Interactions between Sheets of Phonons in Liquid ^{4}He

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We have created two sheets of $\sim$1 K phonons in liquid ^{4}He at $\sim$55 mK such that they intersect each other as they move towards a common point. If the two sheets have a small angle between them, they interact strongly and create a hot line in the liquid helium. This line is continuously fed with energy from the two sheets and loses energy by creating high-energy phonons. If the angle between the sheets is larger than $\sim 30°$ they do not interact but pass through each other. These results give direct evidence for the composition of the sheets: they comprise strongly interacting low-energy phonons which occupy a narrow cone in momentum space.

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Phonon pulses have been created in cold liquid ^{4}He over many years since the initial work of Gernsey and Luszczynski in 1971 [1]. The pulses were found to travel at the velocity of sound, and so it was believed that, to first order, the phonons were independent, noninteracting ballistic wave packets. The liquid ^{4}He was sufficiently cold that the ambient thermal phonons could be ignored. However, it was recognized that phonons could spontaneously decay by the three phonon process (3pp) [2,3], which is allowed by the upward curvature of the dispersion curve [4,5]. This picture was challenged recently by Adamenko *et al.* [6,7] who argued that a number of observations, such as the creation of high-energy phonons in the liquid, could be explained if the phonons in the pulse were treated as strongly interacting and were confined to a narrow cone in momentum space.

It was proposed [6,7] that the phonons in a pulse interact by the 3pp [8] which is very rapid and creates a thermal equilibrium with temperature $T \sim 1$ K within a cone in momentum space, in a time of order of 10^{-10} s [9]. The cone is thought to arise because only phonons with a small angle $2\theta_{3pp}$, typically $\theta_{3pp} = 10°$, between the two momentum vectors, interact by the 3pp. This causes the magnitude of the cone half angle to be $\sim\theta_{3pp}$. The creation of high-energy phonons (h phonons) with energy $\epsilon > 10$ K has been measured [10–12], and can be explained by four-phonon interactions between the low-energy phonons (l phonons) within the pulse [6,7,13]. Despite this indirect evidence for the properties of the phonon pulses, there has been no previous direct confirmation of them. In this Letter we present remarkable experimental results which provide some compelling evidence that the phonons are strongly interacting and occupy a narrow cone in momentum space.

In Fig. 1 we show the signals from two heaters which are separated by an angle $\alpha = 6°$. Signals from separate and simultaneous pulses to the two heaters are shown. The dotted line is the sum of the two separate pulses. It can be seen that the fast l-phonon peak, from simultaneous pulses, is twice as high as that of the sum of the two separate pulses. The slower h-phonon signal, from the simultaneous pulses, is also substantially larger than the sum of the separate pulses.

We define the ratio $R = S_{ij}/(S_i + S_j)$, where i and j are the heater labels, which is the fractional increase of the integral of the the double pulse (S_{ij}) over the sum of the integrals of the two separate pulses (S_i and S_j) (see Fig. 1) during the same time interval. The ratio is equal to 1 if there is no interaction. This ratio is shown as a function of heater power in Fig. 2. We see that for the l phonons $R \sim 2$ and for the h phonons $R \sim 1.3$, but is substantially larger at lower power ($R = 1.6$) than at high power ($R = 0.9$). We see also that the ratio for the faster h phonons is higher than for the slower ones. The results for 50 ns pulses are broadly similar to those for 100 ns.

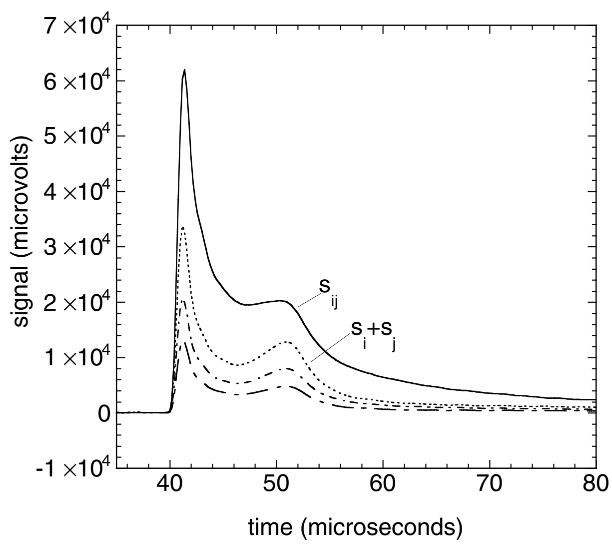

FIG. 1. This shows the bolometer signals for the two separate pulses (dash-dotted lines) and for the double pulse (solid line). The sharp peak at 41 μs is due to the l phonons; the signal after $\approx$44 μs is due to the h phonons. The sum of the signals from the two separate pulses is shown as the dotted line; it can be seen to be about half that of the double pulse. The input pulses are 100 ns and 6.3 mW.

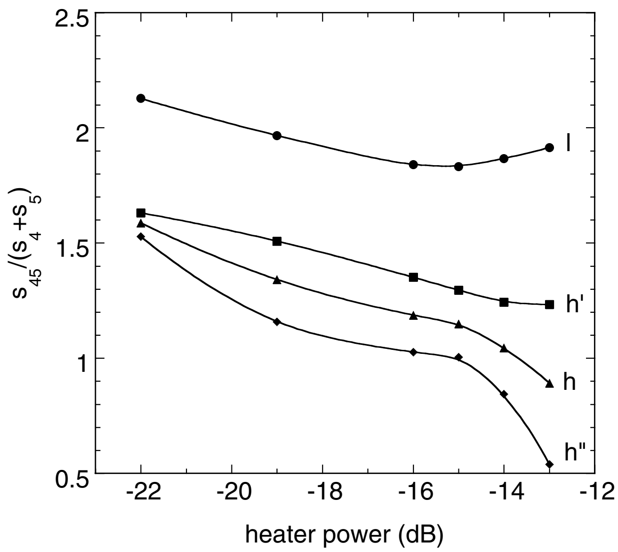

FIG. 2. For heaters with a 6° angle between them, the ratio of the integrated signals from the double pulse to the sum of the integrated signals from the separate pulses is shown as a function of heater power (the reference power is 0.5 W, so -22, -19, -16, and -13 dB correspond to 3.1, 6.3, 12.5, and 25 mW, respectively). The curves labeled l, h′, h, and h″ are integrated over time intervals 40–43, 43–50.4, 43–73, and 50.4–73 μs, respectively. The time 50.4 μs is the propagation time for the fastest h phonon which has energy $\epsilon = 10$ K. The pulse durations are 100 ns.

The results suggest the following questions: (a) where does the extra detected energy come from and (b) why does the extra h-phonon signal arrive earlier than for single pulses? As the double-pulse energy is equal to the sum of the energies in the two separate pulses, the increased detected signal with the double-pulse must be due to a concentration of the injected energy onto the bolometer. We shall now describe how this occurs.

A short heater pulse, $t_p = 100$ ns, creates a sheet of l phonons. The thickness of the sheet is ct_p where c is the velocity of sound; $ct_p = 238 \times 10^{-7} = 24$ μm. At a distance from the heater, the width of the sheet is somewhat larger than the heater width which is 1 mm×1 mm. We have taken 2 mm as a typical width [14]. Figure 3 illustrates the essential elements of the phonon sheets. Figure 3(a) shows a schematic picture of the phonon sheet created by a single heater. The momentum of the phonons in the sheet are in a cone with half angle $\sim\theta_{3pp}$, as shown in Fig. 3(b). The phonons have a temperature which falls in the range $\sim$1.0 to $\sim$0.7 K: as they propagate they cool due to the creation of h phonons [7] and also due to transverse expansion of the sheet [14,15].

Our experiment probes one l-phonon sheet with another similar sheet. The sheets are at a small angle α to each other as shown in Fig. 3(c). The effect of the interaction between the sheets is remarkable. Along the common line of the two sheets a hot line of phonons is created. It will be noticed that the sheets do not extend beyond this hot line because of the strong interactions along the line of contact of the two sheets. This process concentrates the energy in the sheets as they move forward. In the absence of interactions, each sheet would extend behind the other sheet, as shown in Fig. 3(e). The two cones in momentum space overlap, as shown in Fig. 3(d). The phonons in the overlap volume rapidly thermalize within a time of order 10^{-10} s. This creates a new momentum cone, with angle $\sim\theta_{3pp} + \alpha/2$, associated with the hot line, with its axis along the bisecting line of the cone axes from the two sheets.

In Fig. 3(e), we show two sheets with a large angle between them. Then there is no interaction and the two sheets just pass through each other. The two momentum

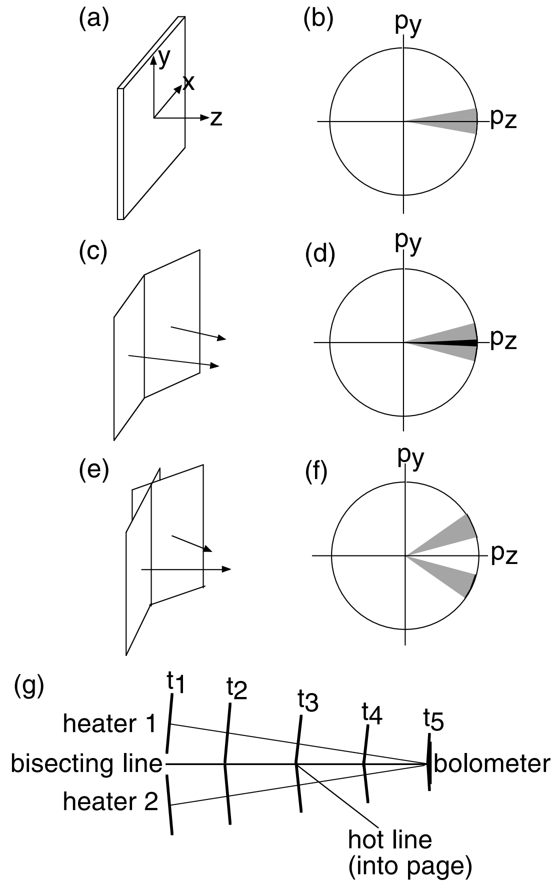

FIG. 3. (a) A schematic of the phonon sheet moving in the z direction. (b) The cone in momentum space occupied by the phonons in the sheet. (c) Two phonon sheets at a small angle to each other; the normals to the two sheets are indicated. (d) The overlap of the two cones in momentum space is shown. (e) Two phonon sheets with a large angle between them. Note that the sheets pass through each other. (f) The two occupied cones separated. (g) A plan view of the two phonon sheets at different times. The sheets are created at time t_1, and at t_2 they start to touch and begin to create the hot line. For $t > t_2$ the area of the sheets decrease and their energy is fed into the hot line. The h phonons, created by the hot line, trail behind the two phonon sheets, but are not shown.

cones, shown in Fig. 3(f), are well separated. The narrow angle of the cone in momentum space will be confirmed by showing that the two sheets do not interact if the angle between the sheets is large enough.

For the experiment, we arranged a set of heaters on an arc of a circle, radius 10 mm, and a detector at the center of the circle. The heaters are 1 mm$\times$1 mm gold films on glass cover slips. Their resistance is $\sim$50 Ω. They can be pulsed individually or in pairs. The pulses are 50 or 100 ns duration with powers in the range 3.1 mW (-22 dB) to 25 mW (-13 dB). The superconducting zinc film detector is held at a constant temperature by an electronic feedback signal which self-heats the zinc and indicates the phonon energy flux [16–18]. The dominant time constant of the detector system is $\approx 1.5\ \mu$s. Many pulses are averaged together to increase the signal to noise ratio using a Tektronix DSA 601A. The helium is isotopically pure [19] and at a temperature $\sim$55 mK.

Figure 3(g) shows a plan view of the spatial positions of the two phonon sheets at a series of times. At time t_1 the sheets are just formed at the heater. At time t_2 the edges of the sheets just touch. This overlap of the sheets starts the creation of a line of higher density phonons, a hot line, along their common line. This is the beginning of the extraordinary process: the interactions between the phonons in the sheets are so strong that the interacting phonons are assimilated into a new population with its own momentum cone, with its axis along the bisecting line. For $t > t_2$ the hot line moves forward along the bisecting line and the sheets get smaller as their energy is fed into the hot line. The hot line does not continuously get hotter as it quickly reaches a temperature where the rate of energy loss due to the creation of h phonons equals the rate of gain of energy from the two sheets. The h phonons are created all the way from the position at t_2 to the bolometer, and those created near the bolometer arrive just after the l phonons. This answers the second question

The higher density of l phonons in the hot line causes the increase in the l-phonon signal. The measurements show that there is as much energy from the hot line as from the area of the two sheets that are separately incident on the bolometer. In separate pulses, the sheets are wider than the bolometer so only a fraction of each phonon sheet is incident on the bolometer. In a double pulse some of this energy that would otherwise miss the bolometer is collected by the hot line. If the phonon sheet is 2 mm$\times$2 mm, then the energy in an extra 0.5 mm^2 area of each sheet falls on the bolometer which has an area of 1 mm^2. This gives a 50% increase in signal. This fraction is almost independent of the angle α between the sheets so long as $\alpha < 2\theta_{3pp}$ and the bolometer is at the center of curvature. This simple geometric argument leaves out the effects of the expansion of the phonon sheet and the hot line. The transverse expansion of the phonon sheet means the separate l-phonon signals are smaller but the hot line still collects the same fraction of the sheets so the ratio $S_{ij}/(S_i + S_j)$ will be larger. The estimate thus agrees with the measurements.

An important feature of our picture is that the l phonons in the sheet are in a narrow cone in momentum space. This can be tested by varying the angle between the two sheets as there should be no interactions between them when the two cones in momentum space are well separated. We define θ_{3pp}, in the 3pp scattering $l_1 \leftrightarrow l_2 + l_3$, as the angle between l_2 and l_1 when l_2 and l_3 have the same energy $\epsilon_1/2$. For a phonon distribution at temperature T, 0.8 of the phonons have energy $\epsilon < 4T$; therefore we use this energy to evaluate θ_{3pp}. For example, using the measurements of the upward dispersion [20], when $T =$ 1 K, $\epsilon_1 = 4$ K and $\theta_{3pp} = 9.7°$.

In reality, the cones do not have a sharp boundary because there are 3pp interactions where $\epsilon_2 \gg \epsilon_3$ which have a larger angle between them. In the example above, the maximum included angle between l_2 and l_3 is 20°, but the probability of this process is much lower than for $\epsilon_2 = \epsilon_3$. These angles scale roughly with T for $T < 1$ K. The phonons in the high-energy tail of the distribution can have somewhat larger angles; the maximum $\theta_{3pp} =$ 12° and the maximum included angle is 24°.

Significant interaction between the sheets requires that $\alpha < 2\theta_{3pp}$. This is confirmed by the two heaters with $\alpha =$ 46° where we find no interaction at all. This is true at the highest power (25 mW), where any deviation from $S_{ij}/(S_i + S_j) = 1$ would be most noticeable: the signals from double pulses are equal to the sum of the signals from two separate pulses.

When the angle between the two heaters is $\alpha = 26°$, the situation is marginal. With low power pulses, 100 μs, 6.3 mW (-19 dB), there is little interaction, but for higher powers, 12.5 ($-$ 16 dB) and 25 mW (-13 dB), there is an increasing interaction shown by the ratio $R = S_{ij}/(S_i + S_j) > 1$. We show this in Fig. 4 which is in stark contrast to Fig. 2. Higher powers increase the temperature of the l phonons. This causes a small increase in θ_{3pp}, which is enough to start the interaction. When $\alpha =$ 26°, only the value of R for the l phonons shows an increase; the h phonons have $R \sim 1$ and even $R < 1$. As α increases, the two phonon sheets overlap nearer the bolometer. The l-phonon density along the hot line reaches its increased dynamic equilibrium value essentially instantaneously, on the time scale of the measurement. Thus, the increase in the l-phonon density is not very sensitive to the position of the initial contact.

This is in contrast to the h-phonon signal from the hot line. Because the h phonons are created over the flight path of the hot line, their signal increases with the length of the flight path, and so decreases with α. Also the h phonons created near the bolometer do not have time to disperse from the l phonons and so can disappear within the l-phonon signal. Both these effects reduce the h-phonon signal as α increases. The ratio $S_{ij}/(S_i + S_j)$

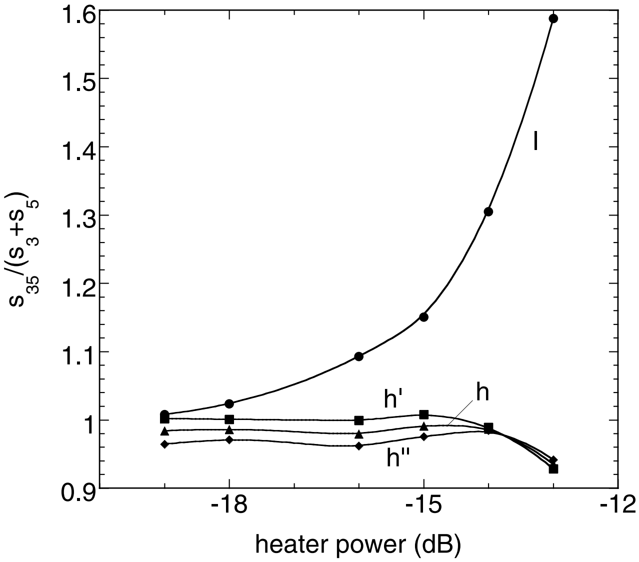

FIG. 4. For heaters with an angle α between them of 26°, the ratio R of the integrated signals from the double pulse to the sum of the integrated signals from the separate pulses is shown as a function of heater power (the reference power is 0.5 W, so -22, -19, -16, and -13 dB correspond to 3.1, 6.3, 12.5, and 25 mW, respectively). The curves labeled l, h′, h, h″ are integrated over time intervals 40–43, 43–50.4, 43–73, and 50.4–73 μs, respectively. The pulse durations are 100 ns.

for the h phonons can be less than 1 due to the h phonons being scattered out of the beam.

In conclusion, we have shown that l-phonon sheets interact when the angle between them is small, but not when it is large. This is direct evidence that the phonons in the sheet occupy a narrow cone in momentum space with a cone angle, typically $\sim 10°$. When the angle between the sheets is small, the interaction between the sheets is strong and leads to a hot line in the liquid helium. This line moves at the velocity $c = 238\ \mathrm{m\,s^{-1}}$ along the bisecting line, between the two normals to the heaters. The hot line is continuously fed with energy from the decreasing area of the sheets, and it loses energy by creating h phonons. The hot line will be in dynamic equilibrium and at an essentially constant temperature over its path. The creation of the hot line collects energy from those parts of the sheets which would otherwise miss the bolometer. This leads to higher l- and h-phonon signals than the sum of the two separate signals.

We see this behavior when the angle between the sheets is 6°. When the angle between the sheets is 26°, there are interactions when the pulse power is high, but not when it is low. In this marginal case, the slightly larger width of the cone at higher powers can initiate interactions which lead to increasingly stronger interactions. When the angle is 46° we find that there are no interactions.

This striking behavior can be understood if the phonons in the sheets are both in a narrow momentum cone and are strongly interacting. This is the most direct evidence to date for the properties of the phonon sheet that has been postulated and used to explain several diverse experimental results.

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