Revisiting the Hanbury Brown–Twiss Setup for Fractional Statistics

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The Hanbury Brown–Twiss experiment has proved to be an effective means of probing statistics of particles. Here, in a setup involving edge-state quasiparticles in a fractional quantum Hall system, we show that a variant of the experiment composed of two sources and two sinks can be used to unearth fractional statistics. We find a clearcut signature of the statistics in the equal-time current-current correlation function for quasiparticle currents emerging from the two sources and collected at the sinks.

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The statistics of indistinguishable particles is manifested in the fate of the common wave function for two particles located at positions r_1 and r_2 under exchange:

$$\Psi(r_1, r_2, r_3, \ldots, r_n) = e^{\pm i\pi\nu}\Psi(r_2, r_1, r_3, \ldots, r_n), \quad (1)$$

where, in general, other particles may be present at positions r_i, $i \neq 1, 2$. The values $\nu = 2n$ and $\nu = 2n + 1$ for integers n correspond to the familiar instances of bosonic and fermionic statistics, respectively. In two dimensions, where the concept of exchange can be unambiguously defined, ν can assume fractional values corresponding to anyonic statistics. A landmark example of this phenomenon occurs for Laughlin states [1] in the fractional quantum Hall (FQH) setup. In this system, the anyonic nature of quasiparticle/quasihole excitations has been demonstrated [2] and, in particular, the gaining of the phase factor $e^{\pm i\pi\nu}$ by quasiparticles [3] and quasiholes [4–6] under exchange, where ν is the filling fraction. Of late, a variety of novel proposals for testing the statistics of edge-state quasiparticles in Laughlin states have come forth [7,8].

In this Letter, we propose a setup consisting of two edge-state quasiparticle sources and two sinks, and the measurement of current-current correlation for currents emerging at the two sources and collected at the sinks. At equal times, the correlator is found to depend only on average values of currents and a factor $\cos\pi\nu$ coming from statistics. As a function of time difference for when currents are correlated, it shows oscillations with a period that depends on the fractional charge of the quasiparticle.

Returning to the common wave function of Eq. (1), one can extract from it various properties, such as filling in of available states, and N-point correlation functions. A quantity sensitive to statistics is the two-particle correlation function

$$g(r_1, r_2) = N(N-1)\int dr_3 \ldots dr_n |\Psi(r_1, r_2 \ldots r_n)|^2, \quad (2)$$

where N is the total number of particles. In systems composed of a single species of noninteracting particles, for fermions $g(r_1, r_2)$ necessarily drops to zero at $r_1 = r_2$ and typically levels off to the uncorrelated value for $r_1 - r_2$ much greater than the mean particle spacing. For uncondensed bosons, wave function symmetrization allows $g(r_1, r_2)$ to reach twice its uncorrelated value. Our purpose here is to study processes that enable the probing of anyonic systems for *their* statistical information. Specifically, two-particle correlations are manifested in events such as the ones shown in Fig. 1, where particles need not scatter, but may merely be detected within a correlation region in time and space to feel the effect of statistics [9].

In fact, in the 1950's, Hanbury Brown and Twiss performed both astronomical and tabletop experiments to measure correlations in light intensities at two detectors, which strikingly reflected bosonic statistics [9,10]. More recently, laboratory experiments measuring analogous correlations in semiconductors and in free space have brought out the fermionic statistics of electrons [11]. Turning our attention to Laughlin states, as anyons are known to exist only within the strongly correlated quantum Hall fluid, simplistic measurements requiring particles to be detected outside the system would fail. Moreover, manipulations of quasiparticles within the bulk are not feasible at present. However, it has been suggested [12] and experimentally ascertained for $\nu = 1/3$ states [13] that weak tunneling of Laughlin quasiparticles between edge states of a single Hall bar produces shot noise characteristic of particles with quantized charge νe. Given the evidence for fractionally charged quasiparticles, we propose a tunneling geometry that

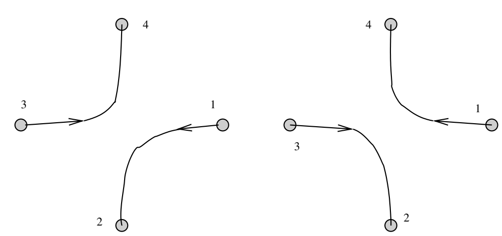

FIG. 1. Indistinguishable particles from sources 1 and 3 can reach sinks 2 and 4 by two possible processes whose probability amplitudes differ by a phase factor that depends on the statistics of the particles.

realizes the events depicted in Fig. 1 for these particles. We show that appropriate current-current correlation measurements in this geometry bring out the fractional statistics of the quasiparticles.

Our proposed setup is as shown in Fig. 2. Four leads at the corners of the Hall bar define four edge states denoted by $\beta = A, B, C, D$. Low-energy excitations of the FQH system correspond to long-wavelength density distortions of the edge. These excitations can be described by the chiral Luttinger liquid model [5,6,14], and thus each of the edge states is characterized by the Hamiltonian

$$H_0^\beta = \frac{1}{4\pi\nu}\int (\partial_x \phi_\beta)^2 dx_\beta, \tag{3}$$

where the bosonic fields ϕ obey the commutation relations $[\phi_\beta(x), \phi_\gamma(x')] = i\pi\nu \mathrm{sgn}(x - x')\delta_{\beta\gamma}$, and their gradients are proportional to density distortions. Here we have set the edge-state velocity to unity. Gates allow for pinching the edge states close to one another [13] to form the cruciform pattern shown in Fig. 2, thus enabling interedge quasiparticle tunneling. For each edge state β, we assume the tunneling to take place from points x_j, where $j = 1, 2, 3, 4$ for $\beta = A, B, C, D$, respectively. Here we require that the region formed by the tunneling points be comparable to the size of the quasiparticles. Unlike in the bulk, a second-quantized description of edge-state quasiparticles is relatively straightforward to formulate. We describe particles at the tunneling points by the creation operators $\psi_j^\dagger = \kappa_j e^{-i\phi_\beta(x_j)}$, where κ denotes Klein factors. The commutation relations for the bosonic fields of Eq. (3) ensure that these quasiparticles, when exchanged with others residing on the same edge, exhibit the statistics of Eq. (1). The Klein factors ensure that they do so when exchanged with particles from neighboring edge states. We pick the convention

$$\psi_j^\dagger \psi_k^\dagger = e^{-i\pi\nu}\psi_k^\dagger \psi_j^\dagger, \tag{4}$$

where $j < k$ for $j = 1, 2, 3$ and $k = 1$ for $j = 4$. Tunneling

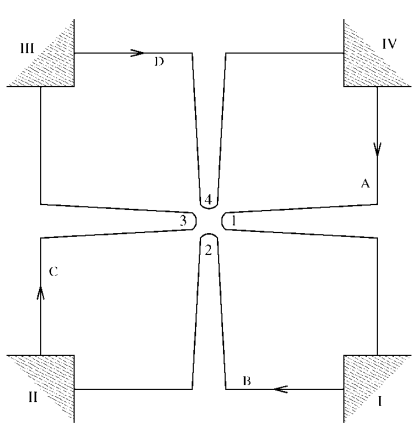

FIG. 2. Setup of Hall bar for measuring two-particle processes. Leads I-IV define edge states A–D. Pinching these edge states together allows for interedge quasiparticle tunneling at points 1–4.

of these quasiparticles to neighboring edge states can be controlled by means of gate voltages. It is described by the tunneling Hamiltonian

$$\mathcal{H}_{jk} = u_{jk}\psi_j^\dagger \psi_k + \text{H.c.}, \tag{5}$$

where H.c. denotes Hermitian conjugation, ψ's are as in Eq. (4), and the u's denote tunable bare tunneling strengths. As a variant of the Hanbury Brown–Twiss experiment, we select the points $m = 1, 3$ as quasiparticle sources and $n = 2, 4$ as sinks by raising the potentials of the edge states A and C with respect to B and D by a voltage V. As all tunneling occurs in a fixed geometry, two-particle correlation functions cannot be studied as a function of spatial separation. However, current-current correlations can be measured as a function of temporal separation. These tunneling currents take the form

$$I_{mn}(t) = \frac{ie^*}{\hbar}(u_{mn}\psi_m^\dagger \psi_n e^{i\tilde{V}t} - \text{H.c.}), \tag{6}$$

where $e^* = \nu e$ is the charge of the quasiparticle [1,6,12], and $\tilde{V} = e^* V/\hbar$. Edge states then carry measurable currents

$$\begin{aligned} I_{\mathrm{I}} &= \frac{\nu e^2}{h}V - I_{12} - I_{14}; \qquad I_{\mathrm{II}} = I_{12} + I_{32}; \\ I_{\mathrm{III}} &= \frac{\nu e^2}{h}V - I_{32} - I_{34}; \qquad I_{\mathrm{IV}} = I_{34} + I_{14}, \end{aligned} \tag{7}$$

where in Fig. 2, I_α are currents going into leads α.

The finite-temperature average values of these currents can be calculated using nonequilibrium Keldysh techniques that treat tunneling perturbatively (see, e.g., Ref. [12]). To summarize the treatment, from Eq. (3), we derive an action for each of the four edge states. Away from the tunneling points x_j, the edge states are described by free fields, whose form is explicitly obtained in terms of the fields at the tunneling points using equations of motion. These free fields are integrated out to obtain an effective action described by fields ϕ_j at points x_j. We then introduce a generating functional in terms of backwards and forwards real-time paths $\phi_j^\pm(t)$, which enables us to obtain expectation values. In equilibrium, the correlation function $C_j \equiv \langle \varphi_j \varphi_j \rangle$ and response function $R_j \equiv \langle \tilde{\varphi}_j \varphi_j \rangle$, where $\phi^\pm \equiv \varphi \pm \frac{1}{2}\tilde{\varphi}$, satisfy the fluctuation-dissipation theorem $C_j(\omega) = \coth(\hbar\omega/2kT)R_j(\omega)$. The effect of tunneling is treated perturbatively.

To second order in tunneling, the technique described above gives the following form for the average currents:

$$\langle I_{mn}(t)\rangle = \nu e\left(\frac{2u_{mn}}{\hbar}\right)^2 \int dt_a \times \sin\tilde{V}(t_a - t) \times \sin F(t_a - t)e^{-f(t_a - t)}, \tag{8}$$

where $F(t) \equiv \nu \int d\omega \omega^{-1} \sin\omega t$ and $f(t) \equiv 2\nu \int d\omega \omega^{-1} \times \coth(\hbar\omega/2kT)(\sin\omega t/2)^2$ originate from response and correlation functions, respectively. Evaluating the above gives for the differential conductance contributions, $dI_{mn}/dV = V^{2\nu-2}\mathcal{G}(e^*V/kT)$, $\mathcal{G}(x \to \infty) \to$ const, which increases with decreasing voltage. Thus, at low temperatures, which is desirable for keeping thermal noise minimal, the voltage V must be held large compared to the bare tunneling strength for the perturbative treatment to remain valid.

We now analyze current correlations of particles emerging from the two sources detected at the two sinks. In principle, a variety of current-current correlators contain information on statistics. As was originally shown by Hanbury Brown and Twiss, even particles from a single source can be distributed into two detectors to exhibit statistical correlations [9,10]. Thus, focusing on one set of source-drain edge states and measuring correlations between the transmitted and reflected currents [15] along the source edge tate, as was done for the integer quantum Hall system [11], can give statistical information. Alternatively, the current-current correlations between currents from one source collected at two different drains can be calculated [15], as has been done explicitly for the FQH setup [8]. Here, we find that a clearcut signature of anyonic statistics comes from events shown in Fig. 1. To extract information on statistics from these events, we propose the measurement of the following time-translation invariant current-current correlator:

$$C(t-t') \equiv \langle \Delta I_{12}(t)\Delta I_{34}(t') + \Delta I_{14}(t)\Delta I_{32}(t')\rangle, \quad (9)$$

where $\Delta I \equiv I - \langle I\rangle$.

The correlator C can be obtained from three sets of measurements. The first would measure correlations $\langle \Delta I_{\rm II}(t)\Delta I_{\rm IV}(t')\rangle$ for currents $I_{\rm II}$ and $I_{\rm IV}$ measured $\Delta t = t - t'$ apart. The other two sets of measurements would be performed in the absence of sources 1 and 3, respectively, realized by controlling the appropriate tunneling strengths u_{mn} by means of gate voltages. It is important to note that these other sets of measurements do not require a change in sample, but can be achieved merely by applying the required gate voltages in a single sample. In each of these instances, currents into leads II and IV would have the form $\tilde{I}_{\rm II} = I_{m2}$ and $\tilde{I}_{\rm IV} = I_{m4}$ with $m = 3$ and $m = 1$, respectively. Then one could measure cross orrelations $\tilde{C}$, for current from one source held at the same potential V as in the first case, into two drains, where $\tilde{C}_m(\Delta t) = \langle \Delta I_{m2}(t)\Delta I_{m4}(t')\rangle$, with $m = 3, 1$, respectively, and $\Delta t = t - t'$. These correlations themselves carry statistical information, but are complicated by the fact that the source edge states are endowed with their own dynamics. Nevertheless, as seen in Ref. [8], one can procure valuable information from them similar to that contained in our sought-after correlator $C(t-t')$ of Eq. (9). This correlator C can now be obtained by subtracting the contributions of the latter two measurements from the first:

$$C(\Delta t) = \langle \Delta I_{\rm II}(t)\Delta I_{\rm IV}(t')\rangle - \tilde{C}_1(\Delta t) - \tilde{C}_3(\Delta t). \quad (10)$$

In fact, Ref. [15] proposes completely analogous sets of measurements in a similar four point tunneling setup in the integer quantum Hall system, and there too, a correlator analogous to C provides key information on statistics.

The correlation can be evaluated in the perturbative Keldysh approach outlined above. To lowest nonvanishing order, i.e., fourth order in tunneling, it takes the form

$$C(\Delta t) = \langle I_{12}(t)\rangle\langle I_{34}(t')\rangle + \langle I_{14}(t)\rangle\langle I_{32}(t')\rangle + C_\diamond(\Delta t). \quad (11)$$

The function $C_\diamond(\Delta t)$ is the piece in the perturbation that connects all points 1–4 of Fig. 2 and thus contains information on the statistics. Explicitly, it is given by

$$C_\diamond(\Delta t) = \cos\pi\nu (e^*)^2 \prod_{m,n} \frac{2u_{mn}}{\hbar}\int dt_a dt_b \times [\cos\tilde{V}(t + t' - t_a - t_b) \times \cos\tilde{F} e^{-\tilde{f}}], \quad (12)$$

where $m = 1, 3$ and $n = 2, 4$. Here $\tilde{F} \equiv 1/2\sum_{j=a,b} \times [F(t-t_j) + F(t'-t_j)]$ and similarly $\tilde{f}$ involve the functions F and f, which appear in Eq. (8). When the time difference Δt is small, i.e., $\hbar/\Delta t \gg kT, e^*V$, one expects the correlations to be maximal [9]. In fact, in this limit and for uniform scattering $u_{mn} = u$ (which we assume from here on), upon evaluating Eq. (12), the current correlation defined in Eq. (9) reduces to the simple and suggestive form

$$C(\Delta t \to 0) = 2[1 + \cos\pi\nu]\langle I_{12}\rangle\langle I_{34}\rangle, \quad (13)$$

where the behavior of the average currents $\langle I\rangle$ is given in Eq. (8). This is consistent with the fermionic limit $C(0) = 0$, $\nu = 1$, which reflects the fact that two electrons cannot be in the same place simultaneously, and the bosonic limit of maximal "bunching" for $\nu = 0$.

The function $C(\Delta t)$ for finite Δt carries telling information on two-particle correlations for edge-state quasiparticles. At $T = 0$, the integral of Eq. (12) can be evaluated using contour integration to give

$$\frac{C_\diamond(\Delta t)}{2\cos\pi\nu} = \left[\frac{4u^2e^*}{\hbar^2}\left(\frac{V}{\epsilon_0}\right)^{2\nu}\frac{e^{-e^*V/\epsilon_0}}{V}\frac{\pi\cos\tilde{V}\Delta t/2}{\Gamma(2\nu)\Gamma(1-2\nu)}\right]^2 \times \left[\mathrm{Re}\left\{e^{-i\tilde{V}\Delta t/2}\int_0^\infty e^{-r}r^{-\nu}(r + i\tilde{V}\Delta t)^{-\nu}\right\}\right]^2, \quad (14)$$

where ϵ_0, the excitation gap for the bulk Hall fluid, acts as a high-energy cutoff. The resulting behavior of the current-current correlations is shown in Fig. 3.

The function $C(\Delta t)$ contains three different aspects of the edge-state quasiparticles. First, central to our problem and akin to the two-particle correlation function of Eq. (2), it shows maximal statistical correlation at $\Delta t = 0$. As shown by the factor of $\cos\pi\nu$ in Eq. (13), it can be either smaller or larger than the uncorrelated value, depending on the (anti)bunching nature of the

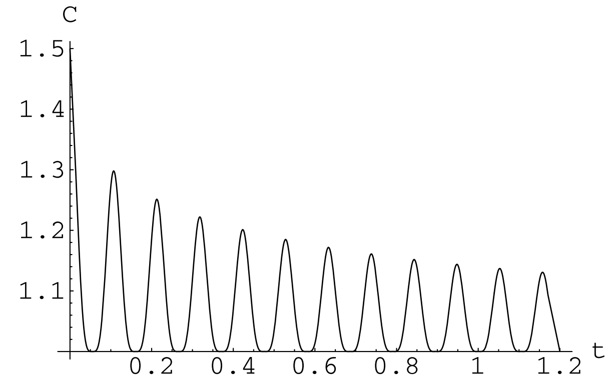

FIG. 3. Normalized correlation function of Eq. (9), $C(\Delta t)/(2\langle I_{12}\rangle\langle I_{34}\rangle)$, at zero temperature, as a function of separation time Δt for filling fraction $\nu = 1/3$. Here we have chosen $\tilde{V} = 60$ in dimensionless units.

quasiparticles. Second, $C(\Delta t)$ reveals oscillations of period h/e^*V and identifies $e^* = \nu e$ as the quantum of quasiparticle charge that couples to the applied voltage. Note that, for Laughlin quasiparticles, statistics and charge are directly related to one another, in that the phase factor acquired under exchange may be interpreted as the Aharnov-Bohm term picked up by the charged quasiparticle [4,5]. However, as the connection between charge and statistics is more complicated for non-Laughlin states [5,16], in these cases $C(\Delta t)$ becomes important in bearing information on both aspects, distinct from one another. While the dependence on charge and statistics ought to hold regardless of the effective theory used to describe the FQH system, the third feature reflects the chiral Luttinger liquid description of the edge state; at large separation time, C decays to the uncorrelated value in the power-law form $C_\diamond(\Delta t) \sim |\tilde{V}\Delta t|^{-2\nu}$, $|\tilde{V}\Delta t| \to \infty$, where the power-law behavior is characteristic of Luttinger liquids. [Note also that the static correlation function of Eq. (2) decays as $g(r_1, r_2) \sim |r_1 - r_2|^{-2\nu}$ within a single edge state, in contrast to the $|r_1 - r_2|^{-2}$ decay appropriate to electrons in a one-dimensional Fermi liquid.] At finite temperatures, as seen from Eqs. (11) and (12), we expect the maximal correlation function $C(\Delta t = 0)$ to cross over to its uncorrelated value at temperatures $kT \approx e^*V$.

In conclusion, we have seen that the principle of extracting information on statistics by means of two-point measurements can be applied equally well to fractional particles realized in current laboratory conditions as to the fermions and bosons found in nature. Characteristic correlations in the detection of two particles at zero separation in space and time, their decay in space or time, and their oscillations over conjugate sets of variables, be they energy and time or position and momentum [9], hitherto observed for fermions and bosons, are seen to be manifest in processes involving anyons. Given the current cutting-edge experimental developments in quantum Hall physics, measurements on edge-state quasiparticles such as the ones proposed here and in other work [7,8] ought to be within experimental reach and thus may provide signatures of fractional statistics for the first time.

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[1] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983); Rev. Mod. Phys. **71**, 863 (1999).
[2] B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).
[3] J. K. Jain, G. S. Jeon, and K. L. Graham, Phys. Rev. Lett. **91**, 036801 (2003).
[4] D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984).
[5] X. G. Wen, Adv. Phys. **44**, 405 (1995).
[6] E. Fradkin, in *Quantum Physics at Mesoscopic Scales*, edited by C. Glattli, M. Sanquer, and J. T. T. Van (EDP Sciences, Les Ulis, 2000).
[7] C. L. Kane, cond-mat/0210621; C. Chamon, D. E. Freed, S. A. Kivelson, S. L. Sondhi, and X. G. Wen, Phys. Rev. B **55**, 2331 (1997).
[8] I. Safi, P. Devillard, and T. Martin, Phys. Rev. Lett. **86**, 4628 (2001).
[9] G. Baym, Acta Phys. Pol. B **29**, 1 (1998).
[10] R. Hanbury Brown and R. Q. Twiss, Philos. Mag. **7**, 663 (1954); Nature (London) **177**, 27 (1956); **178**, 1046 (1956); E. Purcell, Nature (London) **178**, 1449 (1956).
[11] H. Kiesel, A. Renz, and F. Hasselbach, Nature (London) **418**, 392 (2002); M. Henny *et al.*, Science **284**, 296 (1999); W. D. Oliver, J. Kim, R. C. Liu, and Y. Yamamoto, Science **284**, 299 (1999); R. C. Liu, B. Odom, Y. Yamamoto, and S. Tarucha, Nature (London) **391**, 263 (1998).
[12] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **72**, 724 (1994).
[13] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **79**, 2526 (1997); R. de Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahalu, Nature (London) **389**, 162 (1997).
[14] X. G. Wen, Phys. Rev. B **43**, 11 025 (1991); Phys. Rev. Lett. **64**, 2206 (1990); Phys. Rev. B **44**, 5708 (1991).
[15] M. Buttiker, Phys. Rev. B **46**, 12 485 (1992); Y. M. Blanter and M. Buttiker, Phys. Rep. **336**, 1 (2000), and references therein.
[16] For instance, for states described by the composite fermion approach [17], the Aharanov-Bohm phase would have to involve an *effective* magnetic field [3]. Alternatively, in an edge-state description of quasiparticles in these states [see, for example, A. Lopez and E. Fradkin, Phys. Rev. B **59**, 15 323 (1999)], one would require charge and topological sectors, both of which would contribute to the phase factor.
[17] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989); Phys. Today **53**, No. 4, 39 (2000).