Constraining the Variation of Fundamental Constants using 18 cm OH Lines

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We describe a new technique to estimate variations in the fundamental constants using 18 cm OH absorption lines, with the advantage that all lines arise in the same species, allowing a clean comparison between the measured redshifts. In conjunction with one additional transition, it is possible to simultaneously measure changes in α, g_p, and $y \equiv m_e/m_p$. We use the 1665 and 1667 MHz line redshifts in conjunction with those of HI 21 cm and mm-wave molecular absorption in a gravitational lens at $z \sim 0.68$ to constrain changes in the three parameters over the redshift range $0 < z \lesssim 0.68$. While the constraints are relatively weak ($\lesssim 1$ part in 10^3), this is the first simultaneous constraint on the variation of all three parameters. Either one (or more) of α, g_p, and y must vary with cosmological time or there must be systematic velocity offsets between the OH, HCO^+, and HI absorbing clouds.

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Introduction.—The recent claim by Webb *et al.* [1,2] that the fine structure constant α evolves with redshift, with $\Delta\alpha/\alpha = (-1.88 \pm 0.53) \times 10^{-5}$ from $z \sim 1.6$ to today and $\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5}$ for $0.5 < z < 3.5$ (but see [3]), has spurred interest in the possibility that the numerical values of the fundamental constants change with time. Theories that can account for such variations include extra-dimensional Kaluza-Klein theories and superstring theories. In such models, the values of the coupling constants depend on the expectation values of some cosmological scalar field(s); changes in the values of the coupling constants are thus to be expected if this field varies with location and time. Further, depending on the details of the theory, all of the different coupling constants (such as α, the proton g factor g_p, the electron-proton mass ratio $y \equiv m_e/m_p$, the gravitational constant G, etc.) could, in principle, vary simultaneously. For example, Calmet and Fritzsch [4] and Langacker *et al.* [5] find that variations in the value of α should be accompanied by much larger changes (by $\sim$2 orders of magnitude) in the value of y. However, Ivanchik *et al.* [6] constrain the variation in y to be $(3.0 \pm 2.4) \times 10^{-5}$ over the redshift range $0 < z < 3$, comparable to the change claimed in the fine structure constant. A review of the available experimental and observational measurements on the variability of the coupling constants can be found in [7].

One of the main problems in most of the astrophysical techniques used to measure (or constrain) the values of the different constants (e.g., [2,6,8]) is that they involve a comparison between the redshifts of spectral lines of different species (e.g., the HI 21 cm line, millimeter-wave molecular lines, and optical fine structure lines [2,8]). These species are unlikely to all arise at the same physical location in a gas cloud and might thus have systematic velocity offsets relative to each other; the redshift differences may thus be dominated by these effects rather than the measurement errors (i.e., the spectral resolution, which can be quite small, $\sim$few km/s, for HI 21 cm and mm-wave molecular absorption spectra). Conclusions drawn from a comparison between different species might thus well be incorrect.

Clearly, the best way to test the variation of the coupling constants is to use lines originating from a *single species*, but with different dependences on these constants. Further, since many constants may be varying simultaneously, it would be very useful if one could simultaneously measure the changes in a number of constants from this single species, rather than assuming that changes occur in only one of the constants and that the others remain unchanged. We present here a new technique which satisfies both these requirements, using the 18 cm lines of the OH radical.

The ground $2\Pi_{3/2}\, J = 3/2$ state of OH is split into two levels by Λ doubling and each of these Λ-doubled levels further split into two hyperfine states. Transitions between these levels lead to four spectral lines with wavelength $\sim$18 cm. Transitions with $\Delta F = 0$ are called the "main" lines, arising at rest frequencies of 1665.4018 and 1667.3590 MHz, while transitions with $\Delta F = 1$ are called "satellite" lines, with rest frequencies of 1612.2310 and 1720.5299 MHz. Since the four OH lines arise from two very different physical processes, viz. Λ doubling and hyperfine splitting, the transition frequencies have different dependences on the fundamental constants. A perturbative treatment of the OH molecule has been carried out by Dousmanis *et al.* [9](see also [10]); we use the expressions derived in these references to determine the dependence of linear combinations of the four OH line frequencies on α, g_p, and $y \equiv m_e/m_p$, and show that it is possible to simultaneously measure changes in both α and y, if all four line frequencies are known (assuming that g_p does not vary with time). Since OH and HCO^+ column densities are observed to be tightly correlated, both in the Galaxy [11] and out to $z \sim 1$ [12], these species are likely to arise at the same physical location; a comparison between the redshifts of the four 18 cm OH lines and the HCO^+ line should thus allow one to constrain the evolution of all the above three parameters. We use our observations of the OH main lines in the $z \sim 0.6846$ gravitational lens towards B0218+357, in tandem with published HI 21 cm and mm-wave molecular redshifts, to constrain $\Delta y/y$ between $z \sim 0.68$ and today. Finally, as this work was being written, an analysis on the use of OH lines to constrain changes in fundamental constants was also carried out by Darling [13], who, however, considers only variations in the fine structure constant α.

Constraints from radio spectral lines.—Consider two transitions whose rest frame frequencies $\nu_i(z), i = 1, 2$ depend on redshift, due to the evolution of various fundamental constants, such as α, g_p, m_e/m_p, etc. If the lines arise in a source at a "true" redshift z, the *measured* redshift $\hat{z}_i$ of each line is given by

$$(1 + \hat{z}_i)^{-1} = \frac{\nu_i(z)}{(1 + z)\nu_i(0)} = \frac{1 + [\Delta\nu_i(z)/\nu_i(0)]}{1 + z}, \quad (1)$$

where $\Delta\nu_i(z) \equiv \nu_i(z) - \nu_i(0)$; note that $\Delta\nu_i(z) = 0$ if the fundamental constants do not change with time. The first order difference between the two measured redshifts $\Delta z = \hat{z}_1 - \hat{z}_2$ is then

$$\frac{\Delta z}{1 + \bar{z}} = \left[\frac{\Delta\nu_2}{\nu_2(0)}\right] - \left[\frac{\Delta\nu_1}{\nu_1(0)}\right], \quad (2)$$

where $\bar{z}$ is the mean measured redshift. Given two spectral lines (or linear combinations of line frequencies) with different dependences on some fundamental parameter, one can thus use the differences between the measured redshifts to constrain the evolution of the parameter in question.

In the case of the four 18 cm OH lines, the following three independent relations have been shown to be satisfied by the line frequencies [9,10](note that the lines must also satisfy the constraint $\nu_{1665} + \nu_{1667} = \nu_{1720} + \nu_{1612}$):

$$\nu_A \equiv \nu_{1667} + \nu_{1665} = q_\Lambda\left[\left(2 + \frac{A'}{B'}\right)\left(1 - \frac{2 - A/B}{X}\right) - \frac{12}{X}\right], \quad (3)$$

$$\nu_B \equiv \nu_{1667} - \nu_{1665} = \frac{8d(X - 2 + A/B)}{15X}, \quad (4)$$

$$\nu_C \equiv \nu_{1720} - \nu_{1612} = \frac{4}{15X}[2a(2X + 2 - A/B) + 12b + (b + c)(X + 4 - 2A/B)]. \quad (5)$$

Equations (3)–(5) correspond to the energy split due to Λ doubling and to the difference and the sum of the hyperfine splits in the two Λ doubled levels, respectively. Here, $X \equiv [(A/B)\{(A/B) - 4\} + 16]^{1/2}$, A is the fine structure interaction constant, B is the rotational constant, A' and B' are the off-diagonal matrix elements of these operators, $q_\Lambda \approx 4B^2/h\nu_e$ (where $h\nu_e$ is the energy difference between the ground and the first excited electronic state), and, finally, a, b, c, and d are the "hyperfine constants" [9], whose experimental values are $a = 86.012 \pm 0.002$ MHz, $b = -116.719 \pm 0.008$ MHz, $c = 130.75 \pm 0.01$ MHz, and $d = 56.632 \pm 0.004$ MHz [14]. Numerically, $A/B = -7.547$ and $A'/B' = -6.073$ [10]. These quantities have the following dependences on the fundamental constants α, y, $R_\infty \equiv m_e e^4/\hbar^3 c$, and g_p: $A' \propto A \propto \alpha^2 R_\infty$, $B' \propto B \propto y R_\infty$, and $a, b, c, d \propto g_p \alpha^2 y R_\infty$ [10]. For the rotational constant B, we have assumed, as usual (e.g., [15]), that variations in (m_p/M), which are suppressed by a factor $m_p/U \sim 100$ (where M is the reduced mass and U the binding energy) can be ignored. Thus, we have $[A'/B'] \propto [A/B] \propto (\alpha^2/y)$. Replacing the above scalings in Eq. (3) for ν_A, we obtain $\nu_A \propto y^2 R_\infty F(\alpha^2/y)$, where $F \equiv F(\beta)$ is a function which depends only on the ratio $\beta \equiv A/B \propto \alpha^2/y$ and is defined by

$$F(\beta) = \left[\left(2 + \frac{6.073}{7.547}\beta\right)\left(1 + \frac{2 - \beta}{X(\beta)}\right) + \frac{12}{X(\beta)}\right]. \quad (6)$$

Thus,

$$\frac{\Delta\nu_A}{\nu_A} = 2\frac{\Delta y}{y} + \frac{\Delta R_\infty}{R_\infty} + \frac{\Delta F(\beta)}{F(\beta)} \quad (7)$$

$$= 2\frac{\Delta y}{y} + \frac{\Delta R_\infty}{R_\infty} + \frac{\beta}{F}\frac{dF}{d\beta}\left[2\frac{\Delta\alpha}{\alpha} - \frac{\Delta y}{y}\right]. \quad (8)$$

Evaluating the quantity on the right-hand side of the above equation, we obtain

$$\frac{\Delta\nu_A}{\nu_A} = 2.571\frac{\Delta y}{y} - 1.141\frac{\Delta\alpha}{\alpha} + \frac{\Delta R_\infty}{R_\infty}. \quad (9)$$

In similar fashion, Eqs. (4) and (5) yield

$$\frac{\Delta\nu_B}{\nu_B} = 2.442\frac{\Delta y}{y} - 0.883\frac{\Delta\alpha}{\alpha} + \frac{\Delta R_\infty}{R_\infty} + \frac{\Delta g_p}{g_p}, \quad (10)$$

$$\frac{\Delta\nu_C}{\nu_C} = 0.722\frac{\Delta y}{y} + 2.557\frac{\Delta\alpha}{\alpha} + \frac{\Delta R_\infty}{R_\infty} + \frac{\Delta g_p}{g_p}. \quad (11)$$

Equations (9)–(11) govern the way in which a change in one of the fundamental constants affects the line rest frequencies. Combining them in pairs in Eq. (2) yields

$$\frac{\Delta z_{AB}}{1+\bar{z}_{AB}} = \left[\frac{\Delta\nu_B}{\nu_B} - \frac{\Delta\nu_A}{\nu_A}\right] = -0.129\frac{\Delta y}{y} + 0.258\frac{\Delta\alpha}{\alpha} + \frac{\Delta g_p}{g_p}, \quad (12)$$

$$\frac{\Delta z_{AC}}{1+\bar{z}_{AC}} = \left[\frac{\Delta\nu_C}{\nu_C} - \frac{\Delta\nu_A}{\nu_A}\right] = -1.849\frac{\Delta y}{y} + 3.698\frac{\Delta\alpha}{\alpha} + \frac{\Delta g_p}{g_p}. \quad (13)$$

If all four OH lines are detected in absorption in a single cosmological system, we thus have two independent equations relating the differences in measured redshifts to the changes in α, g_p, and $y \equiv (m_e/m_p)$. Most analyses (using other lines) assume that g_p remains unchanged and then estimate the variation of α [8,15]. If one makes the same assumption in the case of the OH lines, Eqs. (12) and (13) immediately allow one to simultaneously solve for changes in both α and (m_e/m_p). Of course, one further equation is needed to simultaneously constrain the evolution of all three constants. Candidates include the HI 21 cm and mm-wave molecular lines. The best of these are likely to be the HCO^+ lines since HCO^+ and OH column densities are found to show a strong correlation (extending over more than 2 orders of magnitude in column density) both in the Galaxy [11] and out to $z \sim 1$ [12]; this suggests that HCO^+ and OH are located in the same region of the molecular cloud. Since the HCO^+ line arises from a rotational transition, we have

$$\frac{\Delta\nu_{\mathrm{HCO^+}}}{\nu_{\mathrm{HCO^+}}} = \frac{\Delta y}{y} + \frac{\Delta R_\infty}{R_\infty}. \quad (14)$$

Equations (12)–(14) all have the same dependence on the Rydberg constant R_∞, which, hence, cancels out (note that R_∞ itself depends on α through the relation $R_\infty \equiv m_e e^4/\hbar^3 c$). One might thus combine HCO^+ absorption redshifts with those derived from the 18 cm OH lines to provide the last equation needed to solve for the evolution of α, y, and g_p. This would allow a simultaneous measurement of all three constants, which has been hitherto impossible. We note that it is also possible to use other OH Λ-doubled transitions to simultaneously constrain the variation of the above parameters; this is discussed elsewhere [16] for the main OH lines. It would also be interesting to carry out an analysis similar to that of Bekenstein [3] to test whether these analyses of the OH lines are also affected by the possibility that the Hamiltonians involved might vary with time (i.e., *dynamical* variability of the different parameters); this is beyond the scope of the present paper. It should also be pointed out that the present calculation is based on an analysis of the OH levels using perturbation theory [9]; more recent analyses [17] use the "effective Hamiltonian" approach, giving rise to higher order effects. While it would be interesting to attempt the present calculation in the latter framework, we emphasise that it should result only in small changes to the coefficients in Eqs. (9)–(11) and does not affect the validity of our approach. Finally, while other "Lambda-doubled" systems could, in principle, be used for a similar analysis, none of these have multiple transitions detected in astrophysical objects (to the best of our knowledge). While it would be interesting to carry out searches for these other transitions, we suspect that OH is yet likely to prove the best candidate because of the strength of its multiple lines.

Application to the $z = 0.6846$ absorber towards B0218+357.—While the above analysis shows that one can use the four 18 cm OH lines (in conjunction with the HCO^+ transition) to constrain the evolution of three separate fundamental parameters, the weaker, satellite 1612 and 1720 MHz lines have thus far not been detected at cosmological distances. Observations are currently being scheduled to carry out deep searches for these lines in the four known OH absorbers at intermediate redshift [12,18,19]. For the present, we will instead use the detected 1665 and 1667 MHz transitions [i.e., Eqs. (9) and (10)] along with the HI 21 cm and millimeter (HCO^+) lines in the $z = 0.6846$ absorber towards B0218+357 to estimate changes in the fundamental parameters (assuming that the lines arise in the same gas cloud). Since the HI 21 cm frequency arises from a hyperfine split and is, hence, proportional to $g_p y \alpha^2 R_\infty$, we have

$$\frac{\Delta\nu_{21}}{\nu_{21}} = \frac{\Delta y}{y} + 2\frac{\Delta\alpha}{\alpha} + \frac{\Delta g_p}{g_p} + \frac{\Delta R_\infty}{R_\infty}. \quad (15)$$

Equations (9), (10), (14), and (15) can now be solved to measure changes in α, g_p, and m_e/m_p. The HI 21 cm redshift is $z_{\mathrm{HI}} = 0.684\,676 \pm 0.000\,005$ [8], while that of the HCO^+ absorption lines is $z_{\mathrm{HCO^+}} = 0.684\,693 \pm 0.000\,001$ [20]. Our new Giant Metrewave Radio Telescope OH absorption spectra towards B0218+357 [19] yield the following redshifts for the sum and the difference of the 1665 and 1667 MHz line frequencies: $z_{\mathrm{sum}} = 0.684\,682 \pm 0.000\,0056$ and $z_{\mathrm{diff}} = 0.685\,780 \pm 0.0067$. A simultaneous solution of Eqs. (9), (10), (14), and (15) then yields $(\Delta\alpha/\alpha) = (-0.38 \pm 2.2) \times 10^{-3}$, $(\Delta y/y) = (-0.27 \pm 1.6) \times 10^{-3}$, and $(\Delta g_p/g_p) = (-0.77 \pm 4.2) \times 10^{-3}$. Since the error on z_{diff} is far higher than the other errors, this dominates the errors on the above estimates and results in relatively uninteresting upper limits on changes in the three constants. We emphasize, however, that, to the best of our knowledge, this is the first time that a simultaneous constraint on the variation of these three fundamental parameters has been obtained in a cosmologically distant object. Further, the weakness of the constraint arises from the relatively small difference between the frequencies of the main lines; this is clearly not a fundamental limitation but depends entirely on the sensitivity of the observations.

Next, if we *assume* that $y \equiv m_e/m_p$ is constant, Eqs. (9) and (14) yield $[\Delta\alpha/\alpha] = -5.7 \pm 3.0 \times 10^{-6}$. This is more than 3σ deviant from the estimate $[2\Delta\alpha/\alpha] = 1 \pm 0.3 \times 10^{-5}$ of Carilli *et al.* [8]. Since the latter analysis assumed that g_p was constant, the difference between the two estimates implies either that the assumptions that y and/or g_p are constant is unjustified or that systematic velocity offsets do exist between the three species.

Further progress can be made if one of the three parameters, α, g_p, and y, is assumed not to change with time (while retaining the assumption that velocity offsets are not significant). We can then avoid having to use the equation governing the difference between the OH redshifts and can thus obtain a far stronger limit on changes in the remaining two quantities. For example, if we assume (as is often done, e.g., [8]) that g_p remains constant, the HI 21 cm and mm-wave molecular line redshifts imply $[\Delta\alpha/\alpha] = (5 \pm 1.5) \times 10^{-6}$ [8]. Combining Eqs. (9) and (14) then yields $[\Delta y/y] = (7.8 \pm 2.4) \times 10^{-6}$. Similarly, if we assume that $y \equiv m_e/m_p$ is constant, Eqs. (9) and (14) yield $[\Delta\alpha/\alpha] = (-5.7 \pm 3.0) \times 10^{-6}$ and $[\Delta g_p/g_p] = (2.2 \pm 0.67) \times 10^{-6}$. Finally, if α remains unchanged, we obtain $[\Delta y/y] = (4.2 \pm 2.2) \times 10^{-6}$ and $[\Delta g_p/g_p] = (1 \pm 0.3) \times 10^{-5}$. The above limits on $\Delta y/y$ are a factor of $\sim$4 stronger than the best earlier limits on changes in this quantity [6]. It is very interesting that all three cases result in a higher than 3σ significance for the variation of at least one of the parameters. This implies either that one (or more) of these parameters indeed varies with cosmological time or that systematic motions between the three species cause the above uncertainties (which only include measurement errors) to be underestimated.

Finally, we note that, while the OH 1667 and 1665 MHz HCO^+ and HI lines have been detected in four absorbers at intermediate redshifts, two of the absorbers (PKS1413+135 and B2 1504+377) are believed to have velocity offsets between the HI and HCO^+ redshifts [8,21]. The last system, at $z \sim 0.889$ towards PKS1830–21, does not at present have OH data of sufficiently high quality to carry out the above analysis.

In summary, we have demonstrated a new technique to simultaneously measure the evolution of the three fundamental constants α, g_p, and m_e/m_p, using 18 cm OH absorption lines in conjunction with one additional transition (which could be an HCO^+ mm-wave line). At present, only the 1665 and 1667 MHz main OH lines have been discovered at cosmological distances; we have used these line redshifts in conjunction with those of HI 21 cm absorption and millimeter-wave molecular lines to constrain the variation of y, g_p, and α between $z = 0.6846$ and today. We argue that one (or more) of the parameters α, y, and g_p must vary with cosmological time, unless systematic velocity offsets exist between the above three species. The constraints placed on changes in the parameter $y \equiv m_e/m_p$ (assuming that either α or g_p are constant) are a factor of $\sim$4 stronger than earlier limits on variations in this parameter.

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