Green Function Retrieval and Time Reversal in a Disordered World

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We apply the theory of multiple wave scattering to two contemporary, related topics: imaging with diffuse correlations and stability of the time reversal of diffuse waves, using equipartition, coherent backscattering, and frequency speckles as fundamental concepts.

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In its early days, multiple scattering of waves was considered to be an unavoidable nuisance. It randomizes the phase, polarization, and wave vector of waves, and thus complicates important applications in imaging, telecommunication, laser action, and remote sensing. Recent developments showed that multiple scattering can actually *enhance* the performance of several applications. We mention in this context the low threshold of random lasers [1], the robust time reversal (TR) of multiply scattered waves [2], with potential applications in communication [3], the sensitivity of diffuse waves to particle motions [4], and the reported retrievals of the Green function from thermal phonons [5] and from diffuse seismic waves [6].

Chaos theory successfully describes time reversal [7] and correlations [5] of elastic waves in chaotic media. Most applications above concern diffuse waves, with their specific questions about statistics, leakage, and dynamics. It is the intention of this Letter to apply multiple scattering theory to these new exciting topics. To our knowledge the first attempts to cross correlate "noisy" signals to retrieve ballistic wave motion come from seismology ("acoustic daylight imaging") [8] and helioseismology [9]. As for time reversal, after many pioneering experiments in Paris [10], Papanicolaou *et al.* [11] explained the self-averaging property of time reversal for broadband signals for (locally) layered random media in a scaling limit of small fluctuations and propagation distances long compared to the wavelength. Our theory makes no assumptions on the scatterers, but applies only in the diffuse regime, i.e., when the medium size L is much bigger than the mean free path ℓ. This excludes 1D and localized media. This criterion is largely fulfilled in 2D time-reversal experiments with high order multiple scattering [2], as well as for some recent seismic coda observations [12].

We consider the simplest case of fluctuations of a scalar wave field $\Psi(\mathbf{r}, t)$ propagating in an infinite random medium after being released by some source $S(t)$ near $t = 0$ far away from the place of measurement. It is customary to describe field correlations by the "Wigner" function [13], $\Psi(\mathbf{r} - \frac{1}{2}\mathbf{x}, t - \frac{1}{2}\tau)\,\Psi(\mathbf{r} + \frac{1}{2}\mathbf{x}, t + \frac{1}{2}\tau)$, whose Fourier transform (FT) with respect to space $\mathbf{x}$ and time τ is better known as the specific intensity $I_{\omega\mathbf{k}}(\mathbf{r}, t)$ of waves with wave vector $\mathbf{k}$ and frequency ω. A *rigorous* statement in multiple scattering theory is that

$$\lim_{t\to\infty} \int d\mathbf{r} \langle I_{\omega\mathbf{k}}(\mathbf{r}, t)\rangle = -\frac{1}{\omega}\mathrm{Im}\,\langle G(\omega, \mathbf{k})\rangle \times \frac{\sum_{\mathbf{k}'} -\frac{1}{\omega}\mathrm{Im}\,\langle G(\omega, \mathbf{k}')\rangle S(\omega, \mathbf{k}')}{\sum_{\mathbf{k}'} -\frac{1}{\omega}\mathrm{Im}\,\langle G(\omega, \mathbf{k}')\rangle}. \tag{1}$$

The structure function of the source $S(\omega, \mathbf{k})$ is the FT of $S(\tau, \mathbf{x}) \equiv \int dt \int d\mathbf{r}\, s(\mathbf{r} - \frac{1}{2}\mathbf{x}, t - \frac{1}{2}\tau)\, s(\mathbf{r} + \frac{1}{2}\mathbf{x}, t + \frac{1}{2}\tau)$, and $\langle G(\omega, \mathbf{k})\rangle$ is the *average* Green's function of the random medium. Equation (1) is a known field-theoretical consequence of flux conservation [14–16]. It relates the specific intensity (a property of the diffuse energy) *at large lapse times* to the spectral function $-\frac{1}{\omega}\mathrm{Im}\,\langle G(\omega, \mathbf{k})\rangle$ of the effective medium [17](a property of the coherent field). Physically, this implies global equipartition of average energy in phase space.

Equation (1) looks like a manifestation of the fluctuation-dissipation (FD) theorem, applied to $\Psi = G \otimes s$ for which thermal equilibrium (TE) implies that $\langle \Psi(\omega)\Psi^*(\omega)\rangle \propto \mathrm{Im}\,\langle G(\omega)\rangle B(\omega)$, with B the Planck function. In TE, formula (1) would even hold without multiple scattering, and at any time. This situation applies to the measurement of the elastic Green's function of an aluminium block by cross-correlating thermal phonons [5]. When translated to space-time, the FD theorem shows that coherent wave paths should in principle be observable with travel times up to the inelastic phonon mean free time.

The diffuse field is not in TE, not even in equilibrium. The physics that multiple scattering and TE share is the equipartition principle [13,18]. Unfortunately, two reasons exist why Eq. (1) is not very useful to retrieve the Green function. First, it applies to the ensemble average of the specific intensity only, which is subject to large mesoscopic fluctuations. This would clearly restrict its use in several imaging problems. Second, Eq. (1) assumes many receivers at different positions to cover whole space. As shown in Ref. [5], the correlation method can be adapted for infinitely many equal sources, which also eliminates the problem of ensemble averaging. The present Letter deals with the other, equally important case of a random field generated by *one* distant source and investigate the applicability to retrieve the Green's function using the *local* time correlation function $\Phi_{\mathbf{r}}(\mathbf{x},\tau) \equiv \int dt\, \Psi(\mathbf{r}-\frac{1}{2}\mathbf{x}, t-\frac{1}{2}\tau)\,\Psi(\mathbf{r}+\frac{1}{2}\mathbf{x}, t+\frac{1}{2}\tau)$. We shall develop a theory for its *statistical average*, and show that $\Phi_{\mathbf{r}}(\mathbf{x},\tau)$ tends to be self-averaging.

It is widely accepted that in dimensions $d=3$, as well as in $d=2$ over not too large distances, the average energy density $\rho_\omega(\mathbf{r},t)$ of the random field near circular frequency ω, released by a source localized in space-time, obeys a diffusion equation [14,16] with source $S(\omega)\delta(\mathbf{r})\delta(t)$. Equation (1) imposes that $S(\omega)$ be the source factor on the righthand side of Eq. (1). Since the density is proportional to the angular integral of the specific intensity $\langle I_{\omega\mathbf{k}}(\mathbf{r},t)\rangle$, the following local expansion of the ensemble-averaged specific intensity is often employed [16],

$$\langle I_{\omega\mathbf{k}}(\mathbf{r},t)\rangle = -\frac{1}{\omega}\mathrm{Im}\langle G(\omega,\mathbf{k})\rangle\left[1 - dD\frac{1}{\omega}\mathbf{k}\cdot\partial_{\mathbf{r}} + \cdots\right] \times S(\omega)\rho_\omega(\mathbf{r},t). \tag{2}$$

The second term in Eq. (2) gives a diffuse flow of energy with diffusion constant D. Let us consider an explosive-type source $s(\omega,\mathbf{k}) = S|\mathbf{k}|$, with power spectrum $S(\omega) = S\omega^2$ that we band-filter in a frequency band B, inside which transport quantities such as D [and thus $\rho_\omega(\mathbf{r},t)$] can be assumed constant. Recalling that the correlation function $\Phi(\mathbf{x},\tau)$ is the double FT of $I_{\omega\mathbf{k}}$, we get the simple result,

$$\langle\Phi_{\mathbf{r}}(\mathbf{x},\tau)\rangle = [\rho_B(\mathbf{r})\partial_\tau - dD\partial_{\mathbf{r}}\rho_B(\mathbf{r})\cdot\partial_{\mathbf{x}} + \cdots] \times [\langle G_B(\mathbf{x},\tau)\rangle - \langle G_B(\mathbf{x},-\tau)\rangle]S. \tag{3}$$

The ensemble-averaged field correlation function is proportional to the time derivative of the band-filtered, ensemble-averaged Green's function $\langle G_B\rangle$ of the random medium, *symmetrized* in the time τ [5], and the time-integrated density $\rho_B(\mathbf{r})$ in the bandwidth B. An *anti-symmetric* part is allowed if a diffuse flux is present. It is likely that this term is the origin of the asymmetry of the correlation function recently observed with seismic Rayleigh waves [6]. The verification of this hypothesis requires a more realistic diffusion model for the Earth's crust [19], that is beyond the scope of this Letter, and for which laboratory experiments will be indispensable [20]. In an open medium of size L and mean free path ℓ, the relative importance of the asymmetric term is of order ℓ/L far away from its boundaries, and thus small if $L \gg \ell$.

If the waves propagate in 3D and without much dispersion, $\langle\mathrm{Im}\, G(\omega,\mathbf{k})\rangle \sim \delta(\omega^2 - k^2)$ and Eq. (3) can easily be generalized for an *arbitrary* source spectrum. We multiply left- and right-handed sides of Eq. (2) by ω and neglect the flow term. A FT with respect to ω, $\mathbf{k}$ gives

$$\partial_\tau\langle\Phi_{\mathbf{r}}(\mathbf{x},\tau)\rangle \to \frac{\rho_B(\mathbf{r})}{4\pi x}\left[S_B\left(\frac{x}{c}+\tau\right) - S_B\left(\frac{x}{c}-\tau\right)\right], \tag{4}$$

with $S_B(\tau)$ the time correlation function of the bandlimited source. In particular, the special result $\langle\Phi_{\mathbf{r}}(0,\tau)\rangle \sim \rho_B(\mathbf{r})S_B(\tau)$ will be needed later to describe time reversal. Relation (4) facilitates the monitoring of source dynamics with distant, diffuse correlations.

Equations (3) and (4) constitute the central result of this Letter, despite their simplicity. Their importance follows from the rest of this Letter. First, we establish that fluctuations around the ensemble average are small if the bandwidth B is large enough, as also found for layered random media [11,21], and first suggested by time-reversal experiments [22]. We will generalize the central result (3) to a random medium containing an *arbitrary* close object. This shows the usefulness of diffuse waves to the inverse problem in disordered media, in a way as first put forward by Claerbout *et al.* [8] for seismic noise. At the end of this Letter we make the link with time reversal and coherent backscattering in disordered media [10].

We start out by calculating the statistical fluctuations of $\Phi_{\mathbf{r}}(\mathbf{x},\tau)$ around its average. To this end let us adopt a source at a large distance $\mathbf{r}$ with power spectrum $S(\omega) = S\omega^2$ that we filter over a bandwidth B. Our most important assumption will be that the bandwidth B is much larger than the Thouless frequency $\Omega_{\mathrm{th}} \sim D/|\mathbf{r}|^2$ of the random medium. It can readily be seen that ($\mathbf{r}^\pm = \mathbf{r} \pm \frac{1}{2}\mathbf{x}$),

$$\Phi_{\mathbf{r}}(\mathbf{x},\tau) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}G(\mathbf{r}^-,0,\omega)G^*(\mathbf{r}^+,0,\omega)S(\omega) \tag{5}$$

in terms of the monochromatic, retarded Green's function $G(\mathbf{r},\mathbf{r}',\omega)$ of the wave equation. From Eq. (3) we find for the average correlation

$$\langle\Phi_{\mathbf{r}}(0,0)\rangle = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{-1}{\omega}\mathrm{Im}\langle G(\omega,0)\rangle S\rho_\omega(\mathbf{r}) \simeq BNS\rho_B(\mathbf{r}),$$

with N the density of states in the bandwidth [14]. In the diffuse regime spatial/frequency fluctuations obey Gaussian (C_1) statistics [23], so that the variance becomes,

$$[\Delta\Phi_{\mathbf{r}}(\mathbf{x},\tau)]^2 = \int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega_2}{2\pi}e^{-i(\omega_1-\omega_2)\tau}eS(\omega_1)S(\omega_2)\langle G(\mathbf{r}^-,0,\omega_1)G^*(\mathbf{r}^-,0,\omega_2)\rangle\langle G(\mathbf{r}^+,0,\omega_2)\,G^*(\mathbf{r}^+,0,\omega_1)\rangle$$
$$\approx \int_B\frac{d\omega_1}{2\pi}|S^2|N(\omega_1)^2\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}e^{-i\Omega\tau}\rho_\omega(\mathbf{r})^2|C^1(\Omega)|^2 \simeq B|S|^2N^2\rho_B(\mathbf{r})^2\Omega_{\mathrm{th}}.$$

We use that the normalized intensity correlation function $C^1(\Omega) \sim \exp(-\sqrt{\Omega/\Omega_{\mathrm{th}}})$ decays rapidly with Ω and that

$\Omega_{\rm th} \ll B, 1/\tau$. In that case is $\Delta\Phi_{\mathbf r}/\langle\Phi_{\mathbf r}\rangle \simeq \sqrt{\Omega_{\rm th}/B} \ll 1$, so that the measurement of $\Phi_{\mathbf r}(\mathbf x, \tau)$ is close to its ensemble average with high probability. The ratio $B/\Omega_{\rm th}$ is interpreted as the number of independent "frequency bits" available in the bandwidth of the waves arriving at the receiver [22].

In the following we will show the possibility to *image* a fixed, close object by means of diffuse correlations of a scalar field Ψ. The formal relation between the Green function G^T of the effective medium *including* the object, the Green function $\langle G\rangle$ of the effective medium alone, and the T matrix T of the object is $G^T = G + GTG$ (we drop the averaging brackets if no confusion can arise). We assert that Eq. (3) is universal so that at a large distance $\mathbf r$ from an explosive-type source,

$$\Phi_{\mathbf r}(\mathbf x, \tau) \propto \rho_B(\mathbf r)\partial_\tau[\langle G_B^T(\mathbf r^-, \mathbf r^+, \tau)\rangle - \langle G_B^T(\mathbf r^-, \mathbf r^+, -\tau)\rangle]. \tag{6}$$

The identity (6) implies that the field correlation of the diffuse waves between two points near the object is equivalent to a time-resolved scattering experiment on the object.

This assertion can be established from the following assumptions: (i) We are in the diffuse field of a distant (explosive) source S; (ii) The object scatters the waves elastically. Equation (2) holds without the object. The solution *with* object can formally be found by "gluing" a single scattering vertex K into the diffuse vertex ρ giving the terms $(\rho + K\rho + \rho K + \rho K\rho)S$, with S the source [24]. In momentum space the vertex K at frequency ω is given by,

$$K_{\mathbf k\mathbf k'}(\mathbf q_1, \mathbf q_2) = T_{\mathbf k+(1/2)\mathbf q_1,\mathbf k'+(1/2)\mathbf q_2}T^*_{\mathbf k-(1/2)\mathbf q_1,\mathbf k'-(1/2)\mathbf q_2} + T_{\mathbf k+(1/2)\mathbf q_1,\mathbf k'+\frac12\mathbf q_2}G^{*-1}_{\mathbf k'-(1/2)\mathbf q_2}\delta_{\mathbf k-(1/2)\mathbf q_1,\mathbf k'-\frac12\mathbf q_2} + T^*_{\mathbf k-(1/2)\mathbf q_1,\mathbf k'-(1/2)\mathbf q_2}G^{-1}_{\mathbf k'+(1/2)\mathbf q_2}\delta_{\mathbf k+(1/2)\mathbf q_1,\mathbf k'+(1/2)\mathbf q_2},$$

with $G_{\mathbf k} = G(\omega, \mathbf k)$. Coherent propagation ρKS from source to object is negligible and $\rho K\rho S$ just slightly modifies the diffuse background ρ. The terms $\rho S + K\rho S$ dominate if the object is less than a mean free path separated from the distant receiver, and give us,

$$\langle\Psi_{\mathbf k+(1/2)\mathbf q_1}\Psi^*_{\mathbf k-(1/2)\mathbf q_1}\rangle = G_{\mathbf k+(1/2)\mathbf q_1}G^*_{\mathbf k-(1/2)\mathbf q_1}\sum_{\mathbf k'}[\delta_{\mathbf k\mathbf k'} + K_{\mathbf k\mathbf k'}(\mathbf q_1, \mathbf q_2)]\frac{-1}{\omega}\,{\rm Im}\,G_{\mathbf k'}\rho(\mathbf q_2)S(\omega). \tag{7}$$

The Ward identity [25],

$$T_{\mathbf p+(1/2)\mathbf q,\mathbf p'+(1/2)\mathbf q} - T^*_{\mathbf p-(1/2)\mathbf q,\mathbf p'-(1/2)\mathbf q} = \sum_{\mathbf k}T_{\mathbf p+(1/2)\mathbf q,\mathbf k+(1/2)\mathbf q}[G_{\mathbf k+\frac12(1/2)\mathbf q} - G^*_{\mathbf k-(1/2)\mathbf q}]T^*_{\mathbf p'-(1/2)\mathbf q,\mathbf k-(1/2)\mathbf q}$$

expressing flux conservation (ii) simplifies this expression. The singular diffusion pole (i) imposes $q_2 \approx 0$ in all T matrices and Green functions. The assertion (6) follows after Fourier transformations with respect to ω,$\mathbf k$, $\mathbf q_1$, and $\mathbf q_2$.

We will finally make the link of Eq. (3) with time-reversal experiments [10] and coherent backscattering (CBS). Consider a source $s(t)$ at point $\mathbf r_S$ in an infinite random medium. At point $\mathbf r_T$ a TR process is carried out between times T_1 and T_2. The TR signal $\Psi(\mathbf r, \tau)$ collected at $\mathbf r_D$ was given by De Rosny *et al.* [7] and Papanicolaou *et al.* [11];

$$\Psi(\mathbf r_D, \tau + 2T_2) = \int_{T_1}^{T_2} dt\, G(\mathbf r_S, \mathbf r_T, \tau + t)G(\mathbf r_T, \mathbf r_D, t) \otimes s(t, \mathbf r_S).$$

If we agree to time reverse the whole signal (T_1 smaller than the first arrival time and T_2 deep inside the coda), a Fourier transformation gives us

$$\Psi(\mathbf r_D, \tau + 2T_2) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega\tau}G(\mathbf r_S, \mathbf r_T, \omega)G^*(\mathbf r_T, \mathbf r_D, \omega) \times s(\omega). \tag{8}$$

The formal equivalence of $\Psi(\mathbf r_D, \tau + 2T_2)$ to $\Phi_{\mathbf r}(\mathbf x, \tau)$ (with $\mathbf r = \mathbf r_S - \mathbf r_T$ and $\mathbf x = \mathbf r_S - \mathbf r_D$) given by Eq. (5), expressing diffuse correlations, is striking. To describe TR only the power spectrum $S(\omega) = |s(\omega)|^2$ of the source has to be replaced by its complex Fourier component $s(\omega)$. Thus Eq. (4) holds for the ensemble average of the TR signal Ψ in 3D near the source, provided that $S_B(t)$ be replaced by the genuine, bandlimited source pulse $s_B(t)$. Equation (4) then describes the autofocalization in space-time: at the source $\mathbf r_S$ the TR signal $s_B(\tau)$ is observed in time, whereas at $\tau = 0$ the TR signal focalizes as $x^{-1}\int_0^{x/c} dt s_B(t)$ at a distance x around the source. Just as we have established for diffuse correlations, the TR signal $\Psi(\mathbf r, \tau)$ is very stable against mesoscopic fluctuations when the bandwidth is much larger than the Thouless frequency ($B/\Omega_{\rm th} \approx 600$ in Ref. [2]).

The link between TR and CBS can be established by applying formula (8) to a disordered half space with a TR machine hidden inside at a depth z_T and a source at a large distance a outside (Fig. 1). We assume $z_T \ll a$. Equation (8) predicts that the TR signal at a distance x from the source is

$$\Psi(\mathbf x, \tau + 2T_2) = s_B(\tau)\int d^2\mathbf r_\parallel\, e^{ik\mathbf x\cdot\mathbf r_\parallel/a}\rho_B(\ell, z_T, \mathbf r_\parallel) \sim s_B(\tau)\,\frac{\ell}{z_T}\exp\!\left(-\frac{z_T k|x|}{a}\right), \tag{9}$$

where we have inserted the diffuse energy propagator $\rho_B(z, z', \mathbf r_\parallel - \mathbf r'_\parallel)$ for the 3D half space [26]. This makes

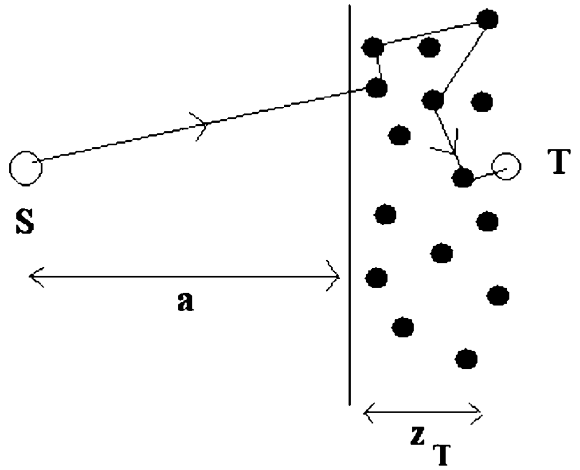

FIG. 1. A setup that establishes the link between coherent backscattering and time reversal. A source **S** emits a signal that is received by a time-reversal machine **T** inside a disordered half-space. The signal that is sent back autofocalizes in space and time near **S** with the spatial line shape of coherent backscattering.

the line shape of the spatial autofocalization in TR *equal* to the one of CBS from a half space, exhibiting the well-known angular triangular cusp caused by long time-reversed wave paths [26]. The angular resolution $\Delta x/a \approx \lambda/z_T$ nicely illustrates that the diffusion process creates an effective aperture of size z_T that increases the quality of spatial focalization, *independent* of ℓ [2]. The peak signal is *stable* against mesoscopic fluctuations if $B \gg 2D/z_T^2$.

In conclusion, we have shown that correlations of diffuse waves can be used to retrieve ballistic waves between two points in space-time. This method is stable against mesoscopic fluctuations, if the operating bandwidth is much larger than the Thouless frequency. We have discussed the possible temporal asymmetry of the field correlations, the role of the power spectrum of the source, as well as the fundamental relation to time reversal and coherent backscattering. The method facilitates a novel passive way of imaging that might find an application in seismology, where active sources are expensive or unpredictable.

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