Experimental Evidence of "Vibrational Resonance" in an Optical System

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The experimental evidence and characterization of "vibrational resonance" in a bistable vertical cavity laser are reported. The system is driven by two periodic forcings, with frequencies differing by several orders and studied in the case of both symmetrical and asymmetrical quasipotentials. The phenomenon shows up in the dynamics of the polarized laser emission as a resonance in the low-frequency response and signal-to-noise ratio, depending on the amplitude of an applied high-frequency modulation. The possibility to use the phenomenon for low-level detection is experimentally demonstrated.

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Bistable systems driven by a periodic signal and noise are known to display a resonancelike behavior at the signal frequency as a function of the applied noise strength [1]. Such a phenomenon, known as a stochastic resonance (SR), has been found in very different physical, chemical, and biological systems [2]. In this context, Landa and McClintock have studied a model [3] where a high-frequency deterministic modulation replaced the added noise. They have numerically shown that the response to the periodic modulation (low frequency, LF) in the absence of noise passes through a maximum, depending on the amplitude of an additional high-frequency (HF) modulation. Such a phenomenon has been named "vibrational resonance" (VR). They have also noted a clear analogy between VR and SR, though the effect of noise on VR was not considered there. Analytical results for VR concerning a bistable oscillator were also given in Ref. [4]. A recent observation of VR in an electronic circuit was presented in Ref. [5], where the study was focused on noise-induced structures. The importance of VR lies not only in the possibility to enhance the performance of a system via the addition of a completely deterministic signal (thus improving the signal-to-internal noise ratio), but also from a dynamical point of view, as it can be considered as a parametric amplification near a critical point. A similar phenomenon has been shown in period-doubling systems [6]. The latter aspect will be studied in detail in a future paper. We point out that the phenomenon of VR is different from so-called deterministic SR where, instead of noise, an external broadband chaotic signal of a deterministic system has been used [7].

In this Letter, we present the experimental evidence and characterization of the phenomenon of VR in an optical system, namely, a vertical cavity surface emitting laser (VCSEL). The VCSEL is known to display a bistability of polarization states of the emitted field which can be controlled by the applied injection current [8]. From a theoretical point of view, dynamics of the polarization switchings in the VCSEL can be locally well described by an overdamped Langevin model with a two-well potential [9,10]. Evidence of SR was recently given in such a system [11]. Different cases of the occurrence of VR were investigated experimentally. Thanks to the flexibility of the system, we were able to observe VR for the case of both nearly symmetrical and strongly asymmetrical quasipotentials. In both cases the laser response at the low frequency passes through a maximum depending on the amplitude of the applied HF signal. Since noise is inevitably present in any real physical systems, we measured also the signal-to-noise ratio at the frequency of the LF signal, which displays the same behavior.

SR and VR are based on the same requirement, namely, the crossing of a barrier in bistable or threshold systems when a sum of amplitudes of two forcings (deterministic or stochastic) at certain times reaches a critical value, corresponding to switching between two states. In the case of added Gaussian noise this requirement takes the form of a specific time matching condition, which corresponds to synchronization between the input periodic signal and the random switching events induced by the noise itself. In the case of a HF deterministic modulation, such a requirement is fulfilled periodically when the sum of the amplitudes of LF and HF signals is large enough to cross a barrier. In this case the time matching condition should be replaced by an amplitude criterion. It can easily be shown that for bistable or threshold systems driven by LF and HF periodic signals (in the absence of any sources of noise), the response at the low frequency has a maximum in the interval

$$\Delta - A_{\rm LF} \lesssim A_{\rm HF} \lesssim \Delta + A_{\rm LF}, \qquad (1)$$

where Δ is a critical value of the amplitude of the HF signal corresponding to crossing a barrier in the absence of the LF signal [12]. Generally, for bistable systems the critical value Δ depends not only on the frequency as shown in [3,13], but also on the level of internal noise of the system.

Bistable and threshold systems are often used as detectors of low-frequency periodic and dc signals. In this context, large efforts have been devoted in the past few years to improve the performance of such systems using the concept of SR [14–17]. Here, we experimentally show that the phenomenon of VR could also be of great interest for the detection of low-level noisy signals.

The experimental investigations were performed using the setup where the phenomenon of SR was studied and characterized in great detail [11]. We studied VR in the laser intensity after polarization selection when two periodic signals with different frequencies were applied to the injection current. One of them was a sinusoidal LF signal with a frequency $\Omega_{LF}/2\pi = 500$ Hz and a subthreshold amplitude A_{LF}. The second one was the HF signal with a frequency $\Omega_{HF}/2\pi = 100$ kHz, much higher than the frequency of the LF signal. In all our investigations we used a square-wave shape of the HF modulation with an on-off time ratio of 2. In this case the amplitude of the HF signal needed for the observation of VR is smaller than in other cases (e.g., a sinusoidal signal or a train of short pulses), though VR has been observed with different types of periodic signals. The amplitude of the HF signal A_{HF} serves here as a control parameter. The injection current was chosen in such a way that the laser operates in the regime of polarization bistability where switching between two states could be induced by applying the deterministic modulation or noise. The laser responses were detected with a fast photodetector and recorded by a digital oscilloscope interfaced with a computer to store and process the data. A detailed description of the experimental setup in the case of SR measurements is given in [11].

For a quantitative characterization of VR we used the standard quantifiers of SR, namely, the signal-to-noise ratio (SNR) and the gain factor G which were obtained from the spectra of the Fourier transformed time series. A total number of 25 000 sampling points were used for each series, corresponding to 50 periods of the LF signal. The statistical indicators were then averaged over a 10 to 50 time series. We define $\mathrm{SNR} = 10\log_{10}\{[I_N(f_s) + I_s(f_s)]/I_N(f_s)\}$, where $I_s(f_s)$ is the power spectrum of the laser response at the frequency of the LF signal, $I_N(f_s)$ is the interpolated level of noise at the low frequency, and $[I_N(f_s) + I_s(f_s)]$ is the superimposed power spectrum of the LF signal and noise background at the low frequency. In what follows we used normalized SNR^n, defined as $\mathrm{SNR}^n = \mathrm{SNR}/\mathrm{SNR}(0)$, where SNR(0) denotes SNR in the absence of the HF modulation or additional noise. The gain factor was defined as $G = [I_s(f_s) - I_N(f_s)]/[I_0(f_s) - I_{N0}(f_s)]$, where $I_0(f_s)$ and $I_{N0}(f_s)$, are the laser responses at the low frequency in the absence of the HF modulation (VR) or noise (SR), respectively.

We performed a first measurement, where the injection current was tuned so that the residence times in the two levels of the polarized laser emission were almost the same [corresponding to a nearly symmetric double-well quasipotential (see [9])]. The amplitude of the LF signal A_{LF} was set to a value smaller than the critical one, corresponding to force switchings between the two states at any semiperiod. We show in Fig. 1 the temporal behavior of the polarized laser emission for different values of the amplitude of the HF signal, increasing from bottom to top (left column). For a comparison purpose, noise-induced switchings in the absence of the HF modulation are shown on the right column of the figure, increasing the noise strength from bottom to top (SR scenario).

In the presence of internal noise only, the laser responses for different HF signal amplitudes are similar to the SR scenario. Indeed, as A_{HF} increases, rare switchings between two states induced by the joint effect of LF and HF signals, and noise [Fig. 1(b), left column], are replaced by more frequent switching events [Figs. 1(c) and 1(d), left column]. A complete synchronization between the input LF signal and output of the system is then obtained [Fig. 1(e), left column]. A further increase of A_{HF} worsens the input-output synchronization [Fig. 1(f), left column]. It is worth noticing that in the VR scenario the time series appear significantly less noisy than in the SR case, indicating a possible improvement in SNR.

Figures 2(a) and 2(b) characterize quantitatively this picture, where G_{VR} and SNR^n_{VR} versus A_{HF} as well as G_{SR} and SNR^n_{SR} versus the noise strength A_N are plotted (subindexes on indicators refer to the VR or SR scenario). One can see from Fig. 2(a) that the dependence of G_{VR} (curve 1) versus A_{HF} displays a well pronounced maximum. This is the distinguished feature of VR [3]. A similar behavior is also observed for SNR^n_{VR} depending

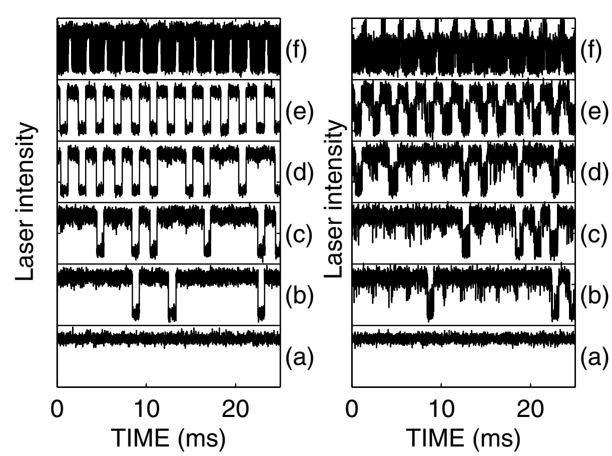

FIG. 1. Time series of the polarized laser intensity for different amplitudes of the HF signal $A_{HF} = 0$, 10.1, 10.2, 10.3, 10.4, 14 mV (increasing from bottom to top, left column) and noise intensities $A_N = 0$, 10.6, 11, 11.4, 14, 20 $\mathrm{mV_{rms}}$ (increasing from bottom to top, right column).

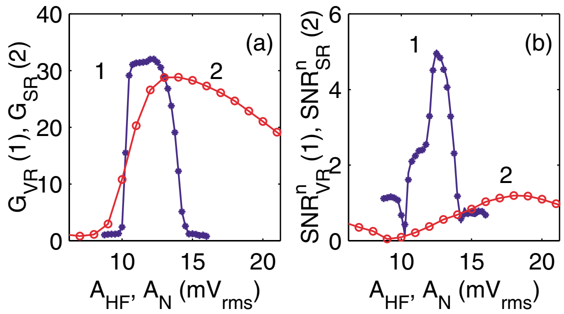

FIG. 2 (color online). Response of the system in the nearly symmetric case. (a) G_{VR} (1) and G_{SR} (2); (b) SNR^n_{VR} (1) and SNR^n_{SR} (2) for the LF signal versus A_{HF} (VR) and A_N (SR). Every point was obtained by averaging over 50 time series (see text).

on A_{HF} [Fig. 2(b), curve 1]. The dip in Fig. 2(b) (curve 1, $A_{HF} \approx 10$ mV) is due to the rare irregular noise-induced switchings shown in Fig. 1(b) and 1(c) (left column) in a very similar way as for the conventional SR. Then, SNR^n_{VR} and G_{VR} rise very quickly as A_{HF} increases. In the region where G_{VR} is practically constant, SNR^n_{VR} slightly depends on A_{HF} up to the certain value of A_{HF} when switchings induced by the HF modulation itself appear in the response. From this value, a further increase of A_{HF} leads to the maximum of SNR^n_{VR}. A very similar behavior of SNR^n_{VR} on A_{HF} is observed in the numerical simulation of an overdamped oscillator driven by two periodic forcings and noise. It is also worth noting that there is a sharper dependence of G_{VR} on A_{HF} and, at the same time, higher values of G_{VR} and SNR^n_{VR} in the case of VR in comparison with SR for these experimental conditions.

A distinctly different picture is observed in the case of a strongly asymmetric quasipotential. We changed the pump current in order to introduce very different residence times for the two levels of the emitted polarization intensity [9]. The corresponding experimental results are presented in Figs. 3 and 4. The laser response to the two-frequency modulation is shown in Fig. 3 for different values of the amplitudes of the HF signal. One can notice the appearance of bursts of HF pulses at the rate corresponding to the low-frequency Ω_{LF}. As A_{HF} increases, a growth of the number of HF pulses occurs in each pulse train. Because of the presence of internal noise in the laser, the number of the HF pulses in each pulse train has small variations [see, e.g., Figs. 3(b) and 3(c)]. The LF signal amplitude A_{LF} was set to be almost the same as in the previous case, but the frequency of the HF signal was taken $\Omega_{HF}/2\pi = 10$ kHz, in order to show more clearly the bursting effect. In fact, in a wide range of HF frequencies up to a cutoff related to the natural frequency response of the system, the phenomenon of VR weakly depends on the HF frequency. This is valid for all the cases studied here. The asymmetric case is very similar

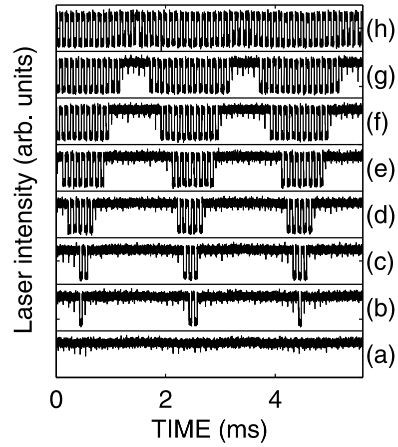

FIG. 3. Time series of the polarized laser intensity in the strongly asymmetric case for different amplitudes of the HF signal $A_{HF} = 0$, 16.8, 17.2, 17.8, 19, 20.5, 22, 23 mV (increasing from bottom to top). $\Omega_{LF} = 500$ Hz; $\Omega_{HF} = 10$ kHz.

to the threshold-crossing systems reported in [18], where the spiking regime of an excitable system was activated adding a suitable amount of noise to a slowly varying modulating signal. In such a case, the resonance was obtained comparing the rate of the spiking bursting events with the frequency of the slow input signal. We will discuss these points in detail elsewhere [12].

The response, as evaluated by the gain factor G_{VR}, has an almost symmetrical bell-shaped form in contrast with a rather sharp dependence in the case of symmetric quasipotential [Fig. 4(a)]. Similar to the previous case, SNR^n_{VR} also displays a maximum, depending on the amplitude of the applied HF modulation. One can note a considerable growth of SNR^n_{VR} with respect to the initial value in the absence of the HF modulation [Fig. 4(b)]. It should be noted also that this case can be analyzed analytically, in the framework of a simple model of the threshold-crossing system with a static nonlinearity [12].

To investigate the possibility of using VR for low-level signal detection, we compare its efficiency for this purpose with conventional SR. In this case, a nearly

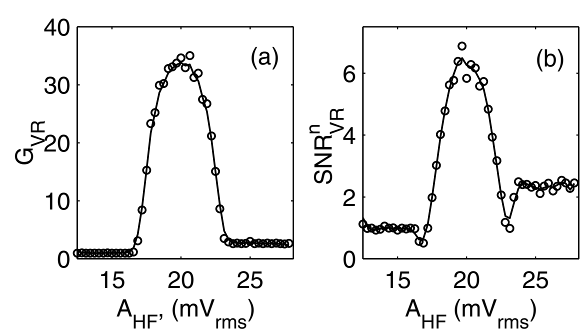

FIG. 4. Response of the system in the strongly asymmetric case. (a) G_{VR} and (b) SNR^n_{VR} versus A_{HF} (see text).

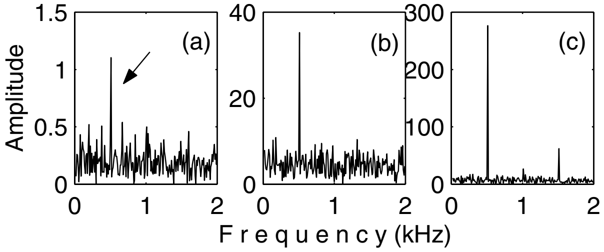

FIG. 5. Low-level detection. Spectra of the laser responses (a) in the absence of the HF signal ($A_{\rm HF} = 0$), (b) for the optimal value of the noise level, and (c) for the optimal amplitude of the HF signal. The arrow indicates the input signal.

symmetric configuration of the bistable laser was used. The amplitude of the LF signal was set to be smaller by 8 times than in previously studied cases and was about 0.019 of the switching threshold. In Fig. 5(a) we show the amplitude spectrum of the laser response in the absence of the HF modulation. The spectra of the laser responses corresponding to the optimal noise level (SR) and the optimal HF amplitude (VR) are shown in Figs. 5(b) and 5(c), respectively. The improvement in detection is evident. A quantitative characterization is presented in Figs. 6(a) and 6(b), where $G_{\rm VR}$ (1) and $\mathrm{SNR}^n_{\rm VR}$ (1) along with $G_{\rm SR}$ (2) and $\mathrm{SNR}^n_{\rm SR}$ (2) versus $A_{\rm HF}$ and A_N are plotted, respectively. Two main peculiarities for the gain factor $G_{\rm VR}$ can be noted. The first is the very high value of $G_{\rm VR}$ (about 290 in the maximum of the gain dependence) as compared with conventional SR ($G_{\rm SR} \approx 25$ in the maximum). The second is the narrow width of the dependence, which is determined by the amplitude of the LF signal. The same features are also observed in the behavior of $\mathrm{SNR}^n_{\rm VR}$ where we were able to increase the initial SNR(0) by about 7 times, whereas $\mathrm{SNR}^n_{\rm SR} \approx 1.3$ in the maximum. Indeed, in our case the amplitude of the LF signal is very low as compared to the threshold; this leads to a strong degradation of SNR in the case of SR [17,19]. Our experimental findings are in agreement with numerical simulations and analytic results; the comparison will be presented elsewhere.

To conclude, we have reported on the experimental observation of the phenomenon of vibrational resonance in a VCSEL operating in the regime of polarization bistability. We have demonstrated the occurrence of VR for both the case of nearly symmetric and strongly asymmetric double-well quasipotentials. Besides, we have also experimentally shown the possibility to apply this phenomenon for an improvement of low-level detection. This result is very general, and can be a promising tool for a variety of systems ranging from signal restoring in optical fiber communication to optimization of threshold sensors in noisy environments.

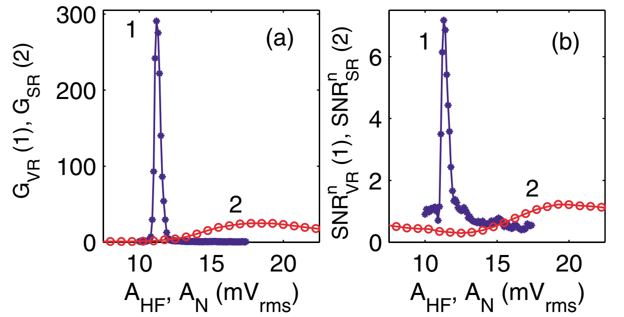

FIG. 6 (color online). Response of the system in the low-level detection. (a) $G_{\rm VR}$ (1) and $G_{\rm SR}$ (2); (b) $\mathrm{SNR}^n_{\rm VR}$ (1) and $\mathrm{SNR}^n_{\rm SR}$ (2) versus $A_{\rm HF}$ and A_N.

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