Concentration of Higher Dimensional Entanglement: Qutrits of Photon Orbital Angular Momentum

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Enhancement of entanglement is necessary for most quantum communication protocols many of which are defined in Hilbert spaces larger than 2. In this work we present the experimental realization of entanglement concentration of orbital angular momentum entangled photons. We investigate the specific case of three dimensions and the possibility of generating different entangled states out of an initial state. The results presented here are of importance for pure states as well as for mixed states.

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Most of the current applications of entanglement in quantum communication such as quantum teleportation [1] and quantum cryptography [2,3] work best for maximally entangled states. However, in practice one always has to deal with nonmaximally entangled or mixed states. Therefore many quantum protocols such as distillation [4,5], purification [6], concentration [7], and error correction [8,9] have been suggested in order to enhance the quality of entanglement. The common idea in most of the protocols is to start with a sample of entangled states having a low quality of entanglement and using local operations and classical communication only to end up in a smaller ensemble with a higher degree of entanglement. There are varying terminologies in the literature. Following Ref. [7] we will use the term concentration for our experimental achievement which was to extract maximally entangled states out of nonmaximally entangled pure states. This method is also referred to as local filtering or the "Procrustean method" [10].

There exists a steadily growing interest in entanglement in higher dimensions since it allows realization of new types of quantum communication protocols [11–14] and provides more security in quantum communication applications [13]. However in order to overcome the problem of uncontrollable interaction of the quantum states with the environment it is important to be in full control of the corresponding distillation and concentration techniques. So far the Procrustean method has been experimentally demonstrated only for entangled qubits, i.e., for two-dimensional systems [5]. Given the motivations discussed above we demonstrate for the first time the experimental realization of quantum concentration in higher dimensions using qutrits entangled in orbital angular momentum (OAM).

In our experiment the entangled qutrits were produced via type-I spontaneous parametric down-conversion using a BBO crystal (beta barium borate) of 1.5 mm thickness pumped by an argon ion laser operating at 351 nm and having 120 mW of light power. We used the energy-degenerated case where both entangled photons had a wavelength of 702 nm.

The light fields of photons having OAM can be described by means of Laguerre-Gaussian (LG_{pl}) modes with two indices p and l. The p index identifies the number of radial nodes observed in the transversal plane and the l index the number of the 2π-phase shifts along a closed path around the beam center. The latter determines the amount of OAM in units of $\hbar$ carried by one photon [15,16]. In all our experiments we considered only LG modes with an index $p = 0$, whereas the l index of the entangled photons varied from $-2, -1, \ldots$ to $\ldots 1, 2$. Since the LG_{pl} modes form an orthogonal basis they can be used to realize discrete higher dimensional entangled systems.

A common technique to produce LG_{0l} modes out of the Gaussian mode is to use computer generated holograms with dislocations in the center [17,18]. Inversely such a hologram in connection with a single-mode optical fiber can be used to identify a certain LG_{0l} mode [19]. It has also been demonstrated that by displacing such a hologram it is possible to create superpositions of the LG_{00} (= Gaussian) and the corresponding LG_{0l} mode with well-defined amplitudes and relative phases. Our holograms were blazed transmission phase gratings with a period of 20 μm which had a diffraction efficiency of about 85%.

In our earlier experiments both have been confirmed: the conservation and the entanglement of the OAM in type-I spontaneous parametric down-conversion [19,20]. The latter was demonstrated for LG_{0-1}, LG_{00}, and LG_{01} modes by violating a generalized Clauser-Horne-Shimony-Holt–type Bell inequality [20], whereas within experimental accuracy conservation was observed for the case where down-converted photons were emitted by an angle of 4° off the pump beam [19]. However strictly speaking perfect conservation should arise only when the generated photons are perfectly collinear with the incident pump beam. It was therefore reasonable to expect that the results on conservation and entanglement of the OAM also hold for the extension to LG_{0-2} and LG_{02}. Thus first we confirmed this issue. Restricting ourselves to the case of a pump beam having no OAM, i.e., an LG_{00} (Gaussian) mode, it was shown that the entangled down-conversion state was given by $C_{00}|00\rangle + C_{11}|11\rangle + C_{22}|22\rangle$, where the numbers in the kets are equivalent to the absolute value of the l index $|l|$ of photon 1 and photon 2, respectively. Using the same techniques as in earlier experiments [19] the down-converted photons on each side were projected onto the respective eigenstates via computer generated holograms. The amplitudes C_{ij} were determined from the coincidence count rates which are a measure for the probabilities. In order to demonstrate the entanglement the state was also measured in a rotated basis, i.e., the down-converted photons were projected onto superpositions of LG modes (Fig. 2, upper row). As discussed in an earlier paper [21] this can be achieved by displaced holograms.

In the second step we actually demonstrated entanglement concentration. This was shown by converting the initial entangled state having nonequal relative amplitudes into a state with equal amplitudes representing a maximally entangled state.

The experimental setup is shown in Fig. 1. The Gaussian (LG_{00}) is focused on the BBO crystal where the entangled pairs are produced. These are emitted from the crystal at an angle of 4° off the pump beam and are coupled into optical fiber couplers via a lens ($f = 25$ cm) on each side. The distances between the crystal and the lenses on each side are given in Fig. 3 and the distance between the crystal and each of the couplers was chosen equally 110 cm.

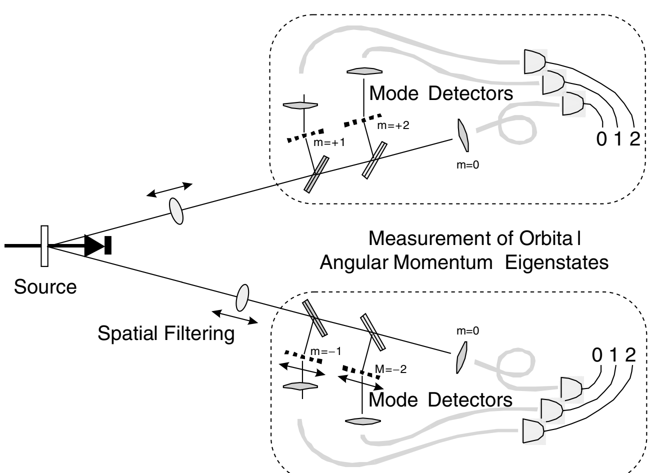

FIG. 1. Experimental setup for the entanglement concentration. After type-I spontaneous parametric down-conversion two lenses are used for spatial mode filtering by which the relative amplitudes between different LG modes are changed. The second part of the setup projects the photons onto the orbital angular momentum OAM eigenstates.

For convenience the mode identification was done using a probabilistic method since the count rates were sufficiently high. However a deterministic mode separator [22,23] would be possible but significantly more complicated. Two nonpolarizing beam splitters, the first one having a transmission to reflectivity ratio of 2:1 and the second one of 1:1, redirected each photon with a probability of $1/3$ to each of three mode detectors. Using the respective holograms and single photon detectors we were able to detect LG_{0-2}, LG_{0-1}, LG_{00}, LG_{01}, and LG_{02} modes. The coincidence logic was designed to record the coincidences between an arbitrary pair of detectors on each side. This gives nine possible coincidence count rates.

It has been experimentally observed [19] that the emission probability for the higher order entangled modes decreases with the index l. Although this fact sets some principle limitations on the entangled states which can be produced by the crystal, as shown by Torres *et al.* [24,25] and Molina-Terizza *et al.* [26] the relative amplitudes depend on the waist size and its position and the length of the crystal. Therefore it is possible to use these parameters to engineer the initial entangled state. However in order to have a maximum collection efficiency of the down-converted photons it is important to adapt their beam parameters to the spatial mode which can be coupled into the single-mode fibers [27]. The waist size of the LG modes grows with their l index; therefore a common setting of the coupling lenses which can couple the LG_{00}, LG_{01}, and the LG_{02} modes all with maximum efficiency does not exist. Thus, in order to measure the initial down-converted state emitted by the crystal we had to proceed in the following way. First the coupling lenses were positioned to have maximal collection efficiency for the LG_{00} mode. Then the lens positions were changed to maximize the collection efficiencies for the LG_{01} and the LG_{02} modes, respectively.

Since the measured amplitudes depend on the positioning of the coupling lenses, it is possible by varying their positions to couple in one mode more effectively than the other one. This method can be considered as a kind of filtering because part of the photon state emitted in the modes for which the collection efficiency of the setup is not optimal is lost.

The refractive index of the monomode fibers determines a certain angle of acceptance for incoming light. The position of the lens is chosen such that the LG mode with the lower emission probability (here $LG_{0|1|}$) has a good overlap with the acceptance mode of the fiber. As a result the LG mode with the higher emission probability (here LG_{00}) has not an optimal overlap with the acceptance mode of the fiber which causes a filtering of the amplitude of this LG mode. The same arrangement with another distance can be used in the other arm of the down-conversion to achieve a filtering of the $LG_{0|1|}$ mode with respect to the $LG_{0|2|}$ mode.

It is also important to mention that as a consequence of entanglement each lens acts as a nonlocal filter on both sides. The filtering action of a lens on one side projects also the corresponding modes on the other side. By varying the distance of the two coupling lenses from the crystal just by the same amount one would be able to equalize the amplitudes of two modes only. It is only by choosing asymmetric positions and by exploiting the entanglement that it is possible to achieve nearly the same coincidence count rates for all three different modes.

In order to identify the state emitted by the crystal we proceeded as described above choosing three different lens positions for collecting the LG_{00}, $LG_{0|1|}$, and $LG_{0|2|}$, respectively, each with the same efficiency. Using detectors with nearly equal detection efficiency the normalized [28] initial state was found to be $\psi_{\mathrm{initial}} = 0,80|00\rangle + 0,44|11\rangle + 0,41|22\rangle$.

The amplitudes were calculated from the coincidence count rates which represent the probabilities for detecting the photon pair in the corresponding LG mode. For convenience we chose the basis for describing the initial state such as having no relative phases between the components. Such a choice is always possible. By scanning all holograms only horizontally these relative phases remain unchanged.

In order to demonstrate that the initial state is entangled it was also measured in bases rotated in Hilbert space. This was done by displacing the holograms. In one beam the LG_{01} and the LG_{02} holograms were displaced while in the other beam the corresponding holograms performed a scan of the mode of the incoming photons. The resulting coincidences are shown in Fig. 2, upper row.

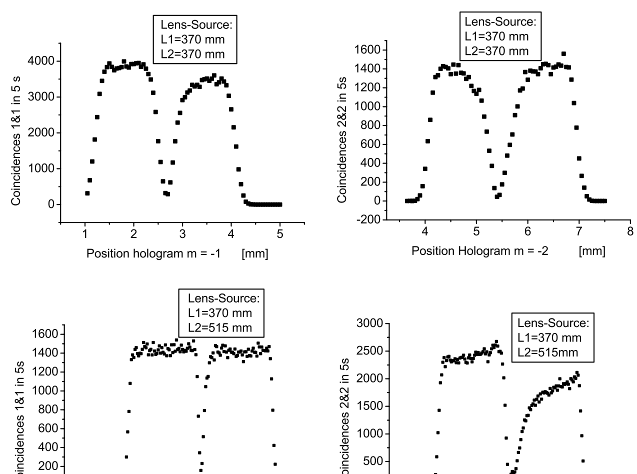

FIG. 2. Measurement of the entangled states in a superposition basis. Upper row: before concentration, lower row: after concentration. In both curves the dip is the signature for entanglement. The measurement of a mixed state in the superposition basis would result in equally distributed coincidences forming curves with no or low contrast.

As shown in an earlier work [19] the high visibility ($\sim 80\%$ typical error $\sim 3\%$) of these curves in the rotated basis can be viewed as a signature of entanglement. Whereas a mixed state would lead to an equally distributed coincidence measurement of the LG modes resulting in curves having low contrast.

As discussed above different lens positions cause different filterings of the initial state and therefore different entangled states can be created out of an initial state. This was demonstrated experimentally for seven different combinations of lens settings (Fig. 3). Each of these lens configurations can be identified with a certain filtering ratio for the LG modes. The filtering ratio for each LG mode is defined as $1 - (C_{\mathrm{LC}}/C_{\mathrm{INT}})$ where C_{LC} and C_{INT} denote the coincidence count rate at a certain lens configuration and the coincidence count rates for the initial state, respectively. It is a quantitative measure therefore how a lens configuration acts as a filter for an LG mode.

The filtered states are not always necessarily more entangled for any lens configuration; however the filtering action of the lens configuration Nr. 5 in Fig. 3 causes a maximal concentration of the initial state emitted by the crystal. The filtering lenses together with the fiber are projecting the incoming state onto the maximally entangled state. This projection lowers the initial amplitudes of the LG modes with higher emission probabilities, i.e., LG_{00} and $LG_{0|1|}$ (see Fig. 3, row Nr. 5). The concentrated state is found to be

$$\psi_{\mathrm{concentrated}} = 0,60|00\rangle + 0,56|11\rangle + 0,57|22\rangle, \qquad (1)$$

which is very close to the three-dimensional maximally entangled state $\psi_{\max} = (1/\sqrt{3})[|00\rangle + |11\rangle + |22\rangle]$.

After the above lens configuration for concentration of the initial state and achieving maximally entangled states is determined experimentally, one could, for example, use the state (1) in order to implement a quantum

Initial State 0,80 |00>+0,44|11>+0,41|22>

Local Filtering ↓

Concentrated State

Lens Configuration			Coincidences in 5 s			Filtered State	Filtering Ratio		
Nr.	L1-Source	L2-Source	0&0	1&1	2&2		0&0	1&1	2&2
1	370 mm	149 mm	1200	105	30	0,95 |00>+0,28|11>+0,15|22>	0,00	0,72	0,90
2	370 mm	332 mm	1000	190	70	0,89 |00>+0,15|11>+0,24|22>	0,17	0,49	0,75
3	370 mm	385 mm	870	370	100	0,81 |00>+0,53|11>+0,27|22>	0,28	0,00	0,68
4	370 mm	430 mm	660	240	50	0,83 |00>+0,50|11>+0,23|22>	0,45	0,35	0,84
5	370 mm	515 mm	340	290	310	0,60 |00>+0,56|11>+0,57|22>	0,72	0,22	0,00
6	420 mm	515 mm	250	190	30	0,73 |00>+0,64|11>+0,25|22>	0,79	0,49	0,90
7	520 mm	515 mm	100	40	190	0,55 |00>+0,35|11>+0,76|22>	0,92	0,89	0,39

FIG. 3. "Procrustean" filtering method. Each lens configuration in the setup cases a specific filtering of the three modes LG_{00}, $LG_{0|1|}$, and $LG_{0|2|}$. In general the filter density is different for each mode and depends on the positions of the two lenses L1 and L2. Specially the lens configuration 5 causes the concentration of the initial state. The concentrated state has nearly equal amplitudes. Typical errors for the amplitudes of the filtered states are about 0, 02.

cryptography protocol with higher alphabets [13]. Since the security of such protocols is affected by the use of nonmaximally entangled states and most of the experimentally available states are not maximally entangled, the concentration can be regarded as a necessary prestage to the implementation of many quantum protocols.

Since only one "initial" state was available to us other lens configurations yielded filtered states which are not necessarily more entangled. However because of the linearity of the filtering process there exist possible initial states for which each of the local filterings in Fig. 3 would cause the concentration of entanglement. For example, given the states $0,26|00\rangle + 0,50|11\rangle + 0,83|22\rangle$ or $0,73|00\rangle + 0,63|11\rangle + 0,27|22\rangle$ as initial states the filtering action of the lens configurations Nr. 1 and Nr. 7 in Fig. 3 would cause the concentration into the maximally entangled state, respectively.

After we had experimentally equalized the amplitudes of the initial state in order to demonstrate the entanglement of the concentrated state (1) it was measured in bases rotated in the Hilbert space.

The measured coincidences are shown in Fig. 2, lower row. One can find that in the $\psi_{\text{concentrated}}$ case which is closer to the maximally entangled state the visibilities are higher. These are defined as $V = (C_{\max} - C_{\min}/C_{\max} + C_{\min})$ where C denotes the coincidence count rates. The corresponding visibilities in Fig. 2 are 86, 4% for the LG_{01} modes and 81, 5% for the LG_{02} in the ψ_{initial} case and 94, 4% for the LG_{01} modes and 87, 3% for the LG_{02} in the $\psi_{\text{concentrated}}$ case. The asymmetry in Fig. 2 lower row right is due to an imperfection of the corresponding hologram [29]. The width of the dips in Fig. 2 is determined by the width of the incident beam on the hologram. The smaller the width of the incident beam on the hologram the more sensitive are the amplitudes of the transformation upon displacement of the hologram.

By exploiting simple experimental techniques theses results clearly show the possibility of extracting maximally entangled states out of nonmaximally entangled ones in higher dimensions. The results presented here are not only of interest for pure states but also for extracting entanglement out of mixed states. Since, as demonstrated by Horodecki *et al.* [4], all inseparable quantum systems can be distilled to singlet states by local filtering and the Bennett *et al.* distillation protocol [6]. Therefore it can be expected that the ideas and techniques presented here will be of importance for future quantum communication networks over long distances which necessarily will need some kind of entanglement enhancement procedure.

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[29] The origin of this imperfection is that the period of the grating is slightly different on both sides of the dislocation. This causes slightly different diffraction angles while the hologram is scanned horizontally and therefore the coupling efficiency is altered. Since this effect also depends on the width of the incident beam on the hologram which is changed upon displacement of the filtering lenses the asymmetry appears stronger in the lower row of Fig. 2.