

## Phase Control of Collective Quantum Dynamics

Mihai Macovei,\* Jörg Evers, and Christoph H. Keitel

*Theoretische Quantendynamik, Physikalisches Institut, Universität Freiburg, H.-Herder-Strasse 3, 79104 Freiburg, Germany*  
(Received 3 June 2003; published 1 December 2003)

The manipulation of the steady-state behavior of a collection of dipole-interacting three-level atoms in a  $V$  or  $\Lambda$  configuration is investigated as a function of the relative phase of two strong coherent driving fields. For larger samples, the phase is shown to be a convenient parameter to rapidly populate or depopulate completely a trapping state of the ensemble. As applications, we present the appropriately prepared atomic sample as an optical switching device and show its virtues in controlling the collective steady-state resonance fluorescence intensity.

DOI: 10.1103/PhysRevLett.91.233601

PACS numbers: 42.50.Fx, 42.50.Lc

Ever since the classic work by Dicke [1], the concept of collectivity for dipole-interacting atoms confined in a region of the order of a typical optical wavelength has received considerable attention [2–12]. Dicke showed that such a sample of  $N$  excited atoms forms a collective dipole moment, which leads to effects such as a fluorescence intensity proportional to  $N^2$  and a quantum dynamics  $N$  times faster than for a single atom. These effects have been verified experimentally [8]. Controlled by coherent sources of light, collections of atomic few-level systems may exhibit interesting steady-state characteristics like jumps and discontinuous behaviors [9–11]. A recent extension to coherent controlling schemes used in quantum optics is based on the relative phase between driving laser fields [13–17]. This typically involves quantum interference between multiple atomic transitions pathways which is known to be a source for many interesting applications [18]. In [13], the phase dependence of the resonance fluorescence spectrum in a single three-level  $\Lambda$ -type atom was investigated. Spectral narrowing and fluorescence quenching in an atomic four-level system was reported in [14]. Also light amplification without inversion may be controlled by the relative phase [15]. The phase control was also shown to be useful in manipulating the single-atom spontaneous emission in different surroundings as in photonic crystals [16].

In this Letter, we put forward a coherent phase-control scheme for a collection of atoms. To this end, we investigate the dependence of the steady-state properties of a sample of atomic three-level systems on the relative phase between the two applied strong driving laser fields. Because of the general approach, our analysis applies both to atoms in  $V$  and in  $\Lambda$  configurations as shown in Fig. 1. By appropriately choosing the relative phase, all atoms may be trapped in the upper atomic state doublet of the three-level systems in the  $V$  configuration, and in the lower doublet for the  $\Lambda$  system. In these trapping states, the total collective fluorescence light is completely suppressed. For different values of the relative phase, these doublet states may also be partially depleted. Increasing the number of atoms involved leads to a more rapid trans-

fer of the atoms into the trapping states or vice versa. This property may be used to build fast optical switching devices conveniently controlled by the relative phase of the two laser fields. As an additional weak probe beam would experience absorption or gain depending on the steady state of the collection of atoms, this fast switching between the ground and the excited states might be used to construct a quantum optical transistor.

To allow for a description of the system using collective operators, we assume the atomic sample to be confined in a region which is small as compared to the wavelengths  $\lambda$  of the atomic transitions. Collisions among the radiators are avoided by considering moderate atomic densities  $d$ , such that  $d\lambda^3 \geq 1$ . The dipole-allowed transitions between the states  $|2\rangle \leftrightarrow |1\rangle$  and  $|3\rangle \leftrightarrow |1\rangle$  are driven at resonance with Rabi frequencies  $2\Omega_2$  and  $2\Omega_3$  by two strong coherent fields with phases  $\phi_2$  and  $\phi_3$ , respectively. In the  $V$  system, the degenerate upper states  $|2\rangle$  and  $|3\rangle$  decay to the ground state with decay rates  $2\gamma_2$  and  $2\gamma_3$ , while two incoherent fields  $2r_{p_2}$  and  $2r_{p_3}$  are used to transfer population from the ground state to both upper states. In the  $\Lambda$  system, the upper state  $|1\rangle$  decays to the degenerate ground states  $|2\rangle$  and  $|3\rangle$  with rates  $2\gamma_2$  and  $2\gamma_3$  and is repumped by two incoherent fields  $2r_{p_2}$  and  $2r_{p_3}$ .

In order to allow for a dependence of the system on the relative phase of the driving fields, spontaneously generated coherence (SGC) or cross-coupling terms are included in the analysis. These terms may be interpreted as arising from virtual collective emissions of photons on

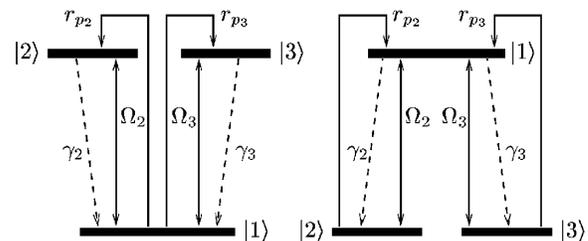


FIG. 1. Bare-state notation of the three-level systems.

one of the transitions followed by a virtual collective absorption of the same photons on the other transition. The notation virtual here refers to the fact that photons involved in these processes cannot be detected. In free space, this effect is strongly dependent on the mutual orientation of the dipole moments  $\vec{d}_{21}$  and  $\vec{d}_{31}$  of the two transitions. It is absent for  $\vec{d}_{21} \perp \vec{d}_{31}$  and is maximal for  $\vec{d}_{21} \parallel \vec{d}_{31}$ . It should be noted that the condition  $\vec{d}_{21} \parallel \vec{d}_{31}$  is rarely met in atomic systems. In order to overcome this difficulty, the atoms may be thought of as interacting with the modified vacuum of a preselected cavity mode in the bad cavity limit. As shown in [19], by proper engineering of the cavity, one may obtain SGC even for initially orthogonal dipole moments. However, our analysis does not depend explicitly on the cavity, such that different schemes which provide nonzero values for SGC may be equally applied. In the usual Born-Marcov, mean-field, dipole, and rotating-wave approximations, for the  $V$  system we obtain the following first order differential equation for the density operator  $\rho$ :

$$\begin{aligned} \dot{\rho} = & - \sum_{\alpha \in \{2,3\}} (i[\Omega_{\alpha} S_{\alpha 1} e^{i\phi_{\alpha}}, \rho] + \Gamma_{\alpha} [S_{\alpha 1}, S_{1\alpha} \rho] \\ & + r_{p_{\alpha}} [S_{1\alpha}, S_{\alpha 1} \rho]) \\ & - \sum_{\alpha \neq \beta \in \{2,3\}} \Gamma_{\alpha\beta} [S_{\alpha 1}, S_{1\beta} \rho] + \text{H.c.} \end{aligned} \quad (1)$$

Here  $\Gamma_{\alpha} = g_{\alpha}^2 / (k - i\Delta_c)$ ,  $g_{\alpha}$  is the coupling between the cavity mode and the atomic transition  $|\alpha\rangle \leftrightarrow |1\rangle$  ( $\alpha \in \{2, 3\}$ ),  $2k$  the rate at which the cavity is losing photons,  $\Delta_c = \omega_c - \omega_a$  the cavity detuning,  $\omega_c$  the cavity mode frequency, and  $\omega_a := \omega_{21} = \omega_{31}$  indicate the atomic transition frequencies.  $\Gamma_{\alpha\beta} = \eta \sqrt{\Gamma_{\alpha} \Gamma_{\beta}}$  is the SGC contribution with  $\eta$  as a control parameter which is unity for maximal SGC contributions and zero if these cross-coupling terms are absent. We define the collective state  $|\alpha\rangle := \sum_{i=1}^N |\alpha\rangle_i$  ( $\alpha \in \{1, 2, 3\}$ ) as the sum over the corresponding single atom states indicated by the subindex  $i$ . Accordingly,  $S_{\alpha\beta} \equiv |\alpha\rangle\langle\beta| := \sum_{i=1}^N |\alpha\rangle_i \langle\beta|_i$  are collective atomic operators given by sums over single atom operators  $|\alpha\rangle_i \langle\beta|_i$  and obey the commutation relation  $[S_{\alpha\beta}, S_{\beta'\alpha'}] = \delta_{\beta\beta'} S_{\alpha\alpha'} - \delta_{\alpha\alpha'} S_{\beta'\beta}$  for  $\{\alpha, \alpha', \beta, \beta'\} \in \{1, 2, 3\}$  as well as the condition  $S_{11} + S_{22} + S_{33} = N$ , where  $N$  is the number of atoms in the sample. In Eq. (1), the first term describes the coupling to the coherent fields. The term proportional to the real part  $\gamma_{\alpha} = \text{Re}(\Gamma_{\alpha})$  of  $\Gamma_{\alpha}$  ( $\alpha \in \{2, 3\}$ ) is due to collective spontaneous emissions in the cavity mode, while the imaginary part is associated with the frequency shift of the atomic levels resulting from the interaction with the vacuum field in the detuned cavity. The incoherent pumping of the excited levels is described by the term proportional to  $r_{p_{\alpha}}$ . The last term of Eq. (1) accounts for the SGC contributions. The corresponding master equation describing a sample of  $\Lambda$ -type atomic systems may be obtained from Eq. (1) by swapping the two indices of each transition operator having  $\alpha$  or  $\beta$  as one of the indices, e.g.,  $S_{\alpha 1} \leftrightarrow S_{1\alpha}$ .

The parameter  $\eta$  also allows one to account for possible external perturbations such as collisions, because these will reduce the SGC contribution. However, as was already shown for particular decoherence sources in collective systems [20], moderate disturbances may be compensated by increasing the number of atoms. In particular, while  $\eta$  need to be above zero, it may be less than unity in our scheme.

In the strong-field limit  $\Omega_{\alpha} \gg N\gamma_{\alpha}, Nr_{p_{\alpha}}$  ( $\alpha \in \{2, 3\}$ ) of interest here, we transfer into the dressed state picture

$$\begin{aligned} |\Psi_1\rangle &= \frac{\Omega_3}{\Omega} |2\rangle - \frac{\Omega_2}{\Omega} |3\rangle, \\ |\Psi_2\rangle &= \frac{1}{\sqrt{2}} \left\{ |1\rangle + \frac{\Omega_2}{\Omega} |2\rangle + \frac{\Omega_3}{\Omega} |3\rangle \right\}, \\ |\Psi_3\rangle &= \frac{1}{\sqrt{2}} \left\{ |1\rangle - \frac{\Omega_2}{\Omega} |2\rangle - \frac{\Omega_3}{\Omega} |3\rangle \right\}, \end{aligned} \quad (2)$$

where  $\Omega = \sqrt{\Omega_2^2 + \Omega_3^2}$ , and we employed also  $\tilde{S}_{\alpha 1} = S_{\alpha 1} e^{i\phi_{\alpha}}$  [ $\tilde{S}_{1\alpha} = S_{1\alpha} e^{i\phi_{\alpha}}$ ] in using Eq. (1) for  $V$ - [ $\Lambda$ -]type atoms and dropped the tilde afterwards. Taking advantage of Eqs. (2) in Eq. (1) and neglecting terms that oscillate with frequencies  $\Omega$  and larger in a secular approximation, one may obtain the following exact steady-state solution (subindex  $s$ ) of the dressed master equation for both the  $V$ - and the  $\Lambda$ -type systems:

$$\rho_s = Z^{-1} \sum_{r=0}^N X_{V(\Lambda)}^r \sum_{m=0}^r |r, m\rangle\langle m, r|. \quad (3)$$

Here  $|r, m\rangle$  are eigenstates of the operators  $R = R_{22} + R_{33}, R_{33}$ , and  $R_{11} + R_{22} + R_{33}$  with eigenvalues  $r, m$ , and  $N$ , respectively.  $R_{\alpha\alpha} = |\Psi_{\alpha}\rangle\langle\Psi_{\alpha}|$  ( $\alpha \in \{1, 2, 3\}$ ) is a collective dressed state population operator and  $Z$  a normalization constant. The coefficients  $X_{V(\Lambda)}$  represent the ratio of dressed decay and pumping rates:

$$\begin{aligned} X_V &= [\gamma_2 + \gamma_3(\Omega_2/\Omega_3)^2 - 2\eta(\Omega_2/\Omega_3)\sqrt{\gamma_2\gamma_3} \\ &\quad \times \cos(\Delta\phi)] / [r_{p_2} + r_{p_3}(\Omega_2/\Omega_3)^2] \end{aligned} \quad (4)$$

for the  $V$  system and  $X_{\Lambda} = X_V^{-1}$  for the  $\Lambda$  scheme.

The dressed steady-state atomic population then yields

$$\langle R_{11} \rangle_s = \frac{NX^{N+2} - (N+2)X^{N+1} + N(X-1) + 2X}{(X-1)[(N+1)X^{N+2} - (N+2)X^{N+1} + 1]}$$

and  $\langle R_{33} \rangle_s = \langle R_{22} \rangle_s = \langle R \rangle_s / 2$  with  $\langle R \rangle_s = N - \langle R_{11} \rangle_s$ . Here we have dropped the index of  $X$  because all discussed populations depend identically on the respective  $X_V$  and  $X_{\Lambda}$  parameters for both systems. The steady-state atomic population in the collective bare state  $|1\rangle$  is given by the relation  $\langle S_{11} \rangle_s = (N - \langle R_{11} \rangle_s) / 2$ .

For simplicity, we restrict the following analysis to the parameter range  $\Omega_2 = \Omega_3, \gamma_2 = \gamma_3 := \gamma_0, r_{p_2} + r_{p_3} := 2r_p$ . The parameter  $X$  may then be written as  $X_V = C\{1 - \eta \cos\Delta\phi\}$  [ $X_{\Lambda} = X_V^{-1}$ ], where  $C = \gamma_0 / r_p$  is the ratio of spontaneous decay and incoherent repumping in the atomic systems and along with  $X_V$  [ $X_{\Lambda}$ ] will turn out

to crucially influence the system behavior. First we investigate the case  $C = 2$ ; i.e., the decay dominates over the repumping. Figure 2(a) shows the steady-state population in the collective dressed state  $|\Psi_1\rangle$  per atom for different numbers of atoms  $N$  versus the relative phase between the two applied strong resonant laser fields for the  $V$ -type system. Figure 3(a) displays the corresponding results for the  $\Lambda$  system. For a single  $V$ -type atom ( $N = 1$ ), the atom may be trapped only in  $|\Psi_1\rangle$  for  $\Delta\phi = n \times 2\pi$  ( $n \in \{0, 1, \dots\}$ ), while for a single  $\Lambda$ -type atom the trapping is not possible with incoherent pumping. However, for a collection of atoms ( $N \gg 1$ ), trapping is possible both for the  $V$  and the  $\Lambda$  systems. With an increasing number of atoms, the range  $\delta(\Delta\phi)$ , for which the collective coherent population trapping effect occurs, grows until in the limit  $N \rightarrow \infty$  the system exhibits jumps between two states with either all or none of the population in the collective state  $|\Psi_1\rangle$ .

This behavior of the steady-state population of the dressed state  $|\Psi_1\rangle$  may be simplified by taking the limit  $N \rightarrow \infty$  of its population per atom  $\langle R_{11} \rangle_s / N$ , which for  $0 \leq X < 1$  is 1, for  $X = 1$  is  $1/3$ , and for  $X > 1$  is 0. Thus the jumps occur at  $X = 1$  or at  $\cos\Delta\phi = (1 - 1/C)/\eta$ . The trapping range is given by  $\delta(\Delta\phi)_V = 2 \arccos[(1 - 1/C)/\eta]$  for the  $V$  system and by  $\delta(\Delta\phi)_\Lambda = 2(\pi - \arccos[(1 - 1/C)/\eta])$  for the  $\Lambda$  system. The discontinuities may be interpreted in terms of phase transitions [11] and are due to the fact that the sample of atoms forms a collective dipole moment which evolves on a time scale  $N$  times faster than the single-atom dipole moment with corresponding changes of the collective dressed steady-state populations [1]. They occur for the  $V[\Lambda]$  system at the points where the dressed spontaneous decay out of [into] state  $|\Psi_1\rangle$  just equal the dressed pumping in the reverse direction.

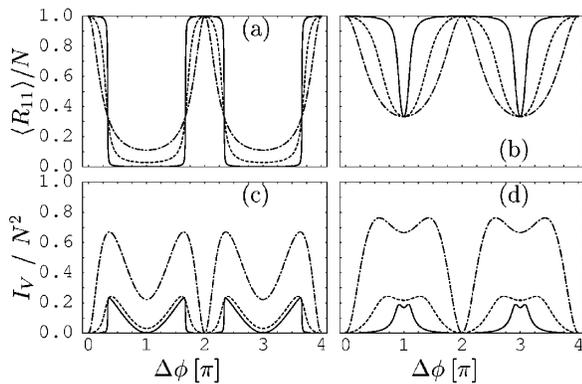


FIG. 2. The dependence of (a),(b) the steady-state population of the collective dressed state  $|\Psi_1\rangle$  and (c),(d) of the total collective steady-state fluorescence intensity as a function of the relative phase  $\Delta\phi := \phi_2 - \phi_3$  for the  $V$  system. The dash-dotted, dashed, and solid curves correspond to  $N = 1$ ,  $N = 10$ , and  $N = 200$ , respectively. Further  $\Omega_2 = \Omega_3$ ,  $\eta = 1$ ,  $\Phi(r) = 1$  and (a),(c)  $C = 2$ , (b),(d)  $C = 0.5$ .

For  $C = 0.5$ , i.e., dominating repumping, the results shown in Figs. 2(b) and 3(b) may be interpreted along the same lines. However, the condition  $X_V \leq 1$  [ $X_\Lambda \geq 1$ ] is always fulfilled. Therefore the collective dressed state  $|\Psi_1\rangle$  is never entirely depleted in the  $V$  system and never fully populated in the  $\Lambda$  case, and no discontinuities are observed. The reason for this is that the repumping of the upper bare states is always strong enough to prevent the population from entirely leaving it.

For the time-dependent dressed population we obtain  $\langle R_{11}(t) \rangle = [a - b \frac{a-R_0}{b-R_0} e^{-|\gamma_m(a-b)t}|] / [1 - \frac{a-R_0}{b-R_0} e^{-|\gamma_m(a-b)t}|]$  for  $N \gg 1$ ,  $X \neq 1$ , with initial condition  $R_0 = \langle R_{11}(0) \rangle$  and  $\gamma_m = \gamma_0(1 - \cos\Delta\phi)(1 - X_V^{-1})$ . If  $X > 1$ , then  $a \rightarrow 0$  and  $b \rightarrow N$ , while for  $X < 1$ ,  $a \rightarrow N$  and  $b \rightarrow 0$ . Thus, due to collective effects, the time required to evolve into steady state for  $X \neq 1$  is proportional to  $N^{-1}$ . For a pencil-shaped sample with length  $10^{-2}$  cm, transversal area  $10^{-6}$  cm<sup>2</sup>,  $\lambda = 10^{-4}$  cm,  $\Omega \sim 10^{11}$  Hz,  $\gamma_0 \sim$  MHz,  $d \sim 10^{12}$  cm<sup>-3</sup>, and phase switching between  $\Delta\phi = 0$  and  $\pi/2$ , switching times of about  $10^{-9}$  s are feasible, including elastic collisions.

To further discuss the system behavior, we now turn to the total steady-state intensity  $I$  of the collective fluorescence light on both transitions  $|2\rangle \leftrightarrow |1\rangle$  and  $|3\rangle \leftrightarrow |1\rangle$  which may be evaluated as  $I = \lim_{t \rightarrow \infty} \langle E^{(-)}(\vec{r}, t) E^{(+)}(\vec{r}, t) \rangle$ , where  $E^{(-)}(\vec{r}, t)$  and  $E^{(+)}(\vec{r}, t)$  represent the positive and negative frequency parts of the amplitude of the electromagnetic field operator  $E(\vec{r}, t)$  and  $\vec{r}$  is the detector position. In the far-zone limit  $r = |\vec{r}| \gg \lambda$  one can express the entire intensity via the collective atomic operators as

$$I_V = \Phi(r) \{ \langle S_{31} S_{13} \rangle + \langle S_{21} S_{12} \rangle + \eta \langle S_{21} S_{13} \rangle e^{-i\Delta\phi} + \text{H.c.} \}$$

for a  $V$ -type atomic sample. For a  $\Lambda$  system the expression for the intensity  $I_\Lambda$  is again obtained by a permutation of the indices in the above expression, e.g.,  $S_{21} \leftrightarrow S_{12}$ .  $\Phi(r)$  is a geometrical factor which we set equal to unity in the following. Employing Eq. (2) and the steady-state solutions Eq. (3) one may obtain the following expression for the collective steady-state intensities

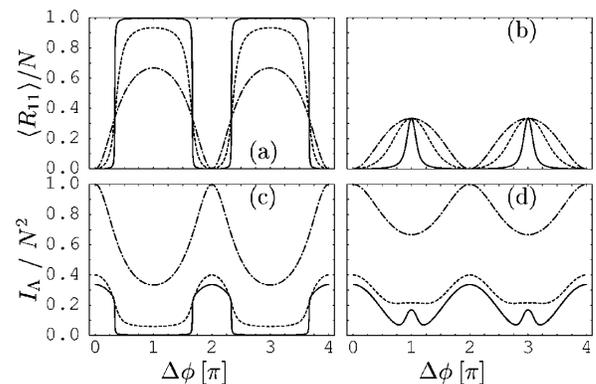


FIG. 3. The same as in Fig. 2 but for the  $\Lambda$  system.

$$[\Omega_{\pm}^2 = \Omega^2 \pm 2\eta\Omega_2\Omega_3 \cos(\Delta\phi)]:$$

$$I_V = \frac{\Omega_-^2}{2\Omega^2}[N(\langle R \rangle_s + 2) - \langle R^2 \rangle_s - 2\langle R \rangle_s] + \frac{\Omega_+^2}{4\Omega^2}\langle R_+ \rangle_s,$$

$$I_{\Lambda} = \frac{\Omega_-^2}{2\Omega^2}[(N+1)\langle R \rangle_s - \langle R^2 \rangle_s] + \frac{\Omega_+^2}{4\Omega^2}\langle R_+ \rangle_s.$$

Here,  $\langle R_+ \rangle_s = 2/3[\langle R^2 \rangle_s + 2\langle R \rangle_s]$ , and

$$\langle R^2 \rangle_s = Z^{-1} \sum_{r=0}^N r^2(r+1)X^r.$$

Figures 2(c) and 2(d) depict the collective steady-state fluorescence intensity per emitted atom  $I_V/N^2$  for a  $V$  system with minima  $\Delta\phi = \pi n$  for  $n \in \{0, 1, 2, \dots\}$ . A comparison with Figs. 2(a) and 2(b) shows that the minima occurring for *even*  $n$  are due to the trapping of the population in the collective dressed state  $|\Psi_1\rangle$ , i.e., in the collective upper bare states  $|2\rangle$  and  $|3\rangle$ . In Fig. 2(c), at phase differences corresponding to *odd*  $n$ , the fluorescent intensity also tends to zero for  $N \gg 1$ . However, at these phase differences the population is not trapped in  $|\Psi_1\rangle$  as for even  $n$ , but with equal weights in the two other dressed states. This means that 0.5 of the atomic population is in the ground bare state  $|1\rangle$ , while 0.25 of the population is in each of the upper bare states  $|2\rangle$  and  $|3\rangle$ . As the inhibition of fluorescence does not occur in the single-atom case, these minima may be associated with subradiant states: The photons emitted by the half of atoms in the excited states are absorbed by the other half in the ground state. For smaller values of  $C$  [see Fig. 2(d)], the dark states at *odd*  $n$  do not occur, and even the local minima at the corresponding phase differences eventually disappear with sufficiently intense repumping for  $C < 0.5$ . Here, the radiation intensity scales linearly with  $N$ , similar to an ensemble of independent radiation emitters.

In the  $\Lambda$  system with decay dominating over repumping, the fluorescence may be suppressed for  $\Delta\phi = \pi(2n+1)$  with  $n \in \{0, 1, 2, \dots\}$  as shown in Fig. 3(c) for  $N \gg 1$ . As for even  $n$  in the  $V$  system, this inhibition is due to the trapping of the population in the collective dressed state  $|\Psi_1\rangle$ , though here only for large  $N$ . For  $\Delta\phi = 2\pi n$ , the collective steady-state intensity is maximal and proportional to the squared number of atoms  $N^2$  [ $I_{\Lambda} = N(2+N)/3$ ]. In this situation, 0.5 of the population is in the upper bare state  $|1\rangle$ , and 0.25 in each ground bare state  $|2\rangle$  and  $|3\rangle$ . This indicates further the existence of a superradiant state. For larger pumping [see Fig. 3(d) with  $C = 0.5$ ], there is again no trapping and the maxima at  $\Delta\phi = \pi(2n+1)$  eventually vanish because the intensity is proportional to  $N$  for  $C < 0.5$ . Here, for *even*  $n$ , the  $N$  dependence is quadratic while it is linear for *odd*  $n$ .

We note that the discussed phase control depends crucially on the presence of incoherent pumping as a counterpart to the phase-dependent dressed decay rates. Thus,

without incoherent pumping the corresponding radiation intensities  $I_V = \Omega_+^2[2+N]N/(6\Omega^2)$  with  $\langle S_{11} \rangle_s = N/2$ ,  $\langle S_{22} \rangle_s = N\Omega_2^2/(\sqrt{2}\Omega)^2$  and  $I_{\Lambda} = 0$  with  $\langle S_{11} \rangle_s = 0$ ,  $\langle S_{22} \rangle_s = N(\Omega_3/\Omega)^2$  (we excluded the special initial-condition-dependent case  $\Omega_2 = \Omega_3$ ,  $\gamma_2 = \gamma_3$ ,  $\Delta\phi = 2\pi n$ ) do depend on  $\Delta\phi$  for the  $V$  system though without phase control of the dressed populations. At best there is the well-known decay-independent coherent population transfer mechanism between states  $|2\rangle$  and  $|3\rangle$  via adapting the intensities of the driving fields appropriately.

In conclusion, the interplay of decay and pumping processes in three-level atoms was shown to allow for rapid coherent phase control of collective population dynamics along with the feasibility of transferring those states at will into subradiant or superradiant ensembles.

Financial support is acknowledged by the Alexander-von-Humboldt Foundation for M. M. and by the German Science Foundation (SFB 276) for J. E. and C. H. K.

---

\*Permanent address: Institute of Applied Physics, Academiei strasse 5, MD-2028 Chisinau, Moldova.

- [1] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
- [2] M. Gross and S. Haroche, Phys. Rep. **93**, 301 (1982).
- [3] A. V. Andreev, V. I. Emel'yanov, and Yu. A. Il'inskii, *Cooperative Effects in Optics. Superfluorescence and Phase Transitions* (IOP Publishing, London, 1993).
- [4] S. John and T. Quang, Phys. Rev. Lett. **74**, 3419 (1995).
- [5] C. H. Keitel, M. O. Scully, and G. Süssmann, Phys. Rev. A **45**, 3242 (1992).
- [6] V. Kozlov *et al.*, Phys. Rev. A **60**, 1598 (1999).
- [7] N. A. Enaki and M. A. Macovei, Sov. Phys. JETP **88**, 633 (1999).
- [8] N. Skribanowitz *et al.*, Phys. Rev. Lett. **30**, 309 (1973); C. Greiner *et al.*, Phys. Rev. Lett. **85**, 3793 (2000).
- [9] R. R. Puri, *Mathematical Methods of Quantum Optics* (Springer, Berlin, 2001).
- [10] M. Lewenstein and J. Javanainen, Phys. Rev. Lett. **59**, 1289 (1987); M. S. Kim, F. A. M. de Oliveira, and P. L. Knight, Opt. Commun. **70**, 473 (1989).
- [11] N. N. Bogolubov, Jr. *et al.*, Phys. Lett. **112A**, 323 (1985); A. S. Shumovsky *et al.*, Opt. Commun. **64**, 45 (1987).
- [12] M. D. Lukin *et al.*, Phys. Rev. Lett. **87**, 037901 (2001).
- [13] M. A. G. Martinez *et al.*, Phys. Rev. A **55**, 4483 (1997).
- [14] E. Paspalakis and P. L. Knight, Phys. Rev. Lett. **81**, 293 (1998).
- [15] J.-H. Wu and J.-Y. Gao, Phys. Rev. A **65**, 063807 (2002).
- [16] T. Quang *et al.*, Phys. Rev. Lett. **79**, 5238 (1997).
- [17] S. Menon and G. S. Agarwal, Phys. Rev. A **57**, 4014 (1998).
- [18] Z. Ficek and S. Swain, J. Mod. Opt. **49**, 3 (2002); J. Evers and C. H. Keitel, Phys. Rev. Lett. **89**, 163601 (2002).
- [19] A. K. Patnaik and G. S. Agarwal, Phys. Rev. A **59**, 3015 (1999); P. Zhou and S. Swain, Opt. Commun. **179**, 267 (2000).
- [20] S. John and T. Quang, Phys. Rev. Lett. **78**, 1888 (1997).