Cooper Pair Cotunneling in Single Charge Transistors with Dissipative Electromagnetic Environment

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We observed current-voltage characteristics of superconducting single charge transistors with on-chip resistors of $R \sim R_Q = h/4e^2 \approx 6.45$ kΩ, which are explained in terms of Cooper pair cotunneling. Both the effective strength of Josephson coupling and the cotunneling current are modulated by the gate-induced charge on the transistor island. For increasing values of the resistance R we found the Cooper pair current at small transport voltages to be dramatically suppressed.

During the last decade superconducting circuits with mesoscopic tunnel junctions have attracted much attention of researchers. Remarkable quantum properties of these circuits, which stem from the interaction between Josephson and charging effects, enable the construction of charge-based qubits [1], sensitive electrometers [2], amplifiers [3], etc. Since the Josephson supercurrent itself is of an elastic nature, tunneling of Cooper pairs in such circuits is dramatically influenced by the Coulomb interaction and dissipation.

The tunneling in the two-junction system (single Cooper pair transistor) with a dissipative environment was recently theoretically studied by Wilhelm *et al.* [4] and experimentally investigated by Kycia *et al.* [5] and Lu *et al.* [6] who used the advantage of locally tunable dissipation associated with the conductance of two dimensional electron gas beneath the transistor structure. These papers were focused on the regime of sequential tunneling of Cooper pairs (STCP) dominating when the electric potential of the transistor island was tuned "in resonance" (see the parameter diagram in Fig. 3 of Ref. [4]). Although a quantum regime of simultaneous pair tunneling across both junctions, i.e., cotunneling of Cooper pairs (CTCP), was roughly evaluated in Ref. [4], it had not been, to our knowledge, studied experimentally.

Under the term "cotunneling" we understand discrete tunneling of *single* Cooper pairs across a whole transistor [7]. Although numerous experiments show that Cooper pair transistors without deliberately attached resistors exhibit either a gate-dependent switching current at ramp-current bias or a current peak at low voltage bias (see, e.g., Ref. [8]), these data are merely related to the supercurrent. Because of unavoidable dissipation leading to fluctuations of the overall Josephson phase ϕ, the observable value of the supercurrent is nonzero at finite transport voltage. Phase ϕ is still a good variable, while the number of transferred Cooper pairs is meaningless, so the term "single" is not applicable to this regime.

When an appreciable impedance is introduced into the network, i.e., $\mathrm{Re} Z = R \sim R_Q = h/4e^2 \approx 6.45$ kΩ, quantum fluctuations of phase ϕ increase dramatically, and this leads to incoherent tunneling of individual pairs across the transistor, i.e., CTCP. In this Letter we investigate the CTCP effect in Al transistors with thin-film Cr local resistors of different values R inserted into the bias leads. We observe the gate and bias voltage dependencies of CTCP current and demonstrate that a significant suppression of CTCP at small bias voltage can be achieved by a further increase in the resistance R.

The Cooper pair transistors were fabricated using the angle evaporation technique for three metal shadows [Cr/Al-1(oxidized)/Al-2] evaporated at different angles through the same mask. Details of the fabrication process can be found elsewhere [9]. We put on the same chip a number of nominally identical symmetrical transistors (with tunnel junctions of type Al/AlO$_x$/Al with dimensions 40 nm $\times$ 80 nm) differing only in the lengths of the Cr microstrips (each 0.1 μm wide and 7 nm thick), ranging from 0.32 to 3 μm. The specific film resistivity was from 5.5 to 7 kΩ per μm of length yielding values of resistance R in the range of 2–20 kΩ.

The measurements were performed in the dilution refrigerator at the base temperature of about 20 mK. The sample was mounted in a copper shielding box. All bias and gate lines were supplied with microwave filters made of Thermocoax cable located in the coldest part of the setup. The electric diagram of the sample bias is shown in the upper inset in Fig. 1. The normal-state tunnel resistance of the transistors, $R_{\mathrm{tr}} = R_1 + R_2$, where $R_{1,2}$ are tunnel resistances of individual junctions, was remarkably uniform and ranged from 40 to 44 kΩ. The superconducting gap and the corresponding values of the Ambegaokar-Baratoff critical current of the junctions (assuming symmetry of the transistors) were estimated to be $\Delta_{\mathrm{Al}} \approx 200$ μeV and $I_c^0 \approx 16$ nA, respectively. These data yield a characteristic Josephson energy of junctions $E_{J1,J2} = E_J \equiv \frac{\Phi_0}{2\pi} I_c^0 \approx 30$ μeV, where $\Phi_0 = \frac{h}{2e}$ is the flux quantum. From the normal-state I-V curves we evaluated a characteristic charging energy of the transistor island $E_c \equiv e^2/2C_\Sigma \approx 150$–160 μeV, where $C_\Sigma = C_1 + C_2 + C_g \approx 530$ aF (typical value) is the total capacitance of the island and $C_{1,2} = C \approx 260$ aF and
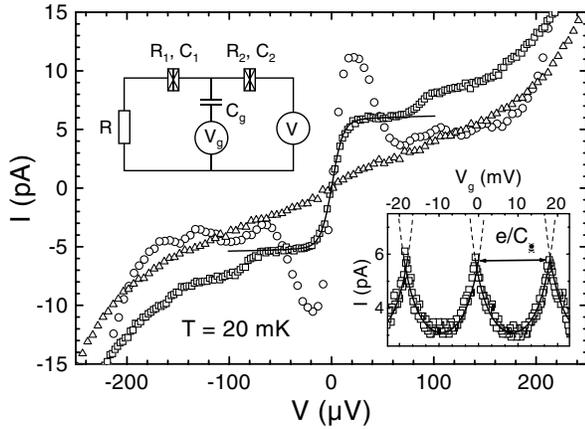
FIG. 1. The electric circuit diagram (upper inset) and *I-V* curves of three Cooper pair transistors with different nominal values of the resistance *R* (circles: 0, squares: 3 kΩ, and triangles: 5 kΩ) at gate voltage values V_g corresponding to maximum of current. The solid line shows computed data assuming a symmetric transistor with $E_J = 26$ μeV and a resistor of $R = 3.3$ kΩ with an effective temperature of 90 mK. The gate dependence in the transistor with $R = 3$ kΩ at $V = 30$ μV is presented in the bottom inset, where the solid (dashed) line shows the data calculated within an experimentally accessible (inaccessible) range.

$C_g \approx 9$ aF $\ll C$ are the capacitances of junctions and gate, respectively. Because of the small C_g, the gate-induced charge is fixed, $Q_0 = C_g V_g + Q_{00}$, where Q_{00} is the background charge. The reasonably small ratio of characteristic energies in our samples, $\lambda_0 = E_J/E_c \approx$ 0.2, allowed us both clear observation and transparent interpretation of the CTCP effect ($\propto \lambda_0^4$). Moreover, linearity and small resistor lengths let us consider them as lumped elements (pure resistors) and neglect in our model their distributed self-capacitance (≈ 60 aF/μm).

I-V characteristics of superconducting single charge transistors are generally featured by several contributions to the net current. These contributions result from the tunneling of Cooper pairs, single quasiparticles, and combining tunneling of both pairs and quasiparticles. At small transport voltage and temperature ($eV, k_B T \ll 2E_c, 2\Delta_{\text{Al}}$), the tunneling of quasiparticles cannot make a notable contribution to the net current because of the exponentially large subgap resistance. In this limit, the Cooper pair (Josephson) current component is dominant. Still, the rare events of quasiparticle tunneling (poisoning of the island) at $|Q_0| \geq e/2$ were possible, and they resulted in 1e-periodic gate dependence of the *I-V* curves (see the bottom inset in Fig. 1). Therefore, slow ramping of the gate voltage (when measuring gate-modulation curves) provided cyclic variations of the polarization charge on the island Q_0 in the range from $-e/2$ to $e/2$. Within this range of Q_0, a change of the island charge by $2e$ would require an energy δE_{ch} ranging between $2E_c$ (at $Q_0 = \pm e/2$) and $4E_c$ (at $Q_0 = 0$). Therefore,

the process of Cooper pair transfer with "landing" on the island, i.e., the STCP, was exponentially suppressed, $\propto \exp(-\delta E_{\text{ch}}/k_B T)$, so only the CTCP contributed to the net current.

Figure 1 shows the *I-V* curves of three samples with resistance *R* of nominally zero value (the actual impedance in this case is of the order of the vacuum impedance $Z_0 \approx 377$ Ω, although its frequency dependence is unknown), 3 and 5 kΩ. One can see a remarkable change from the peak (cf. Fig. 1 in Ref. [8]) to steplike and then to an almost linear shape of the *I-V* curve that can be interpreted as a growth of fluctuations of the total Josephson phase leading to a depression of the supercurrent feature.

A similar behavior of a single junction was described by Averin *et al.* [10] in terms of incoherent tunneling of pairs yielding current $I = 2e[\Gamma(2eV) - \Gamma(-2eV)]$ and a tunneling rate

$$\Gamma(E) = \frac{\pi}{2\hbar} E_J^2 P(E), \qquad (1)$$

where the function $P(E)$ with $E = 2eV$ describes the probability of energy dissipation in the environment network. The experimental observation of this regime was reported by Kuzmin *et al.* [11].

In the case of a transistor, one may consider E_J in Eq. (1) as an effective Josephson coupling dependent on the gate voltage V_g. To support this idea, we observed the shapes of the *I-V* curves similar to those given by the *P* theory for single junctions with Ohmic environment [10,12]. Moreover, the observed shapes were immutable to V_g whose variation led only to scaling of the current.

To model the behavior of our *I-V* curves, we present the Josephson coupling term of the transistor Hamiltonian, $H_J = -E_{J1} \cos\varphi_1 - E_{J2} \cos\varphi_2$, in the form

$$H_J(\phi, \varphi) = -E_J^{\text{tr}}(\phi) \cos[\varphi + \gamma(\phi)], \qquad (2)$$

where $E_J^{\text{tr}} = (E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\phi)^{1/2}$, $\tan\gamma = [(E_{J1} - E_{J2})/(E_{J1} + E_{J2})]\tan(\phi/2)$, and $\phi = \varphi_1 + \varphi_2$ (φ_i is the phase drop across the *i*th junction). The difference phase $\varphi = \frac{1}{2}(\varphi_1 - \varphi_2)$ is a variable conjugate to the island charge $Q = -2ei\frac{\partial}{\partial \varphi}$. Q enters the charging energy term $H_{\text{ch}}(Q) = E_c[(Q + Q_0)/e]^2$. For a given value of operator ϕ, the transistor Hamiltonian $H_{\text{tr}} = H_{\text{ch}}(Q) + H_J(\phi, \varphi)$ is similar to that of a single junction. Taking the fact into account that the commutator $[\phi, H_{\text{tr}}] = 0$, we can diagonalize $H_{\text{tr}}(\phi, \varphi, Q) = H_{\text{tr}}(\phi, q, n)$ in the basis of the Bloch states $|q, n\rangle$, where q is the quasicharge and $n = 0, 1, 2, \ldots$ is the band index [13]. Assuming that only the ground state, $n = 0$, is occupied (i.e., the effects of thermal, photon-assisted and Zener breakthrough excitations of higher bands are negligibly small) and noting that variable q is decoupled from environmental degrees of freedom due to small C_g and fixed by V_g, we present the transistor Hamiltonian as

$$H_{\mathrm{tr}} = E_0[\lambda(\phi), Q_0]. \tag{3}$$

Here E_0 is the ground state energy of an equivalent single junction with variable ratio $\lambda = E_J^{\mathrm{tr}}(\phi)/E_c \le 2\lambda_0$ and $q = Q_0$. E_0 is a 2π-periodic function of ϕ and a $2e$-periodic function of charge Q_0, and its shape depends on the relation between $E_{J1,J2}$ and E_c.

Because of the periodic dependence on ϕ, the Hamiltonian Eq. (3) can be expanded into a Fourier series

$$H_{\mathrm{tr}}(\phi, Q_0) = -\sum_{k=0}^{\infty} E_J^{(k)}(Q_0)\cos(k\phi). \tag{4}$$

For $Q_0 \ne e\,\mathrm{mod}(2e)$ or/and at notable asymmetry of the transistor, $E_{J1} \ne E_{J2}$, the terms with $k = 0$ and 1 are dominating in Eq. (4). This is in accordance with the almost sinusoidal supercurrent-phase relation [14] with effective critical current $I_c^{\mathrm{tr}}(Q_0) = \frac{2\pi}{\Phi_0} E_J^{(1)}(Q_0)$ modulated by gate [see corresponding plots for different values of λ_0 in Fig. 2(a) of Ref. [15]]. In particular, in the symmetric transistor with $\lambda_0 \ll 1$, I_c^{tr} can be found explicitly from Eqs. (4) and (5) of Ref. [15] using the wave functions given by Eq. (A11) of Ref. [13]; i.e.,

$$I_c^{\mathrm{tr}}(Q_0) = \frac{\lambda_0}{4}[1 - (Q_0/e)^2]^{-1} I_c^0. \tag{5}$$

In the degenerate case of $Q_0 = e\,\mathrm{mod}(2e)$ and $E_{J1} = E_{J2} = E_J \lesssim E_c$, the ground state energy Eq. (3) is $E_0 \approx -E_J|\cos\frac{\phi}{2}| + \mathrm{const}$, while the maximum critical current is $I_c^{\mathrm{tr}}(e) \approx 0.5 I_{c1,c2}$. The Fourier coefficients in Eq. (4) are equal to

$$E_J^{(k)}(e) = \frac{4}{\pi}\frac{(-1)^{k-1}}{(2k)^2 - 1} E_J, \qquad k \ge 1, \tag{6}$$

so the amplitudes of the principal (first harmonic) term $E_J^{(1)}(e) \approx 0.42 E_J$ and the ratio $|E_J^{(2)}(e)/E_J^{(1)}(e)| = 0.2$.

The expansion Eq. (4) presents the transistor Hamiltonian as a weighted sum of operators $e^{\pm ik\phi}$, each describing tunneling of a cluster of k Cooper pairs through the transistor in the positive (negative) direction. This presentation enables particular rates for tunneling of k pairs, $\Gamma_k(E)$, to be found and the total current to be calculated via the corresponding sum.

Because of finite impedance R, the phase ϕ couples to the environmental degrees of freedom via an operator of phase on the resistor ϕ_R: $\phi = \frac{2\pi}{\Phi_0} Vt - \phi_R$. Phase ϕ_R is related to the infinite set of bath variables. The effect of this coupling can be treated by means of function P determined by the charge of the tunneling particle (cluster of particles), temperature, and the real part of total impedance Z as seen by the tunneling charge [16].

In our case, the real part of the impedance is equal to

$$\mathrm{Re}Z(\omega) = R/[1 + (\omega R C_{\mathrm{tr}})^2], \tag{7}$$

where the transistor capacitance $C_{\mathrm{tr}} = \frac{C_1 C_2}{C_1 + C_2} \approx \frac{C}{2}$ (corre-

sponding to the characteristic charging energy of $4E_c$) and the tunneling charge is equal to $2ke$. The rates, $\Gamma_k(E) = \frac{\pi}{2\hbar}[E_J^{(k)}]^2 P(E)$, and the total cotunneling current,

$$I_{\mathrm{cot}} = 2e\sum_{k=1}^{\infty} k[\Gamma_k(2\ \mathrm{keV}) - \Gamma_k(-2\ \mathrm{keV})], \tag{8}$$

are found perturbatively, assuming $0.5 E_J/4E_c = \lambda_0/8 \ll 1$. If Q_0 is not close to e, the term with $k = 1$ is essential, and the cotunneling current is given by the formula similar to that derived by Ingold et al. [12] for tunneling across a single junction at $2eV \ll \hbar/RC$,

$$I_{\mathrm{cot}} = \frac{\Phi_0[I_c^{\mathrm{tr}}(Q_0)]^2}{16 E_c} e^{-2\gamma\rho}\rho^{2\rho}\left(\frac{2E_c}{\pi^2 k_B T}\right)^{1-2\rho}$$
$$\times \frac{|\Gamma[\rho - j(eV/\pi k_B T)]|^2}{\Gamma(2\rho)}\sinh\left(\frac{eV}{k_B T}\right), \tag{9}$$

where $\rho = R/R_Q$, Γ is the gamma function, and $\gamma \approx 0.577$ is Euler's constant. The perturbation theory result, Eqs. (8) and (9), is valid if $I_{\mathrm{cot}} \ll \rho I_c^{\mathrm{tr}}$. The latter condition means that the cotunneling events are rare and incoherent, and this is nicely obeyed in our experiment.

Expression (9) pretty well gives the transport and gate voltage dependencies as well as the magnitude of the measured current (see the calculated curves in Fig. 1). The values of the fitting parameters R and E_J are in good agreement with their estimated values (10% and 13% deviation, respectively), while the effective temperature T_{eff} of the resistor was found to be 90 mK (cf. bath temperature of 20 mK). This value matches with $T_{\mathrm{eff}} = 70$–80 mK found in the same setup in measurements of the three-junction Al-Cr single electron pump with local resistors [17] and can be ascribed mostly to a noise effect.

We achieved the CTCP regime in the range of the gate charge $|Q_0| \le \frac{e}{2}$. The gate-modulation depth of the cotunneling current was found to be $d \equiv I_{\mathrm{cot}}(\frac{e}{2})/I_{\mathrm{cot}}(0) \approx 2$, that is in agreement with the value given by Eq. (5); i.e., $d = [I_c^{\mathrm{tr}}(\frac{e}{2})/I_c^{\mathrm{tr}}(0)]^2 = \frac{16}{9} \approx 1.78$ [18]. Approaching the points $Q_0 = \pm e$ one could expect an increase in the cotunneling current by a factor of $(2/\lambda_0)^2 \approx 10^2$, but in these points STCP dominates because effective dissipation for tunneling across either junction is notably smaller than that for CTCP, $\rho \to \rho/4$ [4]. As a manifestation of such behavior in our samples, we observed a current increase by more than 2 orders of magnitude when we applied a fast (~ 1 MHz) gate voltage sweep of amplitude $> e/C_g$. Such a sweep allowed quasiparticle poisoning of the island to be avoided for short time intervals when passing the regions $Q_0 \approx \pm e$.

Finally, an issue of particular interest is the suppression of CTCP at larger dissipation, i.e., at $R > R_Q$ [19]. The increase in R should lead to a blockade behavior of the I-V curves at small V and development of a broad current peak moving to $V = \frac{2e}{C}$. The latter feature can hardly be clearly observed in experiment because at such a large
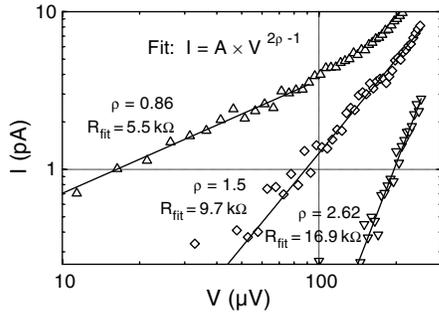
FIG. 2.   The *I-V* curves of three Cooper pair transistors with different values of resistance R (up triangles: 5 kΩ, diamonds: 9 kΩ, and down triangles: 17 kΩ; cf. the fit values R_{fit}) and their power-function fit in accordance with Eq. (10). The fitted values of prefactor $A = I_0 V_0^{1-2\rho}$ were used to restore the values of E_J ($= 33, 34$, and 28 μeV, respectively).

voltage other current components (quasiparticle and combined pair-quasiparticle tunneling) are significant [11]. On the other hand, for small voltages between $k_B T/2e$ and e/C, cotunneling still dominates and for its estimation one can apply the zero temperature expansion of function P [16] and will get

$$I_{\text{cot}} = I_0 \left(\frac{V}{V_0}\right)^{2\rho-1}, \quad \text{where } I_0 = \frac{\pi \rho e^{-2\gamma} \Phi_0 (I_c^{\text{tr}})^2}{32\Gamma(2\rho)E_c} \quad (10)$$

and $V_0 = \frac{2e}{\pi \rho C}$. Thus, the CTCP current obeys the power law; i.e., it is efficiently suppressed at large ρ. This effect was demonstrated and the results are presented in Fig. 2. Although the experimental voltage range was rather limited, the data are in good agreement with the dependence Eq. (10) for parameter values close to those estimated from measurements.

In conclusion, the obtained results on measurements of CTCP are also of practical importance, e.g., for improving the performance characteristics of the Cooper pair pumps. In the latter devices, CTCP may significantly degrade the accuracy of charge pumping [20]. In contrast to the normal junction case, an increase in the number N of junctions in the superconducting pump array seems to be not suitable for suppressing unwanted CTCP [21]. On the other hand, the use of local resistors [17,22] ensures such suppression and, therefore, is promising for the construction of an accurate Cooper pair pump with a reasonable number ($N = 3$–5) of junctions in the array.

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