Multichannel Optical Add-Drop Processes in Symmetrical Waveguide-Resonator Systems

Wei Jiang and Ray T. Chen

Department of Electrical and Computer Engineering and Microelectronic Research Center, University of Texas, Austin, Texas 78758, USA

(Received 7 October 2002; published 17 November 2003)

Multichannel optical add-drop processes are studied in a class of symmetric waveguide-resonator systems. With insight gained from group theory, we analyze these systems and show that they can add or drop multiple wavelengths simultaneously, with 100% efficiency. A new mechanism is presented to reduce the remnant light at the dropped wavelengths in the pass-through port. High-order Butterworth filters can also be achieved in these systems. Built upon conventional or photonic-crystal based structures, these systems can be used in optical communication applications.

DOI: 10.1103/PhysRevLett.91.213901 PACS numbers: 42.82.Et, 42.70.Qs, 42.79.Sz, 42.82.Bq

In today's fiber-optic networks, light of multiple wavelengths propagates along a single optical fiber. Each wavelength of light transmits its own information undisturbed by the other wavelengths. A single-channel optical add-drop multiplexer (OADM) is a device that can add or remove a specific wavelength of light from a fiber. Recently, more and more applications demand OADMs that are able to add and remove multiple wavelengths.

Filters based on photonic crystals (PC) have been discussed for single-channel OADM applications. Fan *et al.* first proposed a structure of two parallel waveguides in a photonic crystal, with two resonators in between [1]. Light of multiple wavelengths comes into one waveguide from a fiber. With a proper design of the resonators, light of a specific wavelength will be completely transferred to the other waveguide, while light of the other wavelengths passes through the original waveguide and is coupled into another fiber. Quantum Green's functions have been used to analyze the light transfer process in this structure. Additionally, simulations are performed to study PC-based single-channel OADMs [2] and demultiplexers [3]. A problem encountered in current simulations is that for many ports the light transfer efficiencies are fairly low. This also results in much light remaining in the pass-through port. Clearly, an analytic theory is needed to explore the characteristics and ultimate performance of PC-based multichannel OADMs and to give direction to the simulation efforts. New system architecture may be needed to overcome the limitations of the old systems.

In this Letter, we propose a class of new structures which can add or drop multiple wavelengths simultaneously. In such a structure that has n-fold symmetry, n pairs of resonators and n waveguides are arranged in a symmetrical manner. An n-fold structure can achieve 100% add and drop of light at $n-1$ different wavelengths. These structures also provide a way of suppressing the remnant light intensity at the pass-through port for the bands of dropped frequencies. Such an improvement in optical isolation is ideal for many applications.

Consider a system having n waveguides on the edges of a regular n-polygon. Inside the polygon, near the middle of each edge, there is a pair of identical cavities each having a single resonant mode. Their modes can be combined to form one even and one odd mode with respect to the mirror plane between them. With the resonators placed symmetrically, the system possesses a symmetry of point group C_{nv}. Figure 1 illustrates the case $n = 3$. An n-fold system is described by a Hamiltonian [1]

$$\begin{aligned} &H = H_0 + V, \qquad H_0 = \sum_{m=0}^{n-1}\sum_k \omega_k |mk\rangle\langle mk| + \sum_{m=0}^{n-1}\sum_c \omega_{mc}|mc\rangle\langle mc|, \\ &V = \sum_{m,m'}\sum_{c,c'}(1-\delta_{m,m'}\delta_{c,c'})V_{mc,m'c'}|mc\rangle\langle m'c'| + \sum_{m,m'}\sum_{k,c}[V_{mc,m'k}|mc\rangle\langle m'k| + V_{m'k,mc}|m'k\rangle\langle mc|], \end{aligned} \tag{1}$$

where $|mk\rangle$ is a propagating mode with wave vector k and frequency ω_k in waveguide m. The mode $|mc\rangle$ is a localized mode of the resonator pair next to waveguide m, $c = e, o$ for the even and odd modes, respectively; ω_{mc} is its frequency. The coefficients $V_{mc,m'c'}$ and $V_{mc,m'k}$ measure the coupling between the corresponding modes. We have neglected the coupling between the propagating modes of different waveguides as discussed by Xu *et al.* [4]. For $n > 2$, the symmetry operations of the group C_{nv} do not commute with each other; therefore, irreducible representations of dimensions higher than unity appear [5]. In simple words, a set of basis functions that are the eigenstates of all symmetry operations does not exist. Compared to the standard basis functions of irreducible representations, the eigenfunctions of C_n operations are found to offer more convenience to analysis. One can readily show that, constructed from $|mk\rangle$, the modes

 0031-9007/03/91(21)/213901(4)$20.00

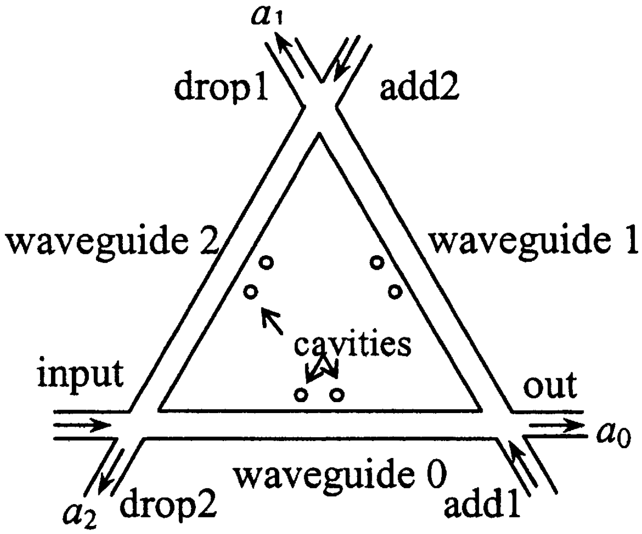

FIG. 1. A sketch of the case $n = 3$. Each waveguide is accompanied by a pair of cavities. Multiple wavelengths of light come in from the input port. Two wavelengths are dropped to the drop1 and drop2 ports, respectively. Light of the corresponding wavelengths is coupled in through the add1 and add2 ports, then merges with the undropped light, and reaches the output port altogether.

$$|\alpha k\rangle = \frac{1}{\sqrt{n}}\sum_{m=0}^{n-1} e^{-i(2\pi/n)\alpha m}|mk\rangle, \qquad \alpha = 0, 1, \ldots, n-1 \tag{2}$$

are eigenfunctions of C_n. One can construct $|\alpha e\rangle$ and $|\alpha o\rangle$ from $|me\rangle$ and $|mo\rangle$ similarly. The reflection M_j by the mirror plane bisecting waveguide j gives

$$M_j|\alpha c\rangle = \pm e^{-i(2\pi/n)2\alpha j}|\overline{\alpha}c\rangle, \qquad c = e, o, \tag{3}$$

where $\overline{\alpha} \equiv -\alpha$, and the plus and minus signs are for e and o, respectively. A similar relation holds for $|\alpha k\rangle$. Note $|\alpha e\rangle$ and $|\alpha o\rangle$ are no longer eigenfunctions of any M_j, which brings difficulties to the analysis. From these, it follows that $V_{\alpha k,\beta k'} = V_{\alpha c,\beta c'} = V_{\alpha k,\beta c} = 0$, for $\alpha \neq \beta$. In terms of the symmetrized basis, the Hamiltonian can be simplified to

$$\begin{aligned} H &= H_0' + V', \\ H_0' &= \sum_{\alpha=0}^{n-1}\sum_k \omega_k|\alpha k\rangle\langle\alpha k| + \sum_{\alpha=0}^{n-1}\sum_{c=c_1,c_2}\omega_{\alpha c}|\alpha c\rangle\langle\alpha c|, \\ V' &= \sum_\alpha\sum_{k,c}[V_{\alpha c,\alpha k}|\alpha c\rangle\langle\alpha k| + V_{\alpha k,\alpha c}|\alpha k\rangle\langle\alpha c|]. \end{aligned} \tag{4}$$

The decoupled $|\alpha c_1\rangle, |\alpha c_2\rangle$ can be expressed as

$$|\alpha c_1\rangle = u_\alpha|\alpha e\rangle + v_\alpha|\alpha o\rangle, \tag{5a}$$
$$|\alpha c_2\rangle = -v_\alpha^*|\alpha e\rangle + u_\alpha^*|\alpha o\rangle, \tag{5b}$$

where u_α and v_α are complex coefficients satisfying $|u_\alpha|^2 + |v_\alpha|^2 = 1$. Their values can be easily solved given $V_{\alpha e,\alpha o}$.

Solving the Lippmann-Schwinger equation [6], one finds the transfer matrix has elements $T_{\alpha' k',\alpha k} = T_{\alpha k',\alpha k}\delta_{\alpha'\alpha}$,

$$T_{\alpha k',\alpha k} = \delta_{k',k} + \sum_{c,c'}\frac{V_{\alpha k',\alpha c'}G_{\alpha c',\alpha c}(\omega_k)V_{\alpha c,\alpha k}}{\omega_k - \omega_{k'} + i\epsilon}, \tag{6}$$

where Green's function is determined by

$$(G^{-1})_{\alpha c',\alpha c}(\omega) = (\omega - \omega_{\alpha c} + i\sigma_{\alpha c})\delta_{c'c} - \Sigma_{\alpha c',\alpha c}, \tag{7}$$

where $\sigma_{\alpha c}$ represents the cavity loss [4,7]. Note that $\sigma_{\alpha c}$ may depend on frequency, because the coupling coefficients $V_{m_1 c,m_2 c'}$ are usually frequency dependent due to the nature of "photonic potential" [8]. The self-energy is

$$\Sigma_{\alpha c',\alpha c}(\omega) = \sum_k V_{\alpha c',\alpha k}\frac{1}{\omega - \omega_k + i\epsilon}V_{\alpha k,\alpha c}. \tag{8}$$

The scattered wave is given by $|\psi_{\alpha k}^{(+)}\rangle = \sum_{k'}T_{\alpha k',\alpha k}|\alpha k'\rangle$. The forward and backward scatterings for $|\alpha k\rangle$ are

$$\langle x = +\infty|\psi_{\alpha k}^{(+)}\rangle = \langle x|\alpha k\rangle + \frac{-iL}{v(\omega_k)}\langle x|\alpha k\rangle\sum_{c,c'}V_{\alpha k,\alpha c'}G_{\alpha c',\alpha c}(\omega_k)V_{\alpha c,\alpha k}, \quad \langle x = -\infty|\psi_{\alpha k}^{(+)}\rangle = \frac{-iL\langle x|\alpha\overline{k}\rangle}{v(\omega_{\overline{k}})}\sum_{c,c'}V_{\alpha\overline{k},\alpha c'}G_{\alpha c',\alpha c}V_{\alpha c,\alpha k}.$$

To further the calculation, we temporarily *assume* $\Sigma_{\alpha c',\alpha c}$ is diagonal. We revisit this issue shortly. We denote the diagonal terms as $\Sigma_{\alpha c,\alpha c}(\omega) = \Delta\omega_{\alpha c}(\omega) - i\gamma_{\alpha c}(\omega)$.

In an OADM, we require no backscattering,

$$V_{\alpha\overline{k},\alpha c_1}V_{\alpha c_1,\alpha k}G_{\alpha c_1,\alpha c_1}(\omega_k) + V_{\alpha\overline{k},\alpha c_2}V_{\alpha c_2,\alpha k}G_{\alpha c_2,\alpha c_2}(\omega_k) = 0. \tag{9}$$

Following Fan *et al.*, accidental degeneracy conditions [7] are adopted, $\omega_{\alpha c_1} + \Delta\omega_{\alpha c_1} = \omega_{\alpha c_2} + \Delta\omega_{\alpha c_2}$, $\gamma_{\alpha c_1} = \gamma_{\alpha c_2}$, and $\sigma_{\alpha c_1} = \sigma_{\alpha c_2}$. Because of these equalities between c_1 and c_2, we hereafter use c to represent c_1 or c_2 unless confusion occurs. Now solving Eq. (9) gives

$$\frac{V_{\alpha\overline{k},\alpha c_2}}{V_{\alpha c_1,\alpha k}} = -\frac{V_{\alpha\overline{k},\alpha c_1}}{V_{\alpha c_2,\alpha k}} = e^{i\eta_{\alpha k}}, \tag{10}$$

where $\eta_{\alpha k}$ is an arbitrary phase angle. This condition also automatically ensures that the self-energy is diagonal.

One can verify that Eq. (10) simultaneously satisfies $\Sigma_{\alpha c_1,\alpha c_2} = 0$ and Eq. (9), two independent equations.

The symmetrized forward scattering amplitude is calculated as

$$A_{\alpha k} = 1 + \frac{-2i\gamma_{\alpha c}}{\omega_k - (\omega_{\alpha c} + \Delta\omega_{\alpha c}) + i(\gamma_{\alpha c} + \sigma_{\alpha c})}, \quad (11)$$

where $\Delta\omega_{\alpha c}$, $\gamma_{\alpha c}$, and $\sigma_{\alpha c}$ are evaluated at ω_k. Now consider the light of a single wavelength coupled into waveguide #0, with unity amplitude. This light can be decomposed into symmetrized modes by Eq. (2); thus the forward scattering amplitude in waveguide m is

$$a_m = \frac{1}{n}\sum_\alpha A_{\alpha k} e^{-i(2\pi/n)\alpha m}. \quad (12)$$

The sum of the scattering amplitudes is found to be independent of the terms $\alpha \neq 0$,

$$\sum_m a_m = A_{0k}. \quad (13)$$

The sum of the intensities is given by

$$\sum_m |a_m|^2 = \frac{1}{n}\sum_\alpha |A_{\alpha k}|^2. \quad (14)$$

If $\sigma_{\alpha c} = 0$ for all α, then all $|A_{\alpha k}| = 1$, total light intensity is conserved after scattering. Generally, in a passive device, all $\sigma_{\alpha k}$'s and $\gamma_{\alpha k}$'s are non-negative, hence $|A_{\alpha k}| \leq 1$, an optical loss occurs.

The equation $a_m = A_{0k_1}\delta_{mm_1}$ characterizes that a complete drop from waveguide #0 to waveguide m_1 occurs at frequency ω_{k_1}. Such a drop occurs if and only if

$$A_{\alpha k_1} = A_{0k_1} e^{i(2\pi/n)\alpha m_1}, \qquad \alpha = 0, 1, \ldots, n-1. \quad (15)$$

Two interesting cases are considered. First, consider an ideal lossless case with $\sigma_{\alpha c} = 0$ for all α. It turns out that the "optical isolation" is very poor in this case. Physically, it means that when a *lossless* drop occurs from waveguide #0 to m_1 at ω_{k_1}, it is impossible to keep the light intensities in other $n-2$ waveguides infinitesimally small over a band of frequency centered at ω_{k_1}. Without loss of generality, consider a drop to waveguide #1. Over a frequency range centered at ω_{k_1}, assume the light intensities are essentially zero in all waveguides except #0 and #1, which yields $a_0 = A_{0k} - a_1$ from Eq. (13). Inverting Eq. (12), one can calculate $A_{\alpha k}$ from the a_m's. Then

$$|A_{\alpha k}|^2 = 1 + 2|a_1|(|a_1| - \cos\delta)\left(1 - \cos\frac{2\pi\alpha}{n}\right) - 2|a_1|\sin\frac{2\pi\alpha}{n}\sin\delta, \quad (16)$$

where $\delta \equiv \arg A_{0k} - \arg a_1$. One readily shows that, for $n \geq 3$, $|A_{\alpha k}| = 1$ cannot hold for all α, when $|a_1|$ varies continuously on the interval $[1-\eta, 1]$ where η is an arbitrarily small, positive number. This contradicts our lossless assumption according to Eq. (14).

The second case we study is that $\sigma_{0k} = 0$ and all other $\sigma_{\alpha c}$'s take non-negative values. This case is no longer lossless, but we see that loss is essentially introduced only in certain frequency ranges in favor of the device performance. To begin with, we show that $\sigma_{0k} = 0$ places a constraint on the phase angle relation between A_{0k} and a_1. In fact, one finds from Eq. (16) that

$$|A_{\alpha k}|^2 + |A_{-\alpha k}|^2 - 2 = 8|a_1|\sin^2\frac{\alpha\pi}{n}(|a_1| - \cos\delta). \quad (17)$$

Since the left-hand side must not be positive, one obtains $\cos\delta \geq |a_1|$. Subject to this constraint, a variable phase $\delta(\omega)$ can be chosen. However, we can show it to be equivalent to the case where $\delta' = 0$ and $a'_m = e^{i\delta}a_m$ for all m when solving for $A_{\alpha k}$'s. Note that the overall phase does not change the filter intensity profile $|a_m|^2$. Also, if δ is a constant, it must be zero since $\max(|a_1|) = 1$; hence we have $A_{0k} = \frac{a_1}{|a_1|}$ by the definition of δ.

For the second case, one can solve for $\tilde{\omega}_{\alpha c} = \omega_{\alpha c} + \Delta\omega_{\alpha c}$, $\sigma_{\alpha c}$ and $\gamma_{\alpha c}$ as functions of frequency, given the desired $a_m(\omega)$ curves. Then one can design the resonators having these characteristics. Consider the system shown in Fig. 1. Assume

$$a_1(\omega) = \frac{-i\gamma_a e^{i\phi_a}}{\omega - \omega_a + i\gamma_a}, \quad (18a)$$

$$a_2(\omega) = \frac{-i\gamma_b e^{i\phi_b}}{\omega - \omega_b + i\gamma_b}, \quad (18b)$$

$$a_0(\omega) = A_{0k} - a_1 - a_2, \quad (18c)$$

where ω_a, ω_b, γ_a, and γ_b are constants. Hereafter, the frequency range where the magnitude of a_1 is appreciable is referred to as band a. A similar reference applies to band b. The appearance of variable phase angles $\phi_a(\omega)$ and $\phi_b(\omega)$ is necessary for the continuity of the solved quantities, at the frequencies between bands a and b. We require these phases to remain constant unless the corresponding amplitudes are negligible. Therefore, they practically have no effect on the delay or other properties of the filter [9].

To solve for $\tilde{\omega}_{\alpha c}$, $\sigma_{\alpha c}$, $\gamma_{\alpha c}$, one substitutes Eqs. (11) and (18) into Eq. (12), noting that ω in Eq. (18) is just ω_k in Eq. (11). Because of the constraints discussed above (constant in-band ϕ_a, ϕ_b, non-negative $\gamma_{\alpha c}, \sigma_{\alpha c}$, and continuity), the solution is not straightforward. Certain optimization algorithms can be used. As an example, we plot one set of solutions for a system with parameters $\gamma_a = \gamma_b$, $\omega_b - \omega_a = 11\gamma_a$ in Fig. 2. However, infinite sets of $\tilde{\omega}_{\alpha c}$, $\sigma_{\alpha c}$, $\gamma_{\alpha c}$ can produce the desired filter. This gives plentiful freedom in design. Such freedom is very desirable when this theory is combined with finite difference time-domain (FDTD) simulations to design a planar light wave circuit. The larger the space of the solutions, the easier some of these solutions can be achieved with simple resonators, such as those formed by varying the diameters of the defect "atoms." A detailed investigation

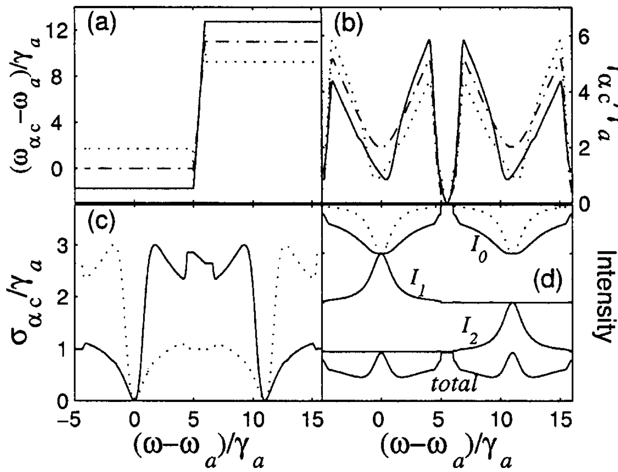

FIG. 2. The solved $\tilde{\omega}_{\alpha c}$, $\sigma_{\alpha c}$, and $\gamma_{\alpha c}$ are plotted in (a)–(c). Dash-dotted, solid, and dotted lines correspond to $\alpha = 0$, 1, and 2, respectively. Reconstructed spectra are shown in (d) on a linear scale, where I_0' (dotted line) is a reference lossless spectrum. Each intensity reaches maximum 1 and minimum 0, except the total intensity.

of the space of the solutions is beyond the scope of this Letter. To obtain the set of solutions in Fig. 2, we have applied the additional constraint that $\tilde{\omega}_{\alpha c}$'s are constant when the magnitudes of the filter transfer functions are appreciable, and we connect $\tilde{\omega}_{\alpha c}$'s of different bands using linear interpolation (other smooth interpolations are possible). In Fig. 2(d), the intensity spectrum $I_m = |a_m|^2$ for each port is reconstructed from the solutions presented in Figs. 2(a)–2(c). One sees 100% drops occur at ω_a and ω_b. Assume that ω_a and ω_b differ by 0.8 nm (centered at 1.55 μm), then the 0.5 and 30 dB bandwidths are 0.05 and 0.93 nm, respectively. The sum of the three spectra shows prominent loss. And the loss penalizes only the pass-through port in the bands of dropped wavelengths, as indicated by a reference lossless pass-through spectrum I_0'. The isolation at the 0.5 dB edges of each passband is enhanced from -9.6 (for I_0') to -25 (for I_0) dB. For an FDTD-aided design, one may start with designing resonators that have spectra closest to the set of exact solutions, then designs the whole system. Without the first step, the algorithm may have no clue to search a big design space and may never converge. The lossless constraint should not be applied across the spectrum, as discussed. Many techniques in traditional filter design [9] can facilitate the OADM design.

Higher order Butterworth functions are highly desirable for an add-drop filter [10]. Our theory is also applied to an $n = 3$ OADM having a third-order Butterworth intensity profile $\frac{\gamma_a^6}{(\omega-\omega_a)^6+\gamma_a^6}$. Figure 3 presents the solved spectra of the two relevant ports in one band, along with a reference lossless spectrum for the pass-through port. Again, the remnant light in the pass-through port is desirably reduced in the dropped frequency ranges.

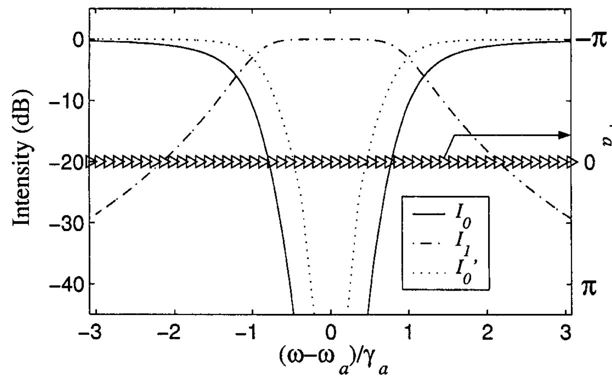

FIG. 3. Reconstructed spectra for the third-order Butterworth case, $\phi_a \equiv 0$ for appreciable $|a_1|$.

Also seen is the obvious flat-top line shape of the drop port compared to the first-order case. The 0.5 to 30 dB bandwidth ratio now increases to 0.22, indicating a much sharper transition between the passband and the stop band.

The crosstalks and losses at the crossings of the waveguides should not be a concern for conventional [11] or PC-based [12] waveguides after optimizing the crossings.

In summary, we have proposed and analyzed a class of waveguide-resonator structures that can be used as multichannel OADMs. Light is shown to completely transfer between different waveguides. Some desirable features of the optical spectra of these structures are also presented.

This work is supported in part by NSF Grant No. DMR-0103134. One of the authors (W. J.) is grateful to Dr. H. Berk's encouragement in the early stage.

[1] S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and H. A. Haus, Phys. Rev. Lett. **80**, 960 (1998).
[2] A. Chutinan, M. Mochizuki, M. Imada, and S. Noda, Appl. Phys. Lett. **79**, 2690 (2001).
[3] A. Sharkawy, S. Shi, and D.W. Prather, Appl. Opt. **40**, 2247 (2001); K. Takano and K. Nakagawa, IEICE Trans. Commun. **E84-B**, 1295 (2001).
[4] Y. Xu, Y. Li, R. K. Lee, and A. Yariv, Phys. Rev. E **62**, 7389 (2000).
[5] M. Hamermesh, *Group Theory and Its Application to Physical Problems* (Addison-Wesley, Reading, 1962).
[6] J. J. Sakurai, *Modern Quantum Mechanics* (Addison-Wesley, Redwood City, CA, 1985).
[7] S. Fan *et al.*, Phys. Rev. B **64**, 245302 (2001).
[8] S. John, Phys. Today **44**, No. 5, 32 (1991).
[9] A.V. Oppenheim and R.W. Schafer, *Discrete-Time Signal Processing* (Prentice Hall, Upper Saddle River, 1999), 2nd ed.
[10] S. Fan *et al.*, Phys. Rev. B **59**, 15 882 (1999); M. J. Khan *et al.*, IEEE J. Quantum Electron. **35**, 1451 (1999).
[11] M. G. Daly, P. E. Jessop, and D. Yevick, J. Lightwave Technol. **14**, 1695 (1996).
[12] S. G. Johnson *et al.*, Opt. Lett. **23**, 1855 (1998).