Radiative Electroweak Symmetry Breaking Revisited

V. Elias,[1,2] R. B. Mann,[1,3] D. G. C. McKeon,[2] and T. G. Steele[4]

[1]*Perimeter Institute for Theoretical Physics, 35 King Street North, Waterloo, Ontario, Canada, N2J 2W9*
[2]*Department of Applied Mathematics, The University of Western Ontario, London, Ontario, Canada, N6A 5B7*
[3]*Department of Physics, University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1*
[4]*Department of Physics & Engineering Physics, University of Saskatchewan, Saskatoon, Saskatchewan, Canada, S7N 5E2*

(Received 22 April 2003; published 16 December 2003)

In the absence of a tree-level scalar-field mass, renormalization-group methods permit the explicit summation of leading-logarithm contributions to all orders of the perturbative series within the effective potential for $SU(2) \times U(1)$ electroweak symmetry. This improvement of the effective potential function is seen to reduce residual dependence on the renormalization mass scale. The all-orders summation of leading-logarithm terms involving the dominant three couplings contributing to radiative corrections is suggestive of a potential characterized by a plausible Higgs boson mass of 216 GeV. However, the tree potential's local minimum at $\phi = 0$ is restored if QCD is sufficiently strong.

DOI: 10.1103/PhysRevLett.91.251601 PACS numbers: 11.30.Qc, 11.10.Hi, 11.15.Tk, 12.15.Lk

Over 30 years ago, Coleman and Weinberg [1] demonstrated how spontaneous symmetry breaking may occur through radiative corrections to a conformally invariant Lagrangian in which no quadratic mass term appears. Such symmetry breaking, in which the scalar-field vacuum expectation value $\langle\phi\rangle$ is the only source of scale, is of particular relevance for the spontaneous breakdown of $SU(2) \times U(1)$ electroweak symmetry, which necessarily requires a mechanism within an embedding theory to keep any such quadratic mass term minimally contaminated by the unification mass scale. The absence of such a mass term implies that this mechanism is exact [2]. We emphasize that such a mechanism, whether exact or nearly so, is a necessary component of the standard model, though the nature of this mechanism (possibly conformal invariance) remains unknown. In the absence of an explicit scalar-field mass term (i.e., the "exact mechanism"), the one-loop (1L) effective potential for $SU(2) \times U(1)$ gauge theory is given by [1]

$$V_{\rm eff}^{(1L)} = \frac{\lambda\phi^4}{4} + \phi^4\left[\frac{12\lambda^2 - 3g_t^2}{64\pi^2} + \frac{3(3g_2^4 + 2g_2^2 g'^2 + g'^4)}{1024\pi^2}\right] \times \left(\ln\frac{\phi^2}{\mu^2} - \frac{25}{6}\right). \quad (1)$$

There are four distinct coupling constants appearing in Eq. (1): the $SU(2)$ coupling constant g_2, the $U(1)$ coupling constant g', the t-quark Yukawa coupling constant g_t, and the quartic scalar-field self-interaction coupling constant λ. The radiative symmetry-breaking scenario of Ref. [1], which preceded the discovery of the massive top quark, led to a value for λ proportional to g_2^4 and a scalar-field mass of order 10 GeV. The presence of a large Yukawa couplant $[g_t^2 \simeq 1.0 \gg g_2^2, g'^2]$ spoils this scenario; the $\mathcal{O}(g_t^2)$ value of λ required for radiative symmetry breaking would be so large that subsequent leading-logarithm terms [e.g., $\lambda^3\phi^4\ln^2(\phi^2/\mu^2)$] would be too large to neglect.

In the present work, we explicitly sum all such leading-logarithm terms within the full perturbative series for the effective potential [3] to examine the viability of radiative electroweak symmetry breaking. We find the potential is minimized for a Higgs mass of 216 GeV, and observe some evidence that this value may be stable after including contributions from subsequent-to-leading logarithms.

If we denote the dominant standard-model couplants as $x \equiv g_t^2/4\pi^2$, $y \equiv \lambda/4\pi^2$, and $z \equiv g_3^2/4\pi^2$ [QCD contributes to leading logarithms past one-loop order], which are much larger than corresponding couplants for g_2, g', and non-t-quark Yukawa couplings, this series is of the form $V_{\rm eff} = \pi^2\phi^4 \sum C_{n,k,\ell,p} x^n y^k z^\ell L^p$, where $L(\mu) \equiv \ln[\phi^2(\mu)/\mu^2]$. The leading logarithms in this series are those terms one degree lower in the power of the logarithm L than in the aggregate power of the couplants $\{x, y, z\}$,

$$V_{LL} = \pi^2\phi^4 S_{LL} = \pi^2\phi^4\left\{\sum_{n=0}^{\infty} x^n \sum_{k=0}^{\infty} y^k \sum_{\ell=0}^{\infty} z^\ell C_{n,k,\ell} L^{n+k+\ell-1}\right\}, \quad (2)$$

$(C_{0,0,0} = 0)$.

The series S_{LL} is determined entirely by one-loop contributions to the renormalization-group (RG) equation, i.e., by those contributions that either lower the power of L by one or raise the aggregate power of the couplants by one [2,4]:

$$\left[-2\frac{\partial}{\partial L} + \left(\frac{9}{4}x^2 - 4xz\right)\frac{\partial}{\partial x} + \left(6y^2 + 3yx - \frac{3}{2}x^2\right)\frac{\partial}{\partial y} - \frac{7}{2}z^2\frac{\partial}{\partial z} - 3x\right] S_{LL}(x, y, z, L) = 0. \quad (3)$$

In Eq. (3), the coefficients of $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ are, respectively, the one-loop beta functions for x, y, z (where $\beta_x = \mu \frac{dx}{d\mu}$); the final term in Eq. (3) is 4 times the one-loop scalar-field anomalous dimension. For example, the leading coefficients $C_{0,1,0} = 1$, $C_{1,0,0} = C_{0,0,1} = 0$, follow from the $\lambda\phi^4/4$ tree-order potential. Upon substitution of Eq. (2) into Eq. (3), one easily sees that $C_{0,2,0} = 3$, that $C_{2,0,0} = -3/4$, and that the remaining four degree-2 coefficients $C_{i,j,2-i-j}$ equal zero, leading to a recovery of the $\{\lambda^2, g_t^2\}$ contributions to the potential (1).

We find it convenient to express the series (2) in the form

$$S_{LL} = yF_0(w, \zeta) + \sum_{n=1}^{\infty} x^n L^{n-1} F_n(w, \zeta), \tag{4}$$

where $w \equiv 1 - 3yL$ and $\zeta \equiv zL$, and where

$$F_n(w, \zeta) = \sum_{\ell=0}^{\infty}\sum_{k=0}^{\infty} C_{n,\ell,k}\left(\frac{1-w}{3}\right)^{\ell}\zeta^k \equiv \sum_{k=0}^{n+1} f_{n,k}(\zeta)\left[\frac{w-1}{w}\right]^k. \tag{5}$$

By using Eq. (3) to obtain sequential partial differential equations relating $F_0 = 1/w$ to $F_1(w, \zeta)$ and $F_2(w, \zeta)$, we are able to solve explicitly for these quantities. For $p \geq 3$ one can show from Eq. (3) that

$$0 = \left[\left((7\zeta^2/2)\frac{d}{d\zeta} + 4p\zeta\right) + \left(2\zeta\frac{d}{d\zeta} + 2(p-1) + 2k\right)\right] f_{p,k}(\zeta) - [(9p-21)/4 + 3k] f_{p-1,k}(\zeta) + 3(k-1) f_{p-1,k-1}(\zeta) - [9(k-1)/2] f_{p-2,k-1}(\zeta) + 9k f_{p-2,k}(\zeta) - [9(k+1)/2] f_{p-2,k+1}(\zeta), \tag{6}$$

where $f_{p,k} \equiv 0$ when $k < 0$ or $k > p + 1$, and where $f_{p,k}(0)$ is finite.

We now examine possible radiative spontaneous symmetry breaking for the RG-improved effective potential $V_{\text{eff}}(\phi) = \pi^2\phi^4(S_{LL} + K)$, where K is a finite ϕ^4 counterterm coefficient. As in Ref. [1], we choose $\mu = \langle\phi\rangle = v$ ($L = \ln[\phi^2(v)/v^2]$), in which case $V'_{\text{eff}}(v) = 0$. The counterterm K facilitates the fourth-derivative renormalization condition $V^{(4)}_{\text{eff}}(v) = V^{(4)}_{\text{tree}}(v) = 24\pi^2 y$ ($= 6\lambda$). For any one-loop effective potential of the form $V^{(1L)}_{\text{eff}}(\phi) = \pi^2\phi^4[y + \sigma L + K]$, this fourth-derivative condition ensures that $K = -25\sigma/6$, as in Eq. (1). For our RG-improved potential, we find it convenient to expand S_{LL} in powers of the logarithm L:

$$S_{LL} = y + B\ln\left(\frac{\phi^2}{v^2}\right) + C\ln^2\left(\frac{\phi^2}{v^2}\right) + D\ln^3\left(\frac{\phi^2}{v^2}\right) + E\ln^4\left(\frac{\phi^2}{v^2}\right) + \ldots. \tag{7}$$

We then obtain from Eqs. (3) and (6) an exact determination of terms up to $\mathcal{O}(L^4)$ within Eq. (7):

$$B = 3y^2 - \frac{3}{4}x^2, \tag{8}$$

$$C = 9y^3 + \frac{9}{4}xy^2 - \frac{9}{4}x^2y + \frac{3}{2}x^2z - \frac{9}{32}x^3, \tag{9}$$

$$D = 27y^4 + \frac{27}{2}xy^3 - \frac{3}{2}xy^2z + 3x^2yz - \frac{225}{32}x^2y^2 - \frac{23}{8}x^2z^2 + \frac{15}{16}x^3z - \frac{45}{16}x^3y + \frac{99}{256}x^4, \tag{10}$$

$$E = 81y^5 + \frac{243}{4}xy^4 - 9xy^3z + \frac{45}{32}xy^2z^2 - \frac{69}{16}x^2yz^2 - \frac{135}{8}x^2y^3 + \frac{531}{64}x^2y^2z + \frac{345}{64}x^2z^3 - \frac{603}{256}x^3z^2 + \frac{207}{32}x^3yz - \frac{8343}{512}x^3y^2 - \frac{459}{512}x^4z + \frac{135}{512}x^4y + \frac{837}{1024}x^5. \tag{11}$$

The procedure for obtaining the Higgs mass, as described below, is insensitive to any terms in the leading-logarithm series (7) past $\mathcal{O}(L^4)$. Consequently, Eqs. (8)–(11) are sufficient to determine the entire leading-logarithm contribution to the scalar-field mass to *all* (contributing) orders in $\{x, y, z\}$. These equations are also obtainable via the method-of-characteristics methodology of Bando *et al.* [5], which has been implemented in conventional (nonradiative) standard-model symmetry breaking by Quiros and collaborators [6]. The conditions $V'_{\text{eff}}(v) = 0$ and $V^{(4)}_{\text{eff}}(v) = 24\pi^2 y$, respectively, imply that $y = -B/2 - K$ and $K = -[\frac{25}{6}B + \frac{35}{3}C + 20D + 16E]$. Given the phenomenological standard-model values for the vacuum expectation value $v = 246$ GeV, the t-quark Yukawa couplant $x(v) = 1/4\pi^2$, the QCD couplant $z(v) = \alpha_s(v)/\pi = 0.0329$, as evolved from $\alpha_s(M_z) = 0.120$ [7], we find these constraints taken together constitute a degree-5 equation for the scalar-field self-interaction couplant y. The only real positive-y solution that yields a positive second derivative (hence, a local minimum) is $y = 0.0538$. Once

y is determined, then B, C, D, and E are also numerically determined. To present order, we can approximate the Higgs field propagator pole with the second derivative of the effective potential at $\phi = v$. One then finds $m_\phi^2 \cong V''_{\rm eff}(v) = 8\pi^2 v^2(B + C) = (216\ {\rm GeV})^2$.

In assessing the viability of this result, it is of interest to consider what one would similarly obtain from the one-loop effective potential augmented by a $\pi^2\phi^4 K$ counterterm. Such a potential is seen to correspond to Eq. (7) with B as given by Eq. (8), but with $C = D = E = 0$. The conditions $V'(v) = 0$, $V^{(4)}(v) = 24\pi^2 y$ are then seen to lead to a solution $y = 0.093$, $m_\phi = 350$ GeV. Such a mass is well outside the $\mathcal{O}(200\ {\rm GeV})$ bound on m_ϕ from corrections to electroweak theory [7]. Moreover, it is easy to demonstrate that this value for y is too large to be meaningful. The contributions of y alone to the β function for its own evolution correspond to the β function of an $O(4)$ symmetric scalar-field theory, which is known to five-loop order [8]:

$$\lim_{\substack{x\to 0\\ z\to 0}} \mu \frac{dy}{d\mu} = 6y^2 - \frac{39}{2}y^3 + 187.85y^4 - 2698.3y^5 + 47\,975y^6 + \ldots. \tag{12}$$

If $y = 0.093$, terms of this series increase after the second term, indicative of a failure to converge. By contrast, terms of the series (12) decrease if $y = 0.0538$. Similar results characterize this same scalar-field theory's anomalous dimension, whose terms decrease monotonically when $y = 0.0538$, but fail to do so when $y = 0.093$. Of course, it is of greater interest to estimate possible corrections to our result to two-loop order. For our parameter values $x = 0.0253$, $y = 0.0538$, $z = 0.0329$, the two-loop contributions to the standard-model beta functions and anomalous scalar-field dimension provide corrections no larger than 17% of their one-loop counterparts. This provides us with further confidence that the 216 GeV Higgs mass prediction will be stable upon summation of subsequent next-to-leading logarithms.

In Fig. 1 we compare the residual scale (μ) dependence of $V_{\rm eff}(\phi) = \pi^2\phi^4(S_{LL} + K)$ obtained via Eqs. (8)–(11) to that of the one-loop effective potential discussed in the preceding paragraph. Such dependence in both potentials occurs explicitly through $L(\mu)$ and implicitly through couplants whose one-loop evolution in μ is anchored to the $\mu = v$ initial values given above [e.g., $x(v) = 1/4\pi^2$]. The $K\phi^4$ counterterms in both potentials are each assumed to be RG invariant [$K(v)\phi^4(v)$], since the subleading logarithm contributions ultimately devolving from such terms are uncontrolled by Eq. (3). For $\mu = \{v/2, v, 2v\}$, the curves exhibit the dependence of the potentials on the RG-invariant initial value $\phi(v)$ for the evolution of $\phi(\mu)$ [$(\mu/\phi)d\phi/d\mu = -3x(\mu)/4$]. Figure 1 shows that summation of leading logarithms substantially reduces the residual scale dependence of the effective potential. Moreover, if we assume such scale dependence to be indicative of next-order corrections, we can expect only modest departures from the $m_\phi = 216$ GeV prediction at $\mu = v$: m_ϕ varies from 208 GeV at $\mu = v/2$ to 217 GeV at $\mu = 2v$. We find such uncertainties in m_ϕ to dominate over much smaller ones deriving from (standard-model) uncertainties in the couplant values $x(v)$ and $z(v)$. We have also verified by numerical calculation of the RG equations that the method-of-characteristics methodology of Bando *et al.* [5,6] yields the same results for the effective potential and the Higgs mass to within a value of 0.2%.

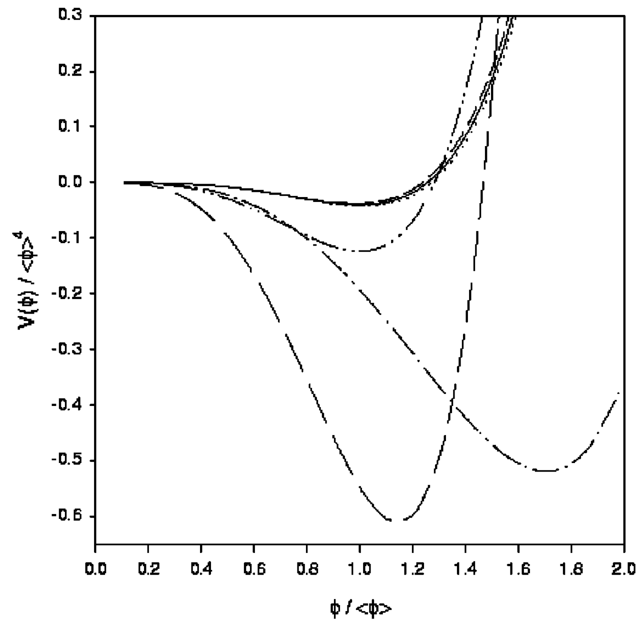

FIG. 1. Residual scale dependence of the standard model effective potential with (upper three curves) and without (lower three curves) summation of leading logarithms, as discussed in the text. For the resummed curves, the solid line represents $\mu = v$, the dashed line represents $\mu = v/2$, and the dotted line represents $\mu = 2v$. For the unsummed curves, the dash–double-dotted curve represents $\mu = v$, the dash–single-dotted curve represents $\mu = v/2$, and the dashed curve represents $\mu = 2v$.

Note also that the scalar-field mass of order 216 GeV we obtain from the aggregate contribution of leading logarithms to the purely radiative breakdown of SU(2) $\times$ U(1) electroweak symmetry is accompanied by a scalar-field interaction couplant $y = 0.0538$ substantially larger than that anticipated from conventional spontaneous symmetry breaking (deriving from a potential with an initially negative quadratic term), in which a 216 GeV Higgs particle would necessarily correspond to a value $y = \lambda/4\pi^2 = m_\phi^2/8\pi^2\langle\phi\rangle^2 = 0.0097$. If electroweak symmetry breaking is purely radiative, then y-sensitive processes such as the $W^+W^- \to ZZ$ scattering cross section [9] will necessarily be larger than anticipated from conventional spontaneous symmetry breaking. Consequently, if an $\mathcal{O}(200\ {\rm GeV})$ Higgs was discovered, a clear signal of radiative symmetry breaking would be a corresponding order-of-magnitude-or-more enhancement of $\sigma(W^+W^- \to ZZ)$ over the value expected from such a Higgs mass.

One of the motivations for summing leading logarithms is to ascertain the negative large-logarithm behavior of the effective potential, behavior corresponding to the zero-field limit of the potential. We do not consider positive large-logarithm behavior because of the intervening Landau singularity at $w = 0$ [Eqs. (4) and (5)], corresponding to $\phi(v) \approx 22v$ [10]. When $|L|$ is very large, we find that $yF_0 \to -1/3L$, $xF_1 \to 2(x/z)/L$, and $x^2LF_2 \to -3(x^2/z^2)/2L$. The large-$|L|$ behavior of subsequent terms in the series (4) can be extracted by noting that the first term on the right-hand side of Eq. (6) dominates the second term when the magnitude of $\zeta(= zL)$ is large, and that $F_p(w, \zeta) \sim \sum_{k=0}^{p+1} f_{p,k}(\zeta)$ in this large-$|L|$ limit $[(\frac{w-1}{w}) \sim 1]$. One then finds after a little algebra that when $|\zeta|$ is large,

$$\left[(7\zeta^2/2)\frac{d}{d\zeta} + 4p\zeta\right]F_p = \frac{9p-21}{4}F_{p-1}, \qquad p \geq 3. \tag{13}$$

In the large $|\zeta|$ limit, we find that $F_2 \sim \sum_{k=0}^{3} f_{2,k}(\zeta) \sim (-3/2)\zeta^{-2}$. Equation (13) implies that $F_p \sim f_p\zeta^{-p}$, where the numerical factors f_p follow from $f_2 = -3/2$ via the recursion relation $f_p = (9p - 21)f_{p-1}/2p$. Note that $x^pL^{p-1}F_p(w, \zeta) \sim (\frac{x}{z})^p f_p/L$ in the large-$|L|$ limit; each term in the series (4) is inversely proportional to L when $|L|$ is large. Moreover, if $(x/z) < 2/9$, the above recursion relation for f_p can be utilized to obtain the closed form series summation $S_{LL} \sim -(\frac{1}{3L})(1 - \frac{9x}{2z})^{4/3}$. For sufficiently strong QCD, this result implies that a standard-model effective potential based upon a massless tree potential exhibits a local *minimum,* rather than a maximum, at $\phi = 0$ (i.e., $L \to -\infty$). Such a conclusion, however, does not follow if x/z is outside its radius of convergence (i.e., if $x/z > 2/9$), as is the case for the empirical standard model [$x(v) \simeq 1/4\pi^2$, $z(v) = \alpha_s(v)/\pi \simeq 0.033$].

Recent work [11] based upon Padé approximants constructed from the QCD β-function series suggests for up to five light flavors that the QCD couplant may exhibit the same double-valued behavior known to characterize $N = 1$ supersymmetric Yang-Mills (SYM) theory, in which coexisting strong-couplant and (asymptotically free) weak-couplant phases evolve toward a common infrared attractive point [12]. If the strong phase is sufficiently strong ($x/z < 2/9$), one can envision a scenario in which the $\phi = 0$ local minimum of preserved SU(2) $\times$ U(1) symmetry is upheld by the strong phase of QCD, but is transformed into a symmetry-breaking minimum at $\phi = v$ if QCD is in its weak phase. Since for the latter case the minimum at $\phi = v$ [$V(v) < 0$] is deeper than the $\phi = 0$ [$V(0) = 0$] minimum occurring when QCD is in its strong phase, the weak phase of QCD is seen to be the preferred one. Thus, if QCD is characterized by two coexisting phases, as is the case for SYM [12], the asymptotic freedom of QCD may be linked to the radiative breakdown of electroweak symmetry.

We are grateful to V. A. Miransky for useful discussions, and to the Natural Sciences & Engineering Research Council of Canada (NSERC) for financial support.

[1] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[2] M. Sher, Phys. Rep. **179**, 273 (1989).
[3] Such an analysis is anticipated in a ϕ^4 model in B. Kastening, Phys. Lett. B **283**, 287 (1992).
[4] T. P. Cheng, E. Eichten, and L.-F. Li, Phys. Rev. D **9**, 2259 (1974); M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B211**, 29 (1983); M. J. Duncan, R. Philippe, and M. Sher, Phys. Lett. **153B**, 165 (1985).
[5] M. Bando, T. Kugo, N. Maekawa, and H. Nakano, Phys. Lett. B **301**, 83 (1993).
[6] J. A. Casas, J. R. Espinosa, and M. Quiros, Phys. Lett. B **342**, 171 (1995); M. Quiros, in *Perspectives on Higgs Physics II*, edited by G. L. Kane (World Scientific, Singapore, 1997), p. 148.
[7] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[8] H. Kleinert, J. Neu, V. Schulte-Frohlinde, K. G. Chetyrkin, and S. A. Larin, Phys. Lett. B **272**, 39 (1991); **319**, 545(E) (1993).
[9] U. Nierste and K. Riesselmann, Phys. Rev. D **53**, 6638 (1996).
[10] A similar ultraviolet singularity can be shown to characterize the summation of leading logarithms in $V_{\rm eff}$ for massless scalar-field electrodynamics, a theory for which radiative spontaneous symmetry breaking is well established.
[11] F. A. Chishtie, V. Elias, V. A. Miransky, and T. G. Steele, Prog. Theor. Phys. **104**, 603 (2000).
[12] I. I. Kogan and M. Shifman, Phys. Rev. Lett. **75**, 2085 (1995); see also V. Elias, J. Phys. G **27**, 217 (2001) regarding D. R. T. Jones, Phys. Lett. B **123**, 45 (1983).