Electromagnetic Meissner Effect in Spin-One Color Superconductors

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It is shown that color-superconducting quark matter, where quarks of the same flavor form Cooper pairs with spin one, exhibits an electromagnetic Meissner effect. This is in contrast to spin-zero color superconductors where Cooper pairs consist of quarks with different flavors.

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Quantum chromodynamics (QCD) is an asymptotically free theory [1] and, thus, quark matter at large quark chemical potential μ is a weakly coupled system. In this case, the dominant interaction between two quarks is single-gluon exchange. Single-gluon exchange is attractive in the color-antitriplet channel. Consequently, at sufficiently low temperatures, the quark Fermi surface is unstable with respect to the formation of Cooper pairs. Since this is analogous to what happens in ordinary superconductors [2], this phenomenon was termed *color* superconductivity [3].

Color superconductivity was studied from first principles in the framework of QCD at weak coupling [4,5] as well as in more phenomenological Nambu–Jona-Lasinio-type models [6]. Both approaches indicate that the color-superconducting state is the true ground state of quark matter at any density beyond the quark-hadron phase transition and at sufficiently low temperature. They also agree in the magnitude of the color-superconducting gap parameter, ϕ, which they predict to be of the order of tens of MeV (for quark Cooper pairs with total spin zero) at densities of the order of 10 times the nuclear matter ground state density. Gap parameters of this order of magnitude may have enormous phenomenological implications, since the transition temperature to the normal conducting phase is typically of the order of ϕ [5,7,8]. For instance, during the evolution of a neutron star, the temperature ranges from a few tens of MeV down to a few keV [9]. If its core is sufficiently dense to consist of quark matter, this quark matter core is then very likely a color superconductor.

Color superconductivity is more complex than ordinary superconductivity, because quarks do not carry only electric, but also color and flavor charges. Two different quark flavors may form Cooper pairs in the color-antitriplet, flavor-singlet, total spin-zero channel (the so-called 2SC phase) [3]. For three different quark flavors, the favored state is the so-called color-flavor locked (CFL) phase [10], with spin-zero Cooper pairs in the color-antitriplet, flavor-antitriplet representation.

A necessary condition for pairing of quarks of different flavors is that their respective Fermi surfaces are equal. In physical systems, however, this condition may be hard to achieve. For instance, compact stellar objects are neutral with respect to electric and color charge. This requires the introduction of separate chemical potentials for quarks which differ in color and flavor [11]. Imposing the constraints of color and electric charge neutrality leads to different values for these chemical potentials. This effect and the mass difference between the light up and down quarks and the heavier strange quark [12] may then lead to different Fermi surfaces for each quark species. If this difference is of the order of, or larger than, the color-superconducting gap parameter ϕ, a color-superconducting state where quarks with different flavors form spin-zero pairs with zero momentum is no longer favored. Then, besides a transition to the normal conducting state [13], there are at least four other possibilities. The first two possibilities are either a displacement [14] or a deformation [15] of the Fermi spheres of the two quark species forming the Cooper pair, breaking translational or rotational invariance, respectively. The third possibility is an interior gap structure for the quark species with the larger Fermi momentum [16].

The fourth possibility is that quarks of the same flavor form Cooper pairs with total spin one [3,5,17–19]. Quarks with the same flavor have the same mass and the same electric charge, and thus pairing is not affected by either a mass difference or a nonzero electric chemical potential which may be required to fulfill the constraint of electric neutrality. Moreover, a potentially nonzero color chemical potential does not destroy the Cooper pairs, because we expect it to be parametrically much smaller than the gap, $\mu_{\rm color} \sim \phi^2/(g\mu) \ll \phi$, where g is the strong coupling constant [20]. (A color chemical potential is necessary in models which treat color as a global charge. In QCD, the role of the color chemical potential is assumed by a constant gluon background field A_a^0 [20].)

According to the results of Refs. [8,18], the gap in spin-one color superconductors is of the order of 20–400 keV (assuming that the gap in the 2SC phase is of the order of 10–100 MeV). The critical temperature is therefore of the order of 10–400 keV [8]. Consequently, when the core of a neutron star cools below this temperature, it could very well consist of quark matter in a spin-one color-superconducting state.

The interesting question is how such a state affects the properties of the star and whether this leads to observable consequences. The best known properties of a neutron star are its radius and mass, which are determined by the equation of state. A recent study [21] shows that radii and masses of compact stellar objects with a color-superconducting quark matter core do not change appreciably from the values expected for ordinary neutron stars. Another property of a neutron star with potentially observable consequences is its magnetic field. Because of an admixture of protons in neutron matter, the core of an ordinary neutron star is a superconductor, and magnetic fields experience the Meissner effect. However, if the core of a neutron star is a spin-zero color-superconductor (for instance, in the 2SC or CFL phase), there is no electromagnetic Meissner effect [22]. Since charge neutrality may favor a spin-one over a spin-zero color-superconducting state, a natural question is whether the electromagnetic Meissner effect is also absent in a spin-one color superconductor. In order to answer this question, in this Letter we study the pattern of symmetry breaking of the local gauge symmetries, and then also present the results of an explicit calculation of the Meissner masses.

In ordinary superconductors, the electromagnetic gauge group $U(1)_{em}$ is broken due to the fact that the electrons in a Cooper pair carry electric charge. This leads to the electromagnetic Meissner effect; i.e., magnetic fields penetrate only a finite distance into the superconductor. The inverse distance can be associated with a nonzero photon mass, the so-called Meissner mass. Since quarks are not only electrically charged but also carry color, besides the electromagnetic $U(1)_{em}$ symmetry also the $SU(3)_c$ gauge symmetry of the strong interaction is broken in a color superconductor. This leads to the color Meissner effect; i.e., the gluons obtain Meissner masses and color-magnetic fields are expelled. The question is whether all eight gluons and the photon become massive. This depends on the particular pattern of how the local symmetries are broken in the color superconductor. If there is a residual local symmetry, the corresponding gauge bosons remain massless. This residual symmetry group of $SU(3)_c \times U(1)_{em}$ leaves the gap matrix Δ invariant,

$$(g_c \times g_{em})\Delta(g_c^T \times g_{em}^T) \overset{!}{=} \Delta, \tag{1}$$

where $g_c \in SU(3)_c$, $g_{em} \in U(1)_{em}$, and T denotes the transpose. In general, the gap matrix Δ is a matrix in color, flavor, and Dirac (spin) space [3–6]. Since pairing occurs in the attractive color-antitriplet channel, the color structure of the gap matrix corresponds to the color-antitriplet $\bar{\mathbf{3}}_c$ representation of the $SU(3)_c$ gauge group. In a spin-one color superconductor, the spin structure of the gap matrix corresponds to the symmetric spin triplet $\mathbf{3}_J$ representation of the $SU(2)_J$ spin group, which is also a representation of $SO(3)_J$. The gap matrix is diagonal in flavor space, since quarks in a Cooper pair carry the same flavor. For the moment, let us consider quark matter with a single flavor only, $N_f = 1$. The case of several quark flavors (where each flavor pairs at its respective Fermi surface) will be discussed below. The gap matrix can be written as

$$\Delta = \Phi_a^i J_a \otimes v^i, \tag{2}$$

where J_a and v_i $(a, i = 1, 2, 3)$ are bases of $\bar{\mathbf{3}}_c$ and $\mathbf{3}_J$, respectively, and Φ_a^i is the order parameter. The form of the order parameter defines the phase of the condensate. As in ^{3}He, there is a multitude of possible phases for spin-one condensates [23]. In this Letter we consider only the polar phase and the color-spin locked (CSL) phase [8,18]. The order parameters in these phases are

$$(\Phi_a^i)_{\text{polar}} \sim \delta_{a3}\delta^{i3}, \qquad (\Phi_a^i)_{\text{CSL}} \sim \delta_a^i. \tag{3}$$

In the polar phase, the condensate points in a fixed direction in real space, which breaks the global spatial symmetry group $SO(3)_J$ to $SO(2)_J$. The condensate also points in a fixed direction in color space, which spontaneously breaks the local $SU(3)_c$ gauge symmetry to a residual $SU(2)_c$ gauge group. From Eq. (1) we deduce that the residual subgroup which leaves the order parameter invariant is generated by the three generators of $SU(2)_c$ [corresponding to T_1, T_2, T_3 of the original $SU(3)_c$] and the generator $\tilde{Q}_{\text{polar}} = Q - 2\sqrt{3}qT_8$, where T_8 is one of the generators of $SU(3)_c$ and $Q \equiv q\mathbf{1}_J$ is the generator of $U(1)_{em}$. The constant q is the electric charge of the single quark flavor considered here (2/3 for u quarks and $-1/3$ for d or s quarks). The generator Q is proportional to the unit matrix in spin triplet space, $\mathbf{1}_J$, since all states of the spin triplet have the same electric charge. Consequently, the symmetry breaking pattern is $SU(3)_c \times U(1)_{em} \to SU(2)_c \times \tilde{U}(1)$, where $\tilde{U}(1)$ is generated by $\tilde{Q}_{\text{polar}}$. The existence of a nontrivial residual gauge symmetry is equivalent to the fact that there are charges with respect to which the Cooper pairs are neutral. These are the two color charges corresponding to the $SU(2)_c$ gauge symmetry and the $\tilde{Q}_{\text{polar}}$ charge corresponding to the $\tilde{U}(1)$ gauge symmetry. The gauge boson of the latter is a superposition of the photon and the eighth gluon of $SU(3)_c$. This superposition is mathematically given by an orthogonal rotation of the original gauge fields by an angle θ. In general, a generator $\tilde{Q} = Q + \eta T_8$ results in a mixing angle given by $\cos^2\theta = g^2/(g^2 + \eta^2 e^2)$, where e is the electromagnetic coupling constant [22]. In our case the mixing angle, θ_{polar}, is determined by this expression with $\eta = -2\sqrt{3}g$. Since $e \ll g$, the mixing angle is small, $\theta_{\text{polar}} \simeq 2\sqrt{3}qe/g \sim q/3$. Consequently, the main contribution to the gauge boson of the local $\tilde{U}(1)$ symmetry comes from the original photon, with a small admixture of the eighth gluon. This justifies calling this gauge boson the "new" photon. There is no electromagnetic Meissner effect, since the new photon can penetrate the color-superconducting phase. This is similar to other color-superconducting phases, for instance, the 2SC phase or the CFL phase [22]. In both cases, there is a

$\tilde{U}(1)$ gauge symmetry and thus no electromagnetic Meissner effect.

In the CSL phase, the order parameter breaks $SU(3)_c \times SO(3)_J$ to the diagonal subgroup $SO(3)_{c+J}$ [18]. This is analogous to the breaking of color-flavor symmetries in the CFL phase, where $SU(3)_c \times SU(3)_f \to SU(3)_{c+f}$. The residual $SO(3)_{c+J}$ and $SU(3)_{c+f}$ groups are global symmetries, and thus not gauged. This similarity between CSL and CFL phases does, however, not extend to the behavior concerning electromagnetism. Unlike the CFL phase, it turns out that for the CSL phase there is *no* nontrivial subgroup of $SU(3)_c \times U(1)_{em}$ that leaves the gap matrix invariant; i.e., the only possible solution to Eq. (1) is $g_c = g_{em} = \mathbf{1}$. Consequently, the symmetry breaking pattern is $SU(3)_c \times U(1)_{em} \to \mathbf{1}$. This is equivalent to the fact that a Cooper pair in the CSL phase is *not* neutral, neither with respect to ordinary electric charge nor with respect to any possible new combination of color and electric charge. This fact has the physical consequence that there is an electromagnetic Meissner effect for the CSL phase [18].

In the following, we confirm the above qualitative arguments by an explicit calculation of the Meissner masses. The Meissner mass is defined as the zero-energy, zero-momentum limit of the spatial (ii) components of the polarization tensor [24]

$$m^2_{ab} \equiv \lim_{p\to 0}\Pi^{ii}_{ab}(0,p), \qquad a,b = 1,\ldots,8,9, \tag{4}$$

where the first eight indices correspond to the gluons, and the ninth index to the photon, $9 \equiv \gamma$. If there is mixing of gluons and photons, the 9×9 Meissner mass matrix m^2_{ab} is not diagonal. The Meissner masses for the physically relevant modes are obtained by diagonalizing this matrix. In weak coupling, the polarization tensor may be computed in one-loop approximation. At zero temperature, only the quark loop contributes. Our calculation follows the method of Ref. [24]; details are deferred to a future paper [25]. It turns out that the gluon part of the mass matrix is diagonal, $m^2_{ab} \equiv \delta_{ab} m^2_{aa}$ for $a, b = 1,\ldots,8$. In Table I we collect all results.

For the polar phase, the vanishing masses for gluons 1, 2, and 3 indicate the unbroken $SU(2)_c$ subgroup. The nonzero Meissner mass $m^2_{8\gamma}$ reflects the mixing of the eighth gluon field, A_8, and the photon, $A_9 \equiv A_\gamma$. In terms of the physically relevant modes, $\tilde{A}_a$, the Meissner mass matrix is diagonal,

$$\sum_{ab} A_a m^2_{ab} A_b \equiv \sum_a \tilde{A}_a \tilde{m}^2_a \tilde{A}_a. \tag{5}$$

For $a = 1,\ldots,7$, $\tilde{A}_a \equiv A_a$ (and, correspondingly, $\tilde{m}^2_a \equiv m^2_{aa}$), whereas the new gluon, $\tilde{A}_8$, and the new photon, $\tilde{A}_\gamma$, are obtained by an orthogonal rotation of A_8 and A_γ,

$$\begin{pmatrix} \tilde{A}_8 \\ \tilde{A}_\gamma \end{pmatrix} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} A_8 \\ A_\gamma \end{pmatrix}, \tag{6}$$

where $\tan(2\vartheta) \equiv 2\, m^2_{8\gamma}/(m^2_{88} - m^2_{\gamma\gamma})$. With the numbers of Table I we find that the rotation angle ϑ is identical to the mixing angle $\theta_{\rm polar}$ found in the above group-theoretical argument. The Meissner masses for the rotated gauge fields are

$$\tilde{m}^2_8 = \left(\frac{1}{3}g^2 + 4q^2e^2\right)\frac{\mu^2}{6\pi^2}, \qquad \tilde{m}^2_\gamma = 0. \tag{7}$$

The massless new photon confirms that there is no electromagnetic Meissner effect in the polar phase, whereas there is a color Meissner effect for the new gluon.

TABLE I. Zero-temperature masses (squared) calculated for the polar phase and the CSL phase of a spin-one color superconductor. The gluon masses are given in units of $g^2\mu^2/(6\pi^2)$, the mixed masses in units of $eg\mu^2/(6\pi^2)$, and the photon masses in units of $e^2\mu^2/(6\pi^2)$. The constants α and β are defined as $\alpha \equiv (3 + 4\ln 2)/27$ and $\beta \equiv (6 - 4\ln 2)/9$.

	m^2_{aa}								$m^2_{a\gamma}$		$m^2_{\gamma\gamma}$
a	1	2	3	4	5	6	7	8	1–7	8	9
Polar	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{\sqrt{3}}q$	$4q^2$
CSL	β	α	β	β	α	β	α	β	0	0	$6q^2$

For the CSL phase, the particular pattern for the gluon masses reflects the residual global $SO(3)_{c+J}$ symmetry which is generated by a combination of the antisymmetric Gell-Mann matrices T_2, T_5, and T_7 and the generators of $SO(3)_J$. In contrast to the polar phase, now all mixed masses are zero, indicating that there is no mixing between gluons and the photon. All gauge bosons have a nonzero Meissner mass. This coincides with the above group-theoretical argument.

Let us now discuss the situation with more than one quark flavor, where the quarks of each flavor separately form spin-one Cooper pairs. In general, each quark flavor has a different chemical potential, μ_i, $i = 1,\ldots,N_f$. In this case, the results of Table I have to be modified. All gluon Meissner masses m^2_{aa} have to be multiplied by a factor $\sum_i(\mu_i/\mu)^2$. In the mixed masses $m^2_{a\gamma}$ we have to replace the single quark electric charge q by a factor $\sum_i q_i(\mu_i/\mu)^2$. Finally, one has to substitute the square of the charge q^2 in the photon masses $m^2_{\gamma\gamma}$ by a factor $\sum_i(q_i\mu_i/\mu)^2$.

For the polar phase, these modifications have the following effect. Only if $\sum_{i,j} q_i(q_i - q_j)(\mu_i\mu_j)^2 = 0$, there is a massless combination of photon and gluon. For a system with two quark flavors, for instance, u and d or u and s, this condition is equivalent to $\mu_u^2\mu_d^2 = 0$ or $\mu_u^2\mu_s^2 = 0$, respectively. This means that, for these two combinations of flavors, there is always a Meissner effect, unless the chemical potential of one of the two quark flavors is zero; i.e., this flavor is absent. For a two-flavor system consisting of d and s quarks, however, the above condition is always trivially fulfilled; i.e., there is no Meissner effect in this case. This is to be expected, since d and s quarks carry the same electric charge and thus appear as one single flavor with respect to electromagnetic interactions. For three flavors, the above condition is equivalent to $\mu_u^2(\mu_d^2 + \mu_s^2) = 0$, and can be fulfilled only if either $\mu_u = 0$ or $\mu_d = \mu_s = 0$. Both cases have been discussed above. We therefore conclude that, for more than one quark flavor, there is an electromagnetic Meissner effect in the polar phase, unless all quarks carry the same electric charge.

In the CSL phase, the mixed masses always vanish, while the results for the gluon masses and the photon mass are modified by the same factors as for the polar phase. Thus, the conclusion that there is no photon-gluon mixing and an electromagnetic Meissner effect in the CSL phase remains valid also for more than one quark flavor.

While spin-zero color superconductors could be of type II at small μ [26,27], a spin-one color superconductor is most likely always of type I because the ratio of the penetration depth to the coherence length is of order $\sim\phi/(e\mu) \sim 10^{-3}\,[100\text{ MeV}/(e\mu)] \ll 1$. Consequently, the magnetic field is completely expelled from the core of a compact stellar object if it is a spin-one color superconductor. This is true unless the magnetic field exceeds the critical field strength for the transition to the normal conducting state. The magnetic field in neutron stars is typically of the order of 10^{12} G. This is much smaller than the critical magnetic field which, from the results of Ref. [27], we estimate to be of the order of 10^{16} G.

In conclusion, a compact stellar object with a core consisting of quark matter in the spin-one color-superconducting state is, with respect to its electromagnetic properties, different from an ordinary neutron star: ordinary neutron star matter is commonly believed to be an electromagnetic superconductor of type II, while a spin-one color superconductor is of type I. The question whether neutron star matter is a type-I or type-II superconductor has recently stirred a lot of attention [28] because it was shown that type-II superconducting matter is incompatible with the observation of pulsars with precession periods of order one year [29]. The presence of spin-one color-superconducting quark matter in the pulsars' core could explain this observation.

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