Are Cluster Magnetic Fields Primordial?

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We present results of a detailed and fully nonlinear numerical and analytical investigation of magnetic field evolution from the very earliest cosmic epochs to the present. We find that, under reasonable assumptions concerning the efficiency of a putative magnetogenesis era during cosmic phase transitions, surprisingly strong magnetic fields 10^{-13}–10^{-11} G on comparatively small scales 100 pc–10 kpc may survive to the present. Building on prior numerical work on the evolution of magnetic fields during the course of gravitational collapse of a cluster, which indicates that precollapse fields of $\sim 4 \times 10^{-12}$ G extant on small scales may suffice to produce clusters with acceptable Faraday rotation measures, we argue that it seems possible for cluster magnetic fields to be entirely of primordial origin.

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Magnetic fields exist throughout the observable Universe. They exist in the interstellar medium, in galaxies, and in clusters of galaxies (for reviews cf. [1]). The origin of galactic- and cluster-magnetic fields is still unknown. A plausible, though by far not convincingly established, possibility is the generation of magnetic seed fields and their subsequent amplification via a galactic dynamo mechanism. Seed fields may be due to a variety of processes (and with a variety of strengths), such as the Biermann battery within intergalactic shocks [2], stellar magnetic fields expelled in planetary nebulae, or during supernovae explosions, either into the intragalactic or in the presence of galactic outflows into the intergalactic medium [3], as well as due to quasar outflows of magnetized plasma [4]. Seed fields may also be of primordial origin with a multitude of proposed scenarios. These include generation during first-order phase transitions (e.g., QCD or electroweak), around cosmic defects, or during an inflationary epoch (with, nevertheless, extremely small amplitudes). For a review of proposed scenarios we refer the reader to [5].

The philosophy in prior studies of primordial magnetogenesis is often (but not always) as follows. After establishing a battery mechanism (e.g., separation of charges and production of currents) and a "prescription" or estimate for the final, nonlinearly evolved magnetic field strength (e.g., equipartition of magnetic energy with turbulent flows), subsequent evolution is approximated by simply assuming frozen-in magnetic field lines into the plasma. Though such an approximation may be appropriate on the very largest scales, it should be clear that this may not be the case on the "integral" or coherence scale of the field. Here, coupling of the magnetic fields to the gas induces nonlinear cascades of energy in Fourier space. The characteristics of initially created magnetic field are thus vastly modified during cosmic evolution between the epoch of magnetogenesis and the present.

The final step in such studies is then often to determine field strengths on some prescribed large scale (e.g., 10 Mpc) typically falling in the range $10^{-30}\ \mathrm{G} \lesssim B \lesssim 10^{-20}$ G, inferring that this may act as seed for a sufficiently efficient dynamo to produce galactic- and cluster-magnetic fields of order 10^{-6} G. This is observed in negligence of the fact that much stronger fields on smaller scales result not only from a variety of astrophysical seeds but from these very same primordial scenarios. Such fields may fill voids of galaxies and may potentially be observable by upcoming technology [6].

The purpose of this Letter is twofold. We have attempted to develop a coherent picture of gross features of nonlinear, cosmic MHD evolution of primordial fields, including all relevant dissipative processes, such as viscosity due to diffusing or free-streaming neutrinos and photons, as well as ambipolar damping. A subset of the results of our numerical and analytical analyses is presented here, whereas details are presented elsewhere [7]. Our study allows us, for the first time, to make predictions for magnetic field energy and coherence length at the present epoch, for broad ranges of initial magnetic configurations, parametrized by spectral index, initial helicity, initial energy, as well as an era of magnetogenesis. Second, drawing on a prior numerical MHD study of the gravitational collapse of a cluster of galaxies [8], we challenge the often-cited conclusion that cluster magnetic fields may not be entirely of primordial origin. Rather, we stress that it seems not clear at the moment if such fields on comparatively small scales may not, after all, produce cluster magnetic fields as widely observed.

In passing we note that there exists a number of analytical [9–15], and numerical [16,17], studies on the evolution of nonhelical and helical primordial magnetic fields, which, nevertheless, for one or the other reason either remain inutile in predicting final field properties or do so only for a specific scenario. The inutility of results is related to facts such as the adoption of an evolutionary model not supported by numerical simulations, an inadequate treatment of viscosity due to photons, or, simply, the analysis being linear in nature or being performed in Minkowski space and not properly transferred to the expanding Universe.

The generation of primordial magnetic fields in magnetogenesis scenarios is generally believed to occur during well-defined periods (e.g., QCD transition). Subsequent evolution of these magnetic fields should therefore be well approximated by a ‘‘free decay’’ without any further input of kinetic or magnetic energy, i.e., as freely decaying MHD. The large number of electrically charged particles in the early Universe allow one to neglect dissipative effects due to finite conductivity. Nevertheless, as already stressed in Ref. [10], dissipative effects arising from the ‘‘imperfectness’’ of the fluid due to neutrino and photon diffusion and free-streaming play an important role in early MHD evolution. Here diffusing particles may dissipate energy due to the presence of shear viscosity, whereas the same may happen in the case of free-streaming particles due to occasional scatterings between these particles and those participating in the flow, yielding a drag force. One may further show that for field strength as considered in this Letter the assumption of fluid incompressibility is appropriate.

To verify theoretical expectations we have performed numerical simulations of incompressible, freely decaying, ideal, but viscous, MHD. These simulations are performed with the help of a modified version of the code ZEUS-3D [18,19] in a nonexpanding (Minkowski) background and on 128^3 to 512^3 grids. Modifications lie in the inclusion of fluid viscosities [20]. It is shown elsewhere [7,16] that conformal or near-conformal invariance of the MHD equations allow for the interpretation of results obtained with Minkowski metric to results applicable for a Friedman-Robertson-Walker metric. Results of such simulations, in particular, the decay of magnetic energy $E_{\rm mag}$ with time, for a variety of physical regimes are shown in Fig. 1.

We have found that results of our simulations may be understood in a comparatively simple manner. In particular, nonlinear MHD processing of the initial spectrum at epoch with Hubble constant $H(T)$ occurs for all scales l, which obey

$$v(l)/l \gtrsim H(T), \qquad (1)$$

irrespective of the Reynolds number Re of the flow. Here $v(l)$ may be written as the Alfvén velocity $v_A(l) = B(l)/\sqrt{4\pi(\varrho + p)}$ when turbulence holds, $\mathrm{Re} \gtrsim 1$, and as $v(l) = v_A(l)\mathrm{Re}(v = v_A, l)$ for viscous MHD ($\mathrm{Re} \lesssim 1$). This holds equally during the photon diffusion ($l_\gamma \ll l$) regime $\mathrm{Re} = vl/\eta$ and the photon free-streaming ($l_\gamma \gg l$) regime $\mathrm{Re} = v/\alpha l$, where η, α, and l_γ are photon shear viscosity, drag coefficient, and mean free path, respectively. Defining $L(T)$, the integral or coherence scale, as the scale where equality applies in Eq. (1),

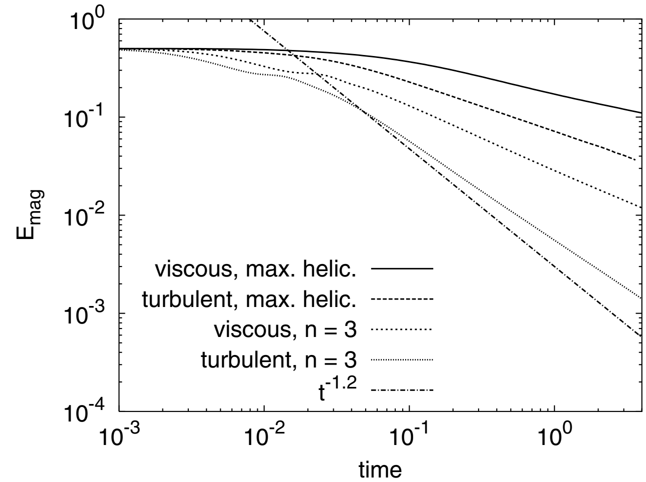

FIG. 1. Decay of magnetic energy in different damping regimes and for different initial conditions as observed in our numerical simulations. The $t^{-1.2}$ line shows the theoretical damping law in the turbulent regime for $n = 3$, $h_g = 0$.

one finds that $L(T)$ is, in fact, the scale containing most of the energy of the flow. This is due to a nonlinear and rapid cascade developing on all scales $l \lesssim L$, with energy in fluid eddies transported down to the dissipation scale $l_{\rm diss}$ and transferred to heat. Since the resultant small-scale spectrum is red and we assume the initial as yet unprocessed large-scale ($l \gtrsim L$) spectrum to be blue, $L(T)$ as the smallest unprocessed scale remains as the magnetic coherence scale of the field at epoch with temperature T. Magnetic energy is then approximately given as the initial energy on scale $L(T)$, in agreement with our simulations.

For fields which are maximally helical, i.e., $\mathcal{H} = \mathcal{H}_{\rm max} \approx \langle B^2(l)\, l\rangle \approx B^2(L)\, L$, Eq. (1) may still be used to obtain the coherence scale of the field. Nevertheless, due to helicity density $\mathcal{H} = (1/V)\int d^3x \mathbf{A}\cdot\mathbf{B}$ (where $\mathbf{A}$ is the vector potential and V is the integration volume) being an ideal invariant in the early Universe, a direct cascade of energy from large scales to small scales is accompanied by an inverse cascade of energy from small scales to large scales. That is, whereas in the absence of helicity the large-scale field remains unprocessed, in the maximally helical case large-scale fields undergo growth even on scales $l \gtrsim L(T)$. Surprisingly, we find that during this process of large length scale magnetic field amplification the initial spectral index n is conserved [17]. The decay of magnetic energy is thus described by the requirement of conservation of helicity, in conjunction with an increase of $L(T)$ described via Eq. (1). Because of a vast increase of $L(T)$ in the early Universe, even initially submaximally helical fields, i.e., $\mathcal{H}_g = h_g\mathcal{H}_{{\rm max},g}$, with $h_g < 1$, ultimately reach a maximal helical configuration. Here, and throughout, an index ‘‘g’’ denotes properties at the magnetogenesis epoch. Parametrizing the initial (comoving) magnetic energy spectrum by $\varrho_B \approx \varrho_{Bg}(l/L_g)^{-n}$, where L_g is the initial (magnetogenesis) coherence scale obeying Eq. (1) and ϱ_{Bg} is the approximate initial magnetic energy, one finds that fields have reached a maximally helical state when $L(T)$ has grown beyond

$$L^{\max} \simeq L_g h_g^{-1/(n-1)}. \qquad (2)$$

This picture may be employed to derive damping exponents, i.e., $E_{\rm mag} \sim t^{-\gamma}$ [21], and compare to those inferred from numerical simulations (cf. Fig. 1). Whereas the comparison is quite favorable in the viscous regime, turbulent decay is observed somewhat slower than predicted. For example, for nonhelical, turbulent MHD with a $n = 3$ spectrum we predict $\gamma = 1.05$. Nevertheless, we argue that this trend, seen also by others [17,22], must be due to limited numerical resolution [7], with resulting predictions for the surviving magnetic fields, given in this Letter, being on the conservative side.

We have undertaken the in practice straightforward but arduous effort to assemble these results and, under the inclusion of all appropriate dissipation terms and for quite general initial conditions, followed the growth of magnetic coherence length and energy density from the very earliest times to the present [7]. Figure 2 shows examples of the growth of $L(T)$ for a number of scenarios of magnetogenesis at the QCD phase transition. The evolution is observed as an alternation between turbulent MHD and viscous MHD. "Viscosity" here is early on due to neutrinos, some time before recombination due to photons, and after recombination due to hydrogen-ion scattering and hydrogen-hydrogen scattering. Particularly notable are phases where the growth of $L(T)$ is halted completely. This occurs either at epochs before recombination in the viscous regime with diffusing photons or neutrinos, as well in part of the regime when those particles are free-streaming or at epochs after recombination, due to the peculiar redshifting of Eq. (1) and/or the effects of hydrogen diffusion and ambipolar diffusion [23]. Note, however, that the growth of $L(T)$ and the concomitant decrease of $B(T)$ during the late phases of viscous MHD with free-streaming photons (neutrinos) may be faster than the growth of $L(T)$ during turbulent MHD. Note also that initial conditions leading to relatively strong magnetic fields at recombination result in a rapid increase of $L(T)$ at $T_{\rm rec} \approx 0.3$ eV, whereas for weaker fields $B \lesssim 10^{-13}$ G a similar jump occurs at reionization [7]. Note that effects due to structure formation are not taken into account here (see below, however).

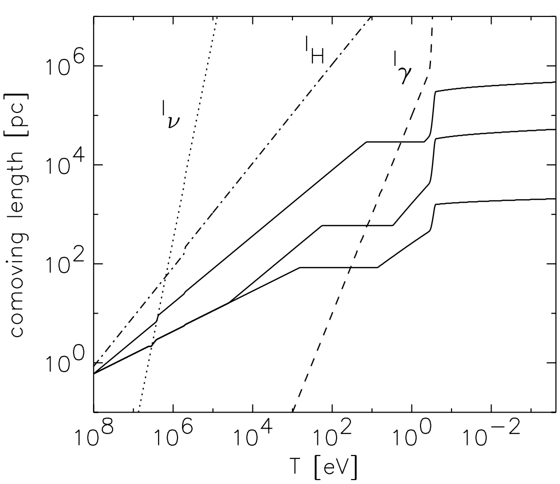

FIG. 2. Evolution of comoving coherence length for different initial magnetic field configurations with $r_g = 0.083$ and $n = 3$ (solid lines from top to bottom: $h_g = 1$, $h_g = 10^{-3}$, $h_g = 0$). The mean free paths of neutrinos and photons are labeled by l_ν and l_γ, and l_H is the Hubble length.

We give here the final (prestructure formation) results on the coherence scale $L(T_0)$ and field amplitude $B(T_0)$, where T_0 is the present cosmic microwave background temperature, derived by employing Eq. (1), as well as retaining the initial field energy due to all scales $l \gtrsim L$ in the submaximally helical case, and conserving helicity density in the maximally helical case. Complete results on L and B for all eras may be found in [7]. Fields which remain still submaximally helical [i.e., $L(T_0) \lesssim L_c^{\max}$] [24] at the present epoch have for coherence length and field strength

$$B(T_0) \approx (1.65 \times 10^{-6}\ {\rm G}) x^{n/(n+2)} \left(\frac{r_g}{0.083}\right)^{1/2} \times \left(\frac{T_g}{100\ {\rm MeV}}\right)^{-n/(n+2)}, \qquad (3)$$

$$L(T_0) \approx L_{gc} x^{-2/(n+2)}, \qquad (4)$$

where $x = 2.30 \times 10^{-9}$ is a small factor and $r_g = (\varrho_B / s_r^{4/3})_g$ is a convenient measure of magnetic energy density ϱ_B in terms of radiation entropy density $s_r = (4/3) g_g (\pi^2/30) T_g^3$ at the magnetogenesis epoch assumed to occur at temperature T_g. Note that, somewhat optimistically $r_g = 0.083$, when magnetogenesis results in magnetic energy density equivalent to the photon energy density shortly after a QCD transition with $g_g \approx 10.75$. The comoving coherence length L_{gc} at the magnetogenesis epoch is given by

$$L_{gc} \approx (0.45\ {\rm pc}) \sqrt{n} \left(\frac{r_g}{0.083}\right)^{1/2} \left(\frac{T_g}{100\ {\rm MeV}}\right)^{-1}. \qquad (5)$$

This yields, for example, for $r_g = 0.083$, $T_g = 100$ MeV, $n = 3$ to the appreciable field strength of $B_c \approx 1.1 \times 10^{-11}$ G. If we were to apply the simulation observed instead of theoretically predicted decay exponent γ, the result would increase approximately to $B \sim 5 \times 10^{-11}$ G. On the other hand, fields which have reached a maximally helical [i.e., $L(T_0) \gtrsim L_c^{\max}$] state at present have

$$B_c \approx (4.69 \times 10^{-12}\ {\rm G}) y, \qquad (6)$$

$$L_c \approx (550\ {\rm pc}) y \sqrt{n} \qquad (7)$$

with $y = (r_g/0.083)^{1/2} (h_g/10^{-8})^{1/3} (T_g/100\ {\rm MeV})^{-1/3}$ and where h_g is the fractional helicity of the maximal one $\mathcal{H}_{\max,g}$ at the generation epoch.

Though on small scales, it is seen that surprisingly strong fields may survive an early Universe magnetogenesis epoch to the present (cf. Fig. 3). This is interesting

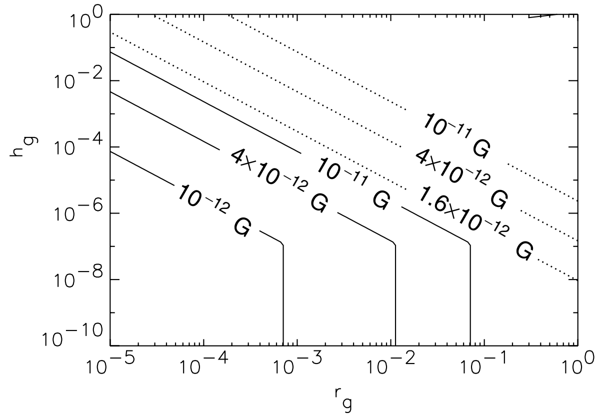

FIG. 3. Final magnetic field strengths in the $r_g - h_g$ parameter space for $T_g = 100$ MeV (solid lines) and $T_g = 100$ GeV (dotted lines), where we assumed a spectral index of $n = 3$.

in light of recent simulations on the formation of clusters of galaxies from slightly overdense and premagnetized regions via the gravitational instability [8]. The authors find that fields of strength $B_c \approx 4 \times 10^{-12}$ G (corresponding to their quoted $B \approx 10^{-9}$ G at simulation starting redshift $z = 15$) yield to clusters whose Faraday rotation measures are essentially indistinguishable from those observed in real clusters [25]. Furthermore, the authors arrive at the intriguing conclusion that this result is virtually independent of whether a homogeneous field is assumed initially, or a field whose energy contribution is dominated by fluctuations on the very smallest scales in their simulations [27]. It is not clear if such an erasure of memory of initial conditions, possibly related to an interplay between the development of shear flows and small-scale turbulence during the course of gravitational collapse, pertains if initial field coherence lengths in the cluster simulations are reduced by a further factor $\sim$100 [due to the comparison between typical coherence lengths in Eqs. (4) and (7) and the spatial resolution, $\sim$100 kpc comoving, of the simulations]. Nevertheless, if so, cosmological magnetic fields generated during early eras, either of moderate magnetic helicity or generated fairly late, could account for present-day observed cluster magnetic fields, and as such in the absence of any further dynamo amplification. We conclude that this interesting possibility seems to deserve further investigation.

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