Gödel's Universe in a Supertube Shroud

Nadav Drukker,* Bartomeu Fiol,† and Joan Simón‡

Department of Particle Physics, The Weizmann Institute of Science, 2 Herzl Street, Rehovot 76100, Israel

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We demonstrate that certain supersymmetric Gödel-like universe solutions of supergravity are not solutions of string theory. This is achieved by realizing that supertubes are Bogomol'nyi-Prasad-Sommerfeld states in these spaces, and under certain conditions, when wrapping closed timelike curves, some world-volume modes develop negative kinetic terms. Since these universes are homogeneous, this instability takes place everywhere in space-time. We also construct a family of supergravity solutions which locally look like the Gödel universe inside a domain wall made out of supertubes, but have very different asymptotic structure. One can adjust the volume inside the domain wall so there will be no closed timelike curves, and then those spaces seem like perfectly good string backgrounds.

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Introduction.—Einstein's equations in general relativity have many solutions that seem unphysical. Some have curvature singularities, which generically lie beyond the regime of validity of the classical theory. Other solutions may have no singularities, but violate causality, by having closed timelike curves. Hawking [1] has presented arguments to support the chronology protection conjecture, namely that, when quantum mechanics is taken into account, backgrounds not having closed timelike curves do not develop them. Even if this is true, it seems to leave open the question of whether backgrounds having closed timelike curves to begin with have to be discarded or not.

The answer to such questions may not be found entirely within classical general relativity. One would like to know that the energy momentum tensor, serving as the source in Einstein's equations, corresponds to reasonable physical matter. Another direction to address the problem is to ask whether there is a way to make sense of physics on spaces with closed timelike curves in classical mechanics, quantum mechanics, or quantum field theory.

Probably best of all is to ask those questions in string theory, which encompasses both general relativity and quantum physics. Just as string theory has managed to resolve certain curvature singularities, mainly timelike, by the addition of extra degrees of freedom, it is natural to wonder about the status of closed timelike curves in such a quantum theory of gravity. Are such backgrounds allowed in string theory? Does string theory realize the chronology protection conjecture? Some recent work in this direction is described in Refs. [2–7].

In string theory, the existence of dynamical extended objects provides us with probes suited particularly well to study nonlocal issues like closed timelike curves. Instead of trying to quantize the string world-sheet theory in one of these backgrounds, finding its spectrum and adding interactions, our strategy will be to look for Bogomol'nyi-Prasad-Sommerfeld (BPS) states corresponding to extended branes in this vacuum and to analyze the dynamics of small fluctuations around them. For a specific subset of backgrounds having closed timelike curves we will find BPS states with a sickness in their effective action, signaling the invalidity of the solution.

Let us be more precise. There are many supergravity backgrounds having closed timelike curves. We will be interested in Gödel-like universes, which were first embedded in five-dimensional supergravity [8]. Some other examples of similar metrics were found in [2,9]. All these solutions are generalizations of a four-dimensional metric studied by Som and Raychaudhuri [10], and are very similar to the original Gödel universe [11].

All these solutions describe rotating spaces, where one can choose the planes of rotation as well as some fluxes that make the solution supersymmetric. We will concentrate on a specific solution of type IIA supergravity with rotation in a single plane, since this is the simplest example of such a metric with closed timelike curves.

This solution [9] has a nontrivial metric in three dimensions, which we parametrize by the time coordinate t, and polar coordinates r and ϕ in the plane. There is a Neveu-Schwarz–Neveu-Schwarz (NS-NS) flux as well as Ramond-Ramond (RR) two-form and four-form fluxes, and we label the extra direction in which there is flux by y. The metric and fluxes are

$$\begin{aligned} ds^2 &= -(dt + cr^2 d\phi)^2 + dr^2 + r^2 d\phi^2 + dy^2 + \sum_{i=4}^{9}(dx^i)^2, \\ H_3 &= -2cr dr \wedge d\phi \wedge dy, \qquad F_2 = -2cr dr \wedge d\phi, \\ F_4 &= 2cr dt \wedge dr \wedge d\phi \wedge dy. \end{aligned} \tag{1}$$

This one parameter solution preserves eight supercharges, and we shall assume positive c without loss of generality. If we consider motion in the periodic ϕ direction at constant r, the corresponding space-time curve becomes timelike for $r > 1/c$, hence the closed timelike curves.

Though this seems like a perfectly valid solution to the equations of supergravity, we will demonstrate that, as it stands, it cannot be a valid solution of string theory. One

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way of realizing this fact is to notice that supertubes [12,13] can exist in the above background. Supertubes are bound states of D0-branes and fundamental strings having nonvanishing D2-dipole moment. They can be realized on the world volume of a cylindrical D2-brane with electric and magnetic fields turned on, such that they induce a nonvanishing angular momentum that balances the tension which would naturally tend to collapse the tube. In our discussion, supertubes extend in the y direction and wrap the angular direction ϕ at fixed radius r. It is not too surprising that this is the most interesting object to probe the above geometry with, since it has the correct rotational symmetry, and couples to all the background fields that were turned on. As will also be made clear below, this object preserves the same supersymmetries as the Gödel-like universe itself. Even though the tension of this supertube is just given by the sum of the D0-brane and fundamental string charges, and it can have an arbitrary size (by adjusting these charges), we will find that for a certain range of charges, some world-volume modes develop a negative kinetic term whenever the radius satisfies $r \geq 1/c$, so that the supertube wraps a closed timelike curve. This sickness is somewhat like that found by considering a probe brane in the repulson background, which led to the enhançon mechanism [14]. Since the dynamics of D-branes captures space-time itself, we interpret this as an inconsistency of the Gödel solution.

The plan of the rest of the Letter is the following. In the next section we shall briefly discuss the probe calculation of a supertube in our Gödel universe, and show that the world-volume theory may become sick at the radius where closed timelike curves appear. Then we describe a family of supergravity solutions that locally look like the Gödel universe, but have a domain wall made out of smeared supertubes separating them from a space that does not have closed timelike curves asymptotically. The domain wall in these solutions can be thought of as a regulator, the initial solution (1) arising in the limit where the domain wall is sent to infinity. One might think it possible to enlarge the region inside the domain wall to get closed timelike curves, but the same problem we encountered in the full Gödel universe prevents this from happening.

We conclude with some discussion of the analogy to the enhançon mechanism and how similar problems show up for other Gödel universes of supergravity, invalidating them too.

Supertubes in a Gödel-like universe.—As mentioned in the introduction, the type IIA Som-Raychaudhuri solution (1) has closed timelike curves. Thus, one may suspect that an extended probe would develop some sickness in this background. Our first goal is then to discuss the possible probes in this background. This task is simplified by the realization that the solution (1) is a particular case of a family of IIA solutions discussed in [13,15]. The authors of [13] prove that in general these solutions admit D0-branes, fundamental strings, and supertubes as probes that do not break any further supersymmetry. Even better, they perform a probe analysis of these solutions, and show that when the solution presents closed timelike curves, the world-volume theory on the supertube is ill defined. Our computation is just a special case of theirs.

The supertube is a cylindrical D2-brane which is extended in the y direction as well as the angular direction ϕ at fixed radius r about the origin. [The metric (1) is actually homogeneous. The addition of the supertube breaks translational invariance, but preserves the rotation around the origin.] The world-volume theory on the supertube is just that of a D2-brane in a curved background, which includes the Dirac-Born-Infeld and Wess-Zumino terms

$$S = -\int e^{-\Phi}\sqrt{-\det(\mathcal{G} + \mathcal{F})} - \int (C_3 + C_1 \wedge \mathcal{F}), \quad (2)$$

where $\mathcal{G}$ is the pullback of the metric and C_1 and C_3 the pullbacks of the RR potentials. The field strength $\mathcal{F}$ includes the gauge fields that are turned on in the brane and those induced by the NS field,

$$\mathcal{F} = F - B_2 = E dt \wedge dy + \mathcal{B} dy \wedge d\phi. \quad (3)$$

E is the electric field and $\mathcal{B} = B - cr^2$ the magnetic field, including the term induced by the background.

The equations of motion are solved by $E = 1$ (which is not critical in the presence of magnetic field) and constant $B > 0$, whereas the radius of the tube is fixed to be the angular momentum J, related to the D0 and fundamental string charge densities $q_0 = B$ and q_s by $r^2 = J = q_0 q_s$. (The positivity of the magnetic field is enforced by supersymmetry.) It is a simple exercise to show that the supertube preserves the same supersymmetries as the background, which is also clear from the analysis of the next section, where the metric (1) is shown to belong to the family of metrics studied in [13].

After adjusting the electric and magnetic fields as above, the Lagrangian density of the D-brane probe is simply the magnetic field $\mathcal{L} = -B$. The supertube tension is just given by the sum of charges it carries, that of D0-branes and fundamental strings, as dictated by supersymmetry. Therefore we can seemingly make the tube as small or as large as we wish.

To verify that, we can start with a very small supertube and slowly increase the radius. There are a number of ways one could envision this happening. For instance, one could imagine a bath of D0-branes and fundamental strings surrounding the supertube, and allowing the supertube to change its radius by exchanging charges with the bath, satisfying the BPS condition at all times. Perhaps more naturally, one can also consider fluctuations of the radius, in time and/or in the y direction, which do not preserve the BPS condition. In this latter case, a nonzero potential would also appear.

This calculation was done in [13] for the background generated by a collection of supertubes, and is trivial to adapt it to our metric. The result is that the expansion of the Lagrangian at small velocities is

$$\mathcal{L} = \mathcal{L}_0 + \tfrac{1}{2}M\dot{r}^2 + \ldots, \qquad (4)$$

where the dots can contain a potential term if the fluctuation does not preserve the BPS condition. $\mathcal{L}_0$ is the static Lagrangian density and

$$M = \frac{(B - cr^2)^2 + r^2(1 - c^2r^2)}{B} \qquad (5)$$

is the mass of the excited mode. This is positive definite for $r < 1/c$, but for larger radii, it becomes negative for a certain range of charges.

How do we interpret this singularity? Usually when we encounter that the metric in moduli space becomes negative we interpret it as meaning that the effective Lagrangian is not valid, because it is missing some light degrees of freedom. One then corrects the effective description by adding extra fields. An example that has some similarities to our case is the enhançon mechanism [14], where the tension of the probe vanishes. This is because the D-brane gets delocalized, effectively forming a domain wall. In our case, the tension remains finite, but still the kinetic term for small fluctuations vanishes signaling that something is missing from the action. It is not exactly clear if and how we should add extra degrees of freedom to the case at hand. Recall that the Gödel universe (1) is homogeneous, so this sickness can appear anywhere in space. What we can safely conclude is that the solution (1) is not a good string theory background.

A domain-wall solution.—In the previous section we argued that the Gödel-type solution (1) cannot be a correct effective description of a string theory background. Nevertheless, it is fairly easy to display supergravity solutions that locally are similar to Gödel, but are free of closed timelike curves. To do so, we start by recalling the family of IIA solutions considered in [13,15],

$$ds^2 = -U^{-1}V^{-1/2}(dt - A)^2 + U^{-1}V^{1/2}dy^2 + V^{1/2}\sum_{i=2}^{9}(dx^i)^2, \qquad B_2 = -U^{-1}(dt - A) \wedge dy + dt \wedge dy,$$
$$C_1 = -V^{-1}(dt - A) + dt, \qquad C_3 = -U^{-1}dt \wedge dy \wedge A, \qquad e^{\Phi} = U^{-1/2}V^{3/4}. \qquad (6)$$

Here U and V are harmonic functions in the eight dimensions spanned by x^i, and A is a Maxwell field. Clearly the IIA Gödel-like solution (1) is recovered if we take $U = V = 1$ and $A = -cr^2d\phi$ (where r and ϕ are polar coordinates in the x^2, x^3 plane).

A very natural question is whether there are more general configurations which are Gödel near the origin, but are asymptotically different, with no closed timelike curves. (The idea of patching another metric outside a finite radius, to construct a solution free of closed timelike curves, was considered for the original Gödel metric in [16].) If we want to retain translation symmetry in $\mathbb{R}^6$ and rotational symmetry in ϕ, any source must be smeared in those directions, so U and V are harmonic functions on the plane with rotational symmetry.

We construct the solution by taking $U = V = 1$ and $A = -cr^2d\phi$ near the origin. At a radius R we put the smeared supertube, and for $r > R$ we choose

$$U = 1 + \frac{Q_s}{2\pi}\ln\frac{r}{R}, \qquad V = 1 + \frac{Q_0}{2\pi}\ln\frac{r}{R}, \qquad A = -cR^2d\phi. \qquad (7)$$

Q_s and Q_0 are, respectively, the fundamental string and D0-brane charge densities, and are still arbitrary. The choice of A is motivated by requiring continuity of the metric through the domain wall.

This space is identical to Gödel near the origin, but has very different asymptotics. For example, it has finite angular momentum, whereas for the original space it diverges. Also, inside the domain wall there are constant magnetic fields for the D0-brane and fundamental string, while outside there are mainly electric fluxes (the rotation of space mixes electric and magnetic fields). One bad feature is that the dilaton grows as $r \to \infty$.

It is easily verified that the solution satisfies the Israel matching conditions [17], with the supertube stress-energy tensor at the junction. Furthermore, the matching fixes the D2-brane density in terms of c. A similar calculation was done for the enhançon in [18].

There is a very good analog to these solutions in electromagnetism—a charged solenoid. Outside there is electric flux, with the potential behaving like a log. The current around the solenoid will induce some magnetic flux inside, but no electric fields.

From this construction it is also clear that the supertube preserves the same supersymmetries as the Gödel metric (1). After all, the latter is just the metric induced by a large collection of the same supertubes, but this can also be verified directly by the same calculations as in [13,15,19].

So far we did not specify the radius where the domain wall is located. Clearly when $R \leq 1/c$ there are no closed timelike curves within the domain wall, but it can also be shown that there are no such curves outside the shell. At most, if $R = 1/c$ there are closed null curves on the shell itself, which might warrant further study.

If we place the shell at a larger radius there will be causality violating curves. It is therefore natural to think of the full Gödel solution as the limit when the domain wall is taken to infinity and the region outside discarded.

This picture may provide a better laboratory for studying the problems of the Gödel universe. In particular, if the radius is only slightly larger than $1/c$, there is only a thin shell of closed timelike curves. So the sickness is not spread over the entire space, and it may be possible to describe a process by which the shell contracts to eliminate the region with closed timelike curves.

In any event, it seems like those spaces cannot be created. A calculation similar to the one carried out in the preceding section will show that, if one tries to increase the size of the region inside the shell, so it exceeds $R = 1/c$, the same problems will appear, indicating that one cannot increase the radius of the shell further.

Discussion.—One of the most striking uses of D-branes in string theory is as probes of space-time, often exposing the limitations of supergravity, and providing a cure to some of its pathological solutions. The enhançon mechanism [14] provides a beautiful example of this. Studying supertubes on the Gödel-like universe (1) teaches us a similar lesson, that the naive supergravity solution is not valid. As opposed to the enhançon case, we find that the space is sick everywhere, and the question of how to "cure" it does not seem well posed.

Nevertheless, the domain-wall solutions we discussed earlier locally look like Gödel universes, and present certain analogies with the enhançon mechanism: in both cases, we keep part of the original metric, and replace the problematic part (the repulson singularity in one case, the region with closed timelike curves in the other) by another solution, separated by a shell formed by the probes themselves. Of course, an obvious difference is that in the enhançon case one keeps the asymptotic form of the metric and changes the interior, while in our case we do the opposite.

In this Letter we concentrated on a specific Gödel universe solution of supergravity. Other solutions were studied in [2,8,9] involving rotations in more planes. It is a simple check to see that all the solutions in types IIA and IIB suffer from the same problems we described. One has to take the same supertube, or in some cases the T-dual objects, and place them so that their circles follow the closed timelike curves, and the rest of the calculation is identical. Therefore all those spaces are not good string theory backgrounds. We expect this to be a general mechanism that eliminates many solutions with closed timelike curves in string theory.

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*Electronic address: drukker@weizmann.ac.il
†Electronic address: fiol@weizmann.ac.il
‡Electronic address: jsimon@weizmann.ac.il

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