Enhancing Acceleration Radiation from Ground-State Atoms via Cavity Quantum Electrodynamics

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When ground-state atoms are accelerated through a high Q microwave cavity, radiation is produced with an intensity which can exceed the intensity of Unruh acceleration radiation in free space by many orders of magnitude. The reason is a strong nonadiabatic effect at cavity boundaries and its interplay with the standard Unruh effect. The cavity field at steady state is still described by a thermal density matrix under most conditions. However, under some conditions gain is possible, and when the atoms are injected in a regular fashion, squeezed radiation can be produced.

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One of the most intriguing results of modern quantum field theory is the proof by Davies *et al.* [1], and others [2], that ground-state atoms, accelerated through vacuum, are promoted to an excited state just as if they were in contact with a blackbody thermal field. These studies [1,2] predict that a (two-level) ground-state atom, having transition frequency ω, and experiencing a constant acceleration a, will be excited to its upper level with a probability governed by the Boltzmann factor $\exp(-2\pi\omega/\alpha)$, where $\alpha = a/c$, c is the speed of light in vacuum. Unfortunately, even for large acceleration frequency $\alpha \approx 10^8$ Hz [3], and microwave frequency $\omega \approx 10^{10}$ Hz [4], this factor is exponentially small, $\sim 10^{-200}$, and is not of experimental interest.

Thus we were motivated to study a simple *gedanken* experiment based on a model consisting of a high Q "single mode" cavity through which we pass accelerated two-level atoms, as in Fig. 1. We find that the radiation is thermal (in the typical case) and the effective Boltzmann factor is now given by $\alpha/2\pi\omega$. For the above example, $\alpha/2\pi\omega \sim 10^{-3}$; hence, it is many orders of magnitude larger than that for the usual Unruh effect and is potentially observable. The reason for such a strong enhancement is a fast nonadiabatic switch of the interaction of atoms with the field at the boundaries of the cavity. Moreover, this nonadiabatic boundary contribution, in most cases, prevails over the standard Unruh effect.

The envisioned experiment can be described as a kind of "acceleration radiation" mazer [5,6]. In the ordinary maser, stimulated emission is the mechanism for the production of radiation. In the present case, the physics of the emission process is intimately association with the center-of-mass motion (taken in the z direction).

One scheme for accelerating [7] the atoms uses a particle accelerator with, e.g., hydrogenlike ions. In such a case, ordinary (i.e., not Unruh) radiation emitted by accelerated charged particles must be taken into account. Atoms can be accelerated in a strong gravitational field through a cavity. Other means of operation via periodically driven atoms are also possible, as in Figs. 1(b) and 1(c), and are discussed later. For the moment, we simply assume the trajectories given by, e.g., Eq. (2) and neglect the quantization of translational motion and recoil effect.

Our main results are contained in Eqs. (4), (5), (6a), (6b), (7a), (7b), (8a), (8b), and (9). We find that the

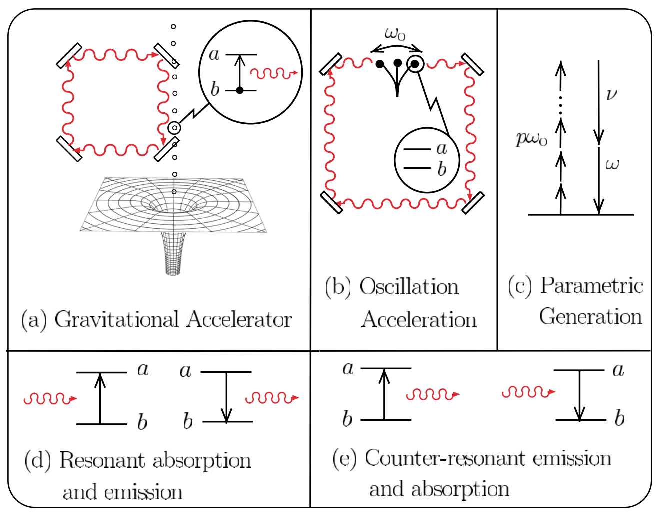

FIG. 1 (color). (a) Atoms in the ground state $|b\rangle$ are accelerated through small holes in the corner reflectors of a microwave (or optical) cavity by, e.g., a strong gravitational field. This is depicted as a unidirectional, single mode, ring cavity to convey the idea. (b) "Vibrating reed" piezoelectrically driven oscillator containing a two-level atom is placed in the cavity yielding strong mazer action. (c) Parametric conversion of vibronic energy $p\hbar\omega_0$ into photon and atom energies $\hbar\nu$ and $\hbar\omega$, respectively. (d) An atom is excited (deexcited) as it simultaneously absorbs (emits) a photon in a resonant process. (e) The counter-resonant processes that are usually neglected as compared to the resonant processes in the "rotating wave" approximation; i.e., an atom is excited (deexcited) as it simultaneously emits (absorbs) a photon.

acceleration radiation is generated by a kind of parametric process [8] in which both the atomic polarization (the idler) and the radiation (the signal) are excited by extracting energy from the atomic center-of-mass motion (the pump). Such processes are intimately related to the counter-rotating terms in the Hamiltonian and are discarded in the rotating wave approximation.

This provides a simple picture for the generation of acceleration radiation. The photons emitted are real. The generation of radiation by the counter-rotating terms is interesting, but, perhaps, no more bizarre than the earlier demonstration of mazer emission [5] due to scattering of atoms off the cavity interface [9]. Furthermore, we find that the radiation may even be squeezed when $S_{1,2}$ in Eqs. (8a) and (8b) are nonvanishing. Calculation details and experimental implications will be given elsewhere (in preparation).

As in the quantum theory of the laser [10,11], the (microscopic) change in the density matrix of a cavity mode due to any one atom, $\delta\rho^i$, is small. The (macroscopic) change due to ΔN atoms is then $\Delta\rho = \sum_i \delta\rho^i = \Delta N \delta\rho$. Writing $\Delta N = r\Delta t$, where r is the atomic injection rate, we have a coarse grained equation of motion: $\frac{\Delta\rho}{\Delta t} = r\delta\rho$. The change $\delta\rho^i$ due to an atom injected at time τ_i is

$$\delta\rho^i = -\frac{1}{\hbar^2}\int_{\tau_i}^{\tau_i+T}\int_{\tau_i}^{\tau_i+\tau'} \mathrm{tr}_{\mathrm{atom}}[\hat{V}(\tau'), [\hat{V}(\tau''), \rho^{\mathrm{atom}}(\tau_i) \otimes \rho(t(\tau_i))]]d\tau' d\tau'', \tag{1}$$

where T is the proper time of flight through the cavity and $\mathrm{tr}_{\mathrm{atom}}$ denotes the trace over atom states. The time τ is the atomic proper time, i.e., the time measured by an observer riding along with the atom. The cavity proper time $t(\tau)$ and the atomic trajectory of the atom as it passes through the cavity, $z(\tau)$, are given by [12]

$$t(\tau) = t_0 + \frac{1}{\alpha}\sinh(\alpha\tau), \qquad z(\tau) = \frac{c}{\alpha}[\cosh(\alpha\tau) - 1], \tag{2}$$

where $t_0 = t(\tau = 0)$ is the moment of time in the laboratory (cavity) frame when the atom starts its acceleration. Let $g(\tau) = \mu E'/\hbar$ be the atom-field coupling frequency which depends on the atomic dipole moment μ and the electrical field E' in the frame of the atom. In the case of a running wave with a wave vector $\mathbf{k}$, $k_z = \mathbf{k}\cdot\mathbf{v}/v$, the interaction Hamiltonian in the atomic frame is

$$\hat{V}(\tau) = \hbar g(\tau)[\hat{a}_k e^{-i\nu t(\tau) + ik_z z(\tau)} + \mathrm{H.c.}][\hat{\sigma}e^{-i\omega\tau} + \mathrm{H.c.}]. \tag{3}$$

For simplicity, consider the copropagating atom and field, $k_z = \nu/c$, so that $E' = \sqrt{(c-v)/(c+v)}E$. Since $v = c\tanh(\alpha\tau)$ for a uniformly accelerated particle, we have $E' = e^{-\alpha\tau}E$ and $g(\tau) = ge^{-\alpha\tau}$. The operator $\hat{a}_k$ is the annihilation operator for the running wave, while $\hat{\sigma}$ is the atomic lowering operator. Inserting Eq. (3) into Eq. (1) and using Eq. (2), we obtain the results (4), (5), (6a), (6b), (7a), (7b), (8a), and (8b).

In the case of random injection times, the equation of motion for the density matrix of the field is

$$d\rho_{n,n}/dt = -R_2[(n+1)\rho_{n,n} - n\rho_{n-1,n-1}] - R_1[n\rho_{n,n} - (n+1)\rho_{n+1,n+1}], \tag{4}$$

where $R_{1,2}$ are defined in the following. If $R_1 > R_2$, there is a steady state solution which is thermal [10]

$$\rho_{n,n} = e^{-\hbar\nu n/k_B \mathcal{T}_c}(1 - e^{-\hbar\nu/k_B\mathcal{T}_c}), \tag{5a}$$

$$\bar{n} = \sum_n n\rho_{nn} = \frac{1}{e^{\hbar\nu/k_B\mathcal{T}_c} - 1}, \qquad e^{-\hbar\nu/k_B\mathcal{T}_c} = \frac{R_2}{R_1}, \tag{5b}$$

where an effective temperature of the field in the cavity is $\mathcal{T}_c = \hbar\nu/k_B \ln[R_1/R_2]$. Thus, spontaneous emission of randomly injected ground-state atoms in the cavity results in thermal statistics of the mode excitation. Note that the thermal statistics of the atomic excitation in the standard Unruh effect in free space is due to spontaneous emission into a vacuum field reservoir with a continuous spectrum of modes.

Absorption and emission coefficients $R_{1,2} = r|gI_{1,2}|^2$ are determined by the amplitude $ge^{-i\nu/\alpha}I_{1,2} = -\frac{i}{\hbar}\int_{\tau_i}^{\tau_i+T} V_{1,2}d\tau$ of the matrix elements $V_1 = \langle a, 0|\hat{V}|b, 1\rangle$ and $V_2 = \langle a, 1|\hat{V}|b, 0\rangle$ of the interaction Hamiltonian (3), respectively. In the particular case $\tau_i = 0$, we find

$$I_1(\omega) = \int_0^T \exp\left[i\frac{\nu}{\alpha}e^{-\alpha\tau} + i\omega\tau - \alpha\tau\right]d\tau. \tag{6a}$$

It is convenient to write this as

$$I_1(\omega) = \left[\int_{\tau^*}^T d\tau - \int_{\tau^*}^0 d\tau\right]\exp\left[i\frac{\nu}{\alpha}e^{-\alpha\tau} + i\omega\tau - \alpha\tau\right], \tag{6b}$$

where $\tau^* = -\infty - i\pi/2\alpha$. We carry out the first integral by changing the variable of integration to $x = -i(\nu/\alpha)e^{-\alpha\tau}$ and assume that $e^{-\alpha T} \approx 0$. In such a case the first integral is proportional to the ordinary gamma function defined as $\Gamma(z) = \int_0^\infty e^{-x}x^{z-1}dx$. The second integral may be adequately approximated by integration by parts in the limit that $\frac{\alpha}{\nu} \ll 1$ and $\frac{\alpha}{\omega} \ll 1$. We find

$$I_1(\omega) = \frac{i}{\nu}\left(\frac{\alpha}{\nu}\right)^{-i(\omega/\alpha)} e^{\pi\omega/2\alpha}\Gamma\left(1 - \frac{i\omega}{\alpha}\right) - \frac{ie^{i(\nu/\alpha)}}{\nu - \omega}\left[1 + \mathrm{O}\left(\frac{\alpha\omega}{(\nu-\omega)^2}\right)\right]. \tag{7a}$$

The corresponding integral for the emission of radiation $I_2(\omega)$ is equal to $I_1(-\omega)$. We proceed to calculate $R_1 \propto |I_1(\omega)|^2$ and $R_2 \propto |I_2(\omega)|^2$ by noting that $\Gamma(1 + i\omega/\alpha)\Gamma(1 - i\omega/\alpha) = (\pi\omega/\alpha)/\sinh(\pi\omega/\alpha)$.

We find that in the limit $\nu \gg \omega \gg \alpha$ the emission/absorption ratio is $R_2/R_1 \simeq \alpha/(2\pi\omega)$, which is an enhancement by many orders of magnitude as compared to the exponentially small value $R_2/R_1 = \exp(-2\pi\omega/\alpha)$.

For arbitrary values of parameters, the absorption and emission amplitudes can be calculated as

$$I_{1,2}(\omega) = \frac{i}{\nu}\left(\frac{\alpha}{\nu}\right)^{\mp i(\omega/\alpha)} e^{\pm\pi\omega/2\alpha}[\Gamma(z, ue^{-\alpha T}) - \Gamma(z, u)], \tag{7b}$$

where $z = 1 \mp i\frac{\omega}{\alpha}$, $u = -i\frac{\nu}{\alpha}e^{-\alpha\tau_i}$, and $\Gamma(z, u) = \int_u^\infty e^{-x}x^{z-1}dx$ is the incomplete gamma function.

The above analysis clearly shows that the mechanism of the field and atom excitation in cavity QED is the same as for the Unruh effect in free space and is nothing but a nonadiabatic transition due to the counter-rotating term $\hat{a}_k^+\hat{\sigma}^+$ in the interaction Hamiltonian (3), i.e., V_2. The reason for an enhanced excitation in the cavity is the relatively large amplitude for a quantum transition $|b, 0\rangle \to |a, 1\rangle$ due to the sudden nonadiabatic switching on of the interaction. As a result of this rapid turn on, the initial state $|b, 0\rangle$ is no longer an eigenstate of the Hamiltonian. Now, a linear superposition of the excited states of atom and field makes up the dressed [13] ground state of the interacting system $\psi_0 = |b, 0\rangle - \frac{g(\tau)}{\nu'+\omega}|a, 1\rangle$ as well as the dressed excited state $\psi_1 = |a, 1\rangle + \frac{g(\tau)}{\nu'+\omega}|b, 0\rangle$.

The amplitude of the bare excited state $|a, 1\rangle$ in ψ_0 is of the order of $C \sim \mu E'/\hbar(\omega + \nu')$. The latter corresponds to the atomic excitation probability $\rho_{aa}^{\text{atom}} = |C|^2 \sim |\mu E'/\hbar(\omega + \nu')|^2 \sim |gI_2|^2$. This can also be found directly from the density matrix equation for the atom, via the atomic counterpart to Eq. (1) with a trace over the photon states instead of the tr_{atom}. This probability has the same origin and value as the well-known Bloch-Siegert shift of a two-level atomic transition [13], $\Delta\omega/\omega = [\mu E'/\hbar(\omega + \nu')]^2$, due to counter-rotating terms.

Clearly, the second term in Eqs. (6b) and (7a) represents the contributions from boundaries to the nonadiabatic transition amplitudes [14]. In the absence of the boundaries, the emission integral $I_2(\omega) = I_1(-\omega)$ in Eqs. (6a), (6b), (7a), and (7b) becomes exponentially small $\sim \exp(-\pi\omega/\alpha)$ for the small parameter $\alpha/2\pi\omega \ll 1$ since there are no stationary-phase points in the integration interval. The absorption integral I_1 does have a point of stationary phase when the atomic frequency ω is brought into resonance with the field due to the time-dependent Doppler shift of the mode frequency [15] $\nu'(\tau) = \nu\exp(-\alpha\tau)$. This fact explains why the related exponential factor effectively disappears from the absorption integral (7a), $|e^{\pi\omega/2\alpha}\Gamma(1 - i\omega/\alpha)| \simeq (2\pi\omega/\alpha)^{1/2}$, when $\alpha \ll 2\pi\omega$. As a result, if there are no edge effects, we obtain the same excitation factor $R_2/R_1 = \exp(-2\pi\omega/\alpha)$ as in the Unruh effect (in free space). This means that in order to observe the standard Unruh result one has to extend the mode profile $g(z)$ near the boundaries, i.e., eliminate nonadiabatic boundary contributions [second term in Eq. (6b)].

The nonadiabatic nature of the Unruh effect can be demonstrated most clearly by following explicit derivation of the Unruh factor as a probability of the nonadiabatic transition [16] $\psi_0 \to \psi_1$ from the dressed ground state. Indeed, the Schrödinger equation $i\hbar d\psi/d\tau = H\psi$ in the two-level case $\psi = c_0\psi_0 + c_1\psi_1$ yields $dc_1/d\tau + (iE_1/\hbar + \langle\dot{\psi}_1|\psi_1\rangle)c_1 = -c_0\langle\dot{\psi}_0|\psi_1\rangle$. The difference between the eigenenergies is, to the first order, $E_1 - E_0 = \hbar(\omega + \nu')$. For small nonadiabatic coupling $-\langle\dot{\psi}_0|\psi_1\rangle = \frac{d}{d\tau}(\frac{g(\tau)}{\omega+\nu'}) \ll \omega + \nu'$, the perturbation solution is $|c_1|^2 = |\int_{\tau_i}^{\tau}\exp[i\int_{\tau_i}^{\tau'}(\nu' + \omega)d\tau'']\frac{d}{d\tau'}(\frac{g(\tau')}{\omega+\nu'})d\tau'|^2$. If we now make the assumption of an adiabatic switching (on and off) of the interaction $g(\tau)$ as in standard Unruh effect treatments, then after integration by parts the latter integral is reduced to the integral $I_2(\omega) = I_1(-\omega)$ in Eqs. (6a) and (6b) but in the infinite limits, i.e., without edge effects. This yields the standard Unruh factor $|c_1|^2 \propto \exp(-2\pi\omega/\alpha)$. This derivation clearly shows the dramatic effect of boundary contributions leading to a large amplitude $\sim g(\tau)/(\omega + \nu')$ of the atomic excited state $|a\rangle$. Only if we eliminate the edge effects by adiabatic switching of the interaction do we retrieve the exponentially small excitation factor.

The surprising result is that in the cavity the excitation factor $\exp(-\hbar\nu/k_B\mathcal{T}_c) \equiv R_2/R_1 = \alpha/2\pi\omega$ is determined by the first power of the same nonadiabaticity parameter $\alpha/2\pi\omega$. The reason for this effect is the existence of a true resonance, i.e., a stationary-phase point, in the absorption coefficient [first term in Eqs. (6b) and (7a)]. As mentioned earlier, this yields a resonance between the atomic transition frequency and the Doppler-shifted frequency of the field seen by the atom, $\omega + \frac{d}{d\tau}(\frac{\nu}{\alpha}e^{-\alpha\tau}) \simeq 0$.

Another feature of the cavity acceleration radiation is squeezing. If the atoms are injected at regular intervals of times, $t_{0i} = \pi m_i/\nu + t_\phi$, where m_i is an integer, all atoms have the same phase with respect to the cavity mode, $\Phi = \sum_{j=1}^{\Delta N} e^{-2i\pi m_j - 2i\nu t_\phi}/\Delta N = e^{-2i\nu t_\phi}$, and instead of Eq. (4) we find

$$\dot{\rho}_{n,n} = -R_1[n\rho_{nn} - (n+1)\rho_{n+1,n+1}] - R_2[(n+1)\rho_{n,n} - n\rho_{n-1,n-1}] + [-S_1\sqrt{(n+1)(n+2)}\rho_{n+2,n} - S_2\sqrt{(n-1)n}\rho_{n,n-2} + (S_1 + S_2)\sqrt{(n+1)n}\rho_{n+1,n-1} + \text{H.c.}]. \tag{8a}$$

In this case the analysis is similar to the analysis of a polarization injected laser [9], and the radiation density matrix is far from being thermal due to squeezing factors

$$S_{1,2} = rg^2\Phi e^{-2i\nu/\alpha}\int_{\tau_i}^{\tau_i+T}d\tau'\int_{\tau_i}^{\tau'}d\tau'' e^{i(\nu/\alpha)e^{-\alpha\tau'}\mp i\omega\tau' - \alpha\tau'}e^{i(\nu/\alpha)e^{-\alpha\tau''}\pm i\omega\tau'' - \alpha\tau''}. \tag{8b}$$

It is also possible to implement a more powerful resonant emission by ground-state atoms in a cavity, e.g., when the center of mass of the atom is oscillating as $z(\tau) = z_0\cos(\omega_0\tau)$. This can be viewed as another example of mazer action.

In such a case, the density matrix of a cavity mode is again found to obey Eqs. (8a) and (8b), but now

$$R_{1,2} \cong rg^2 J_p^2(kz_o)/(\gamma + \alpha)^2, \tag{9}$$

where p is an integer, $J_p(x)$ the Bessel function, γ the effective atomic decay rate, and the squeezing terms $S_{1,2}$ are governed by cross terms which go as $[rg^2/(\gamma+\alpha)^2]J_pJ_0$ [17]. Since this is a resonant parametric process [Fig. 1(b)], the absorption ($p = 0$, $\omega = \nu$) and emission ($p \neq 0$, $\omega + \nu = p\omega_0$) coefficients (9) are larger than for counter-rotating interactions, Eqs. (6a), (6b), and (7a), by a resonant factor $[\nu/(\gamma+\alpha)]^2$ [18].

In conclusion, our simple model demonstrates that the ground-state atoms accelerated through a vacuum-state cavity radiate real photons. For small acceleration $a < 2\pi\omega c$, the excitation Boltzmann factor $\exp(-\hbar\nu/k_B\mathcal{T}_c) \sim \alpha/2\pi\omega$ is much larger than the standard Unruh factor $\exp(-2\pi\omega/\alpha)$. The physical origin of the field energy in the cavity and of the internal energy in the atom is the work done by an external force driving the center-of-mass motion of the atom against the radiation reaction force. Both the present effect (in a cavity) and the Unruh effect (in free space) originate from the transition of the ground-state atom to the excited state with simultaneous emission of photon due to the counter-rotating term $\hat{a}_k^+\hat{\sigma}^+$ in the Hamiltonian (3). The enhanced rate of emission into the cavity mode comes from the second term in Eqs. (6b) and (7a)–the nonadiabatic transition at the cavity boundaries; the standard Unruh excitation comes from the first term in Eqs. (6b) and (7a)–the nonadiabatic transition in free space due to the time dependence of the Doppler-shifted field frequency $\nu' = \nu e^{-\alpha\tau}$, as seen by the accelerating atom.

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[14] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, Oxford, 1977).

[15] The Doppler-shifted frequency of the co- or counter-propagating wave is $\nu' = d(\nu t(\tau) - k_z z(\tau))/d\tau = \nu e^{\mp\alpha\tau}$.

[16] For the review on nonadiabatic transitions see, e.g., V. V. Zheleznyakov, V. V. Kocharovsky, and Vl. V. Kocharovsky, Sov. Phys. Usp. **26**, 877 (1983).

[17] This effect is similar to squeezing induced by oscillatory mirror motion; see, e.g., V. V. Dodonov, Phys. Rev. A **58**, 4147 (1998), and references therein.

[18] Remarkably, parametric gain is possible in cavity QED via counter-resonant emission by ground-state atoms with random injection times. For such gain to occur, the time of flight T should be tuned to a set of specific values to ensure that the atom emits into the cavity mode more energy than it takes away, $R_2 > R_1$. The counter-propagating mode is more favorable for gain since the absorption can then be small while the gain remains as large as for the copropagating mode. The gain is easily seen in the case of a constant velocity: $\frac{R_2}{R_1} = |\frac{\nu'-\omega}{\nu'+\omega}|^2 |\frac{1-e^{-i(\nu'+\omega)T}}{1-e^{-i(\nu'-\omega)T}}|^2$, $\frac{\nu'}{\nu} = (\frac{\nu-\mathbf{k}\cdot\mathbf{v}}{\nu+\mathbf{k}\cdot\mathbf{v}})^{1/2}$. One can tune the time of flight to get the proper interference factors: $e^{-i(\nu'-\omega)T} \to 1$, $|e^{-i(\nu'+\omega)T} - 1| \sim 1$. Similar tuning is used in some electronic devices, e.g., klystrons.