splitting in antinucleon spectra. In most cases, the pseudospin orbit splittings for

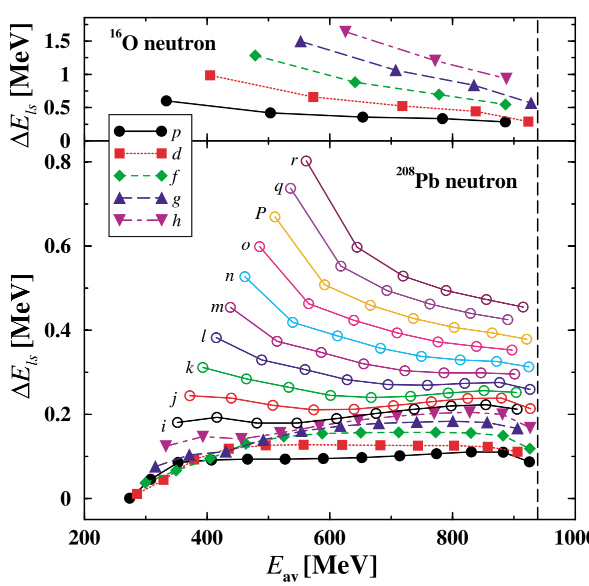

FIG. 2 (color online). Spin-orbit splitting $\epsilon_A(nl_{l-1/2}) - \epsilon_A(nl_{l+1/2})$ in antineutron spectra of ^{16}O and ^{208}Pb versus the average energy of a pair of spin doublets. The vertical dashed line shows the continuum limit.

TABLE II. Energies (in MeV) of some pseudospin doublets in neutron spectrum of ^{208}Pb.

$(n+1)s_{1/2}$	$nd_{3/2}$	ΔE	$(n+1)p_{3/2}$	$nf_{5/2}$	ΔE
895.046	898.152	−3.106	904.603	908.520	−3.917
920.168	920.914	−0.746	929.995	930.709	−0.714
938.878	938.455	0.423	$(n+1)f_{7/2}$	$nh_{9/2}$	ΔE
$(n+1)d_{5/2}$	$ng_{7/2}$	ΔE	925.638	927.984	−2.346
914.962	918.517	−3.555	$(n+1)g_{9/2}$	$ni_{11/2}$	ΔE
938.484	938.292	0.192	936.078	936.572	−0.494

particles are larger than 0.4 MeV, and for deeply bound states, it can reach even values around 4 MeV.

In general, the spin-orbit splitting decreases with the state approaching the continuum limit. But for very deeply bound antineutron p, d, f, and g states in ^{208}Pb, the spin-orbit splitting is smaller. This might be due to the competition between the centrifugal barrier and the spin-orbit potential in Eq. (6). In order to investigate this in more detail, we calculated the expectation value of the spin-orbit potential,

$$\mathrm{SOP} = -\int dr F(r)^2 \frac{1}{4M_-^2(\epsilon)} \frac{\tilde{\kappa}}{r} \frac{dV_-}{dr}. \tag{7}$$

Since the upper amplitudes of the two spin doublets are nearly equal to each other (cf. Figs. 4 and 5 below), we expect the difference, ΔSOP, gives the main part of $\Delta\epsilon$ of a pair of spin doublets. In Fig. 3 we present ΔSOP as a function of the average energy for spin doublets in ^{208}Pb. The variational trend of ΔSOP is roughly in agreement with that of $\Delta\epsilon$. Particularly, for deeply bound states, $\Delta\mathrm{SOP} \sim \Delta\epsilon$.

Wave functions of pseudospin doublets in single nucleon spectra have been studied extensively in the literature [5]. The lower amplitudes of pseudospin doublets are

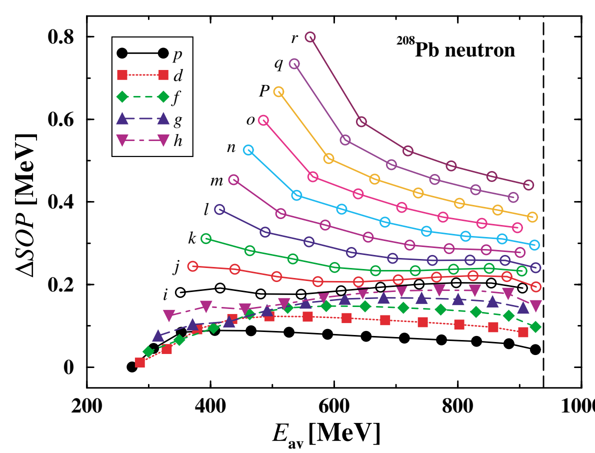

FIG. 3 (color online). Difference of the integration of the spin-orbit potential ΔSOP versus the average energy for spin doublets in ^{208}Pb. The vertical dashed line shows the continuum limit.

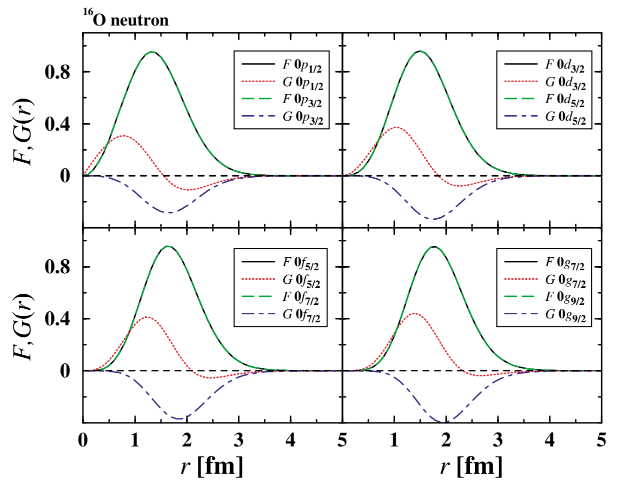

FIG. 4 (color online). Radial wave functions of some spin doublets in the antineutron spectrum of ^{16}O.

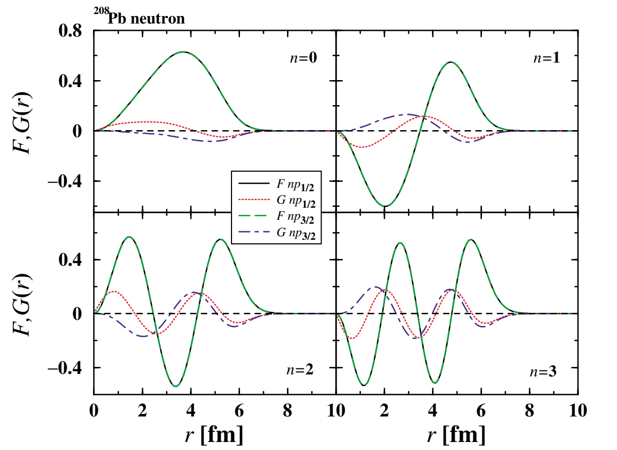

FIG. 5 (color online). Radial wave functions of some spin doublets in the antineutron spectrum of ^{208}Pb.

found to be close to each other. Since the spin symmetry in the antinucleon spectrum is much more exact than the pseudospin symmetry in the single nucleon spectrum, we expect that the upper amplitudes of the spin doublets coincide with each other even more.

In Figs. 4 and 5, we show radial wave functions $F(r)$ and $G(r)$ for several antinucleon spin doublets in ^{16}O and ^{208}Pb. The dominant components $F(r)$ are nearly exactly identical for the two spin partners. On the other hand, the small components $G(r)$ of the two spin partners show dramatic deviations from each other. The relation between the node numbers of the upper and lower amplitudes given in Eq. (4) is seen in Figs. 4 and 5.

We mention that polarization effects caused by the antinucleon are not taken into account in our calculations. They change both the vector and the scalar potentials [7], thus making the spin symmetry a bit worse. However, these effects are never taken into account in discussions of spin and pseudospin symmetries, and we did not include the imaginary part of the optical potential of the antinucleons. The annihilation probability of the antinucleon in the nucleus is, of course, very large and makes it very difficult to observe the small spin-orbit splitting of the antinucleon levels experimentally.

In summary, we discussed the relation between the (pseudo)spin symmetry in single (anti)particle states and the external fields where the (anti)particle moves. We present the single antinucleon spectra in atomic nuclei as examples and find an almost exact spin symmetry. The origin of the spin symmetry in antinucleon spectra and the pseudospin symmetry in nucleon spectra have the same origin, but the former is much more conserved in real nuclei. We performed RMF calculations for some doubly magic nuclei. Even in a very light nucleus, ^{16}O, the spin symmetry in the antinucleon spectrum is very good. An investigation of wave functions shows that the dominant components of the Dirac spinor of the antinucleon spin doublets are almost identical.

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