Thermoelectric Rotating Torus for Fusion

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(Received 10 July 2003; published 6 November 2003)

A plasma toroid is rotated toroidally to supersonic speeds by external means. The input power maintains the rotation and also heats the plasma. The thermoelectric effect from the resulting temperature gradient creates and maintains a poloidal magnetic field against resistive decay, confining the plasma in steady state. The shear in the rotation keeps the plasma stable to MHD kinks and interchanges. Such a system has two novel advantages as a fusion device: there are no strong electromagnets needed to create the confining magnetic field, and there is effectively no limit on the field strength and, hence, no limit on the plasma pressure contained. The system has to be of a large aspect ratio, to minimize centrifugal effects, and a weak, external vertical magnetic field is needed to balance the radial hoop force.

DOI: 10.1103/PhysRevLett.91.195002 PACS numbers: 52.58.–c, 52.30.–q, 52.55.–s

In magnetized plasma, thermoelectric currents can be driven across the magnetic field if an electron temperature gradient is maintained across the field. Theoretical studies have shown that external heating power that maintains a hot central temperature can concomitantly maintain an encircling magnetic field against resistive diffusion [1–5]. The magnetic field can, in fact, be built up from a weak, seed field to a steady state value, characterized by the magnetic energy density, $B^2/2\mu_0$, being of the same order as the thermal energy in the plasma pressure, p. In this Letter, we argue that this thermoelectric effect, in conjunction with the relatively recent idea that supersonic sheared plasma flow can suppress MHD instability [6], can be used to conceptualize a system for thermonuclear fusion that, if realizable, would offer some novel advantages over conventional fusion approaches.

Consider the toroidal magnetized plasma depicted in Fig. 1 (toroidal Z-pinch configuration). The magnetic field is purely poloidal. It contains plasma pressure according to the MHD force balance equation

$$\vec{\nabla}(p + B^2/2\mu_0) = \vec{B} \cdot \vec{\nabla} \vec{B} / \mu_0. \qquad (1)$$

Since the magnetic field on axis vanishes, the maximum plasma pressure is of the order of the maximum magnetic energy density. The plasma is heated on axis, thus maintaining a hot central core and creating for a cross field temperature gradient. The temperature gradient, in turn, maintains the magnetic field against resistive diffusion by the thermoelectric effect, as described in Ref. [5] wherein details of such toroidal equilibrium are given. As also shown in Ref. [5], this magnetic field could be built up from a much weaker field, as long as the initial, seed field is enough to magnetize electrons (~ 10 G for fusion temperatures).

It is well known that the plasma configuration described above is unstable to MHD kink modes and interchange modes. To suppress these, the plasma is to be forced to rotate toroidally. Recent work [6] has shown that for sheared toroidal rotation, peaked at the center and no slip at the boundaries, kinks and interchanges are stabilized provided the central sonic Mach number exceeds a value of about 4. The resulting plasma is laminar in most of the discharge. Input power is needed to maintain the plasma toroidal momentum, depicted by P in Fig. 1. The momentum losses are dissipated as heat; this same power can then be shown to be sufficient to heat the plasma (as we describe in greater detail later).

The rapid toroidal plasma rotation has associated strong centrifugal forces, tending to sling the plasma to the outboard side of the torus and to distort the magnetic field. To minimize this tendency, the pressure and magnetic forces must exceed the outward centrifugal force. Since the pressure and the magnetic energy density are of the same order, this requirement can be expressed by solving the equation [7] $\vec{B} \cdot \vec{\nabla} p = \vec{B} \cdot (nMu_T^2 \hat{R}/R)$ and

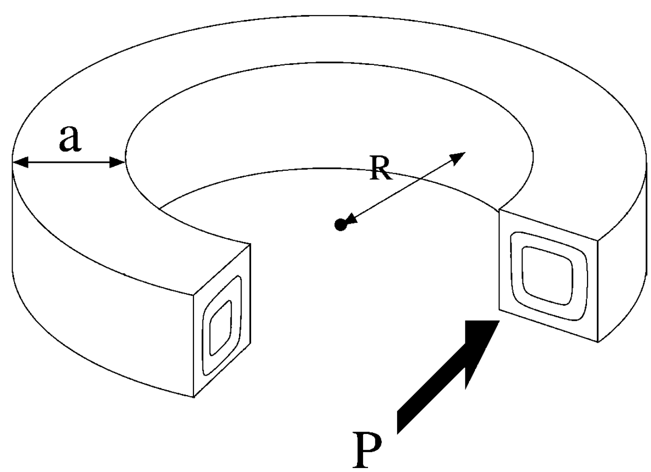

FIG. 1. Toroidal plasma chamber with poloidal magnetic field. External momentum input, P, forces toroidal rotation as well as heats the plasma.

demanding that the pressure be almost constant on a field line. Here, $\hat{R}$ is the toroidal direction, u_T is the toroidal speed, and M is the species mass. If this condition is satisfied, it translates to a condition on the Mach number given by $M_s^2 < R/a$, where a and R are the minor and major radii, as shown in Fig. 1, and $M_s \equiv (Mu_T^2/T)^{1/2}$ is the sonic Mach number. Since we need $M_s \sim 4$, the system must be of a large aspect ratio, greater than ~ 16.

The system as just described constitutes, in essence, the fusion reactor concept: input power maintains the sheared rotation against viscosity, the flow keeps the plasma laminar, the viscous heating maintains the temperature, the temperature gradient maintains the magnetic field, and the magnetic field confines the plasma pressure. Two significant and advantageous departures from conventional concepts are discernible at this stage: (1) there are practically no current-carrying external coils required to make the magnetic field, most of which is thermoelectrically generated; (2) since the magnetic field is not generated by external coils, there is no engineering limit on the magnetic field and, so, practically, no limit on the central plasma pressure containable by the system. To be sure, a coil system to create a weak seed magnetic field is needed, and recirculating power is necessary to maintain the plasma rotation. In addition, the system has to be of a large aspect ratio, adding to cost considerations. A weak, external vertical field is also necessary for overall force balance of the toroid. Nonetheless, the advantage stemming from no large electromagnet systems required could offset the large aspect ratio expense. In addition, the high plasma pressure allowed in principle opens up the possibility of using advanced fuels and high power density systems.

There are further possible advantages: (3) if the seed field can be created by noninductive means, the entire system can be made steady state (a steady state weak current needs to be driven on the magnetic axis, in accordance with a version [5] of Cowling's theorem [8]); (4) the large velocity shear can also suppress microturbulence, possibly allowing classical cross field confinement. Some major unresolved issues—the thermoelectric effect itself, handling of alpha particle power from fusion, and momentum drivers—are discussed later.

A somewhat related concept has been proposed by Winterberg [4]. This involves a linear Z pinch, maintained by the thermoelectric effect, with a thin rod accelerated down the axis to a speed of a few km/s. As discussed below, our required rotation speeds are much higher and our system is toroidal and potentially steady state.

We now examine toroidal momentum force balance, energy balance, and power considerations for this system. Let F be the effective force needed to maintain the toroidal flow u_T against viscosity, i.e., $nMu_T(\mathrm{Vol})/\tau_M = F$, where τ_M is the momentum confinement time and Vol is the plasma volume. The input power P is Fu_T. The momentum balance equation can then be expressed as

$$nMu_T^2(\mathrm{Vol})/\tau_M = P. \quad (2)$$

In steady state, the input power is dissipated via viscosity as heat. The heat is lost by plasma material heat losses or radiation. Thus, heat balance is expressed as

$$3nT(\mathrm{Vol})/\tau_E = fP - P_{\mathrm{rad}}, \quad (3)$$

where we assume that the electron and ion temperatures are each equal to T, that τ_E is the heat confinement time, f is the fraction of the input power that shows up as plasma heat, and P_{rad} is the radiated power.

We first establish a representative point in parameter space for a fusion power system. Assume a D-T (deuterium-tritium) plasma. Let the target temperature be $T = 10$ keV and the target Mach number $M_s = 4$, which fixes the rotation speed u_T. Let $R/a = 20$, as required to minimize centrifugal effects. If the density is n, unspecified as yet, the magnetic field is then determined, according to the thermoelectric scaling, to be of the order of $(2\mu_0 nT)^{1/2}$. Assume that, at these values of temperature and Mach number, the velocity shear suppresses microturbulence, resulting in classical confinement of momentum and heat: thus let [9] $\tau_M \sim \tau_E \sim a^2/(\rho_i^2 \nu_{ii})$, where ρ_i is the ion Larmor radius and ν_{ii} is the ion-ion collision frequency. Assume further that the heating fraction f is unity and that the radiated power is solely bremsstrahlung. There are now three parameters left, P, n, and a, and the two transport equations, (2) and (3). A final target parameter is Q, the energy multiplication ratio of the fusion power, P_{fusion}, to the input power, P. If a certain Q is desired, it would seem that there is no freedom in the system of equations and unknowns in that P, n, and a should be uniquely determined for given Q, T, and M_s. We find, however, because of the peculiar structure of the equations here that Q is determined completely once T and M_s are fixed. In particular, for $T = 10$ keV and $M_s = 4$, Q is fixed to be 26, independent of P, n, and a. In addition, we find that the equations can be recast in the natural variables na^2 and Pa, and that a unique solution exists in these natural variables. In particular, again for $T = 10$ keV and $M_s = 4$, we find $na^2 = 5\ \mathrm{m}^{-1}$ and $Pa = 60$ MWm. In addition, since $B^2 \sim nT$, we find $Ba = 1.5$ Tm.

To summarize, for $T = 10$ keV and $M_s = 4$, we find $Q = 26$. Then, if $a = 1$ m, we find $n = 5 \times 10^{20}\ \mathrm{m}^{-3}$, $P = 60$ MW, and $B = 1.5$ T. Since Pa, na^2, and Ba are constant for fixed T and M_s, if we changed a to be 0.5 m, we find $P = 129$ MW, $n = 20 \times 10^{20}\ \mathrm{m}^{-3}$, $B = 3$ T. Clearly, this behavior is an artifact of the equations, in particular, the assumption of bremsstrahlung, among others. Nonetheless, it is indicative of the parameters that can be expected given the model. The system exhibits a runaway character: suppose the temperature is low so that bremsstrahlung is small. Now, if T increases slightly, the confinement time $\tau_E \sim T^{1/2}$ increases, thus causing T to increase even further; the runaway continues until bremsstrahlung finally caps it. Indeed, the operating point is a stable point.

Attaining the target Mach number of ~4 needs discussion. Dividing Eq. (3) into Eq. (2), we find the following expression for the Mach number M_s:

$$M_s^2 = 3(\tau_M/\tau_E)/(f - P_{\rm rad}/P). \quad (4)$$

If $\tau_M = \tau_E$ and $f = 1$ as assumed, then the Mach number is capped at $3^{1/2}$ if the radiated power is negligible. This would result in poor confinement, since MHD instabilities will create turbulence if the Mach number is not sufficiently high. This would prohibitively raise the power requirement to attain high temperature. A sufficient level of radiation, enough to raise the Mach number so that the confinement improves, is needed to get to high temperature. At fusion temperatures, bremsstrahlung would set the desired operating point, as discussed above. However, at these temperatures, alpha particle heating also has to be factored in.

Alpha particles produced from fusion add to the heating on the right-hand side of (3). There is no simple way to extract a corresponding momentum source from the alphas. Thus, alpha heating would lower the Mach number, degrading the confinement. Beyond $T = 4.3$ keV, the alpha power will exceed the bremsstrahlung losses. Thus, if the Mach number is to be kept high, most of the alpha power would have to be extracted, for example, by radiative means. The ambient bremsstrahlung should then maintain the system at the operating point calculated above. Alpha particle handling clearly needs investigation. In addition, synchrotron radiation losses need closer consideration. Our assumption that velocity shear will suppress microinstabilities clearly also needs closer scrutiny: while ion Larmor radius scale instabilities will likely be suppressed, the same may not hold at the electron scales, rendering electron transport turbulent. This has the advantage, however, of decoupling momentum and heat transport, relatively degrading the latter and making for operation at higher Mach numbers.

More fundamentally, the existence of the thermoelectric effect in high temperature magnetized plasma is itself an open question. As reported in Ref. [2], this effect has been observed in low temperature plasmas in a strong axial magnetic field with axial heating ($T \sim 10$ eV, $n \sim 10^{22}$ m^{-3}, $B \sim 6$ T). To our knowledge, there have been no experimental observations of thermoelectricity in high temperature, confined plasmas, although the bootstrap current in tokamaks [10] may well be the counterpart of this effect for the case of poloidal and toroidal fields. As discussed in Ref. [5], the thermoelectric effect is a transport term in the Braginskii-Chapman-Enskog ordering, possibly with a collisionless counterpart [3]. As such, small-scale electron turbulence could greatly influence resistivity and thermoelectricity in plasmas, easily rendering the theoretical extrapolations herein inapplicable.

Another question for further investigation is the momentum drive for toroidal rotation. Neutral beam injection, widely shown to rotate tokamaks, is a possibility. However, beam penetration considerations aside, it is not clear that neutral beams are able, even in principle, to rotate plasmas to supersonic speeds. Radio frequency or other drives need examination.

An interesting question is why use sheared toroidal flow to suppress MHD instability, as opposed to a toroidal magnetic field. This is worth investigating, especially for an interim approach, or to assess thermoelectricity: as mentioned, the bootstrap current in tokamaks [10] may well be interpreted as the counterpart to this effect. Using the toroidal magnetic field as the stabilizer, however, has the drawback that external coils would be needed. More so, the engineering limit on the toroidal field will imply a stability limit on the poloidal field and a corresponding limit on the maximum plasma pressure: with rotational stabilization, there are apparently no such limits on pressure.

As per the virial theorem, the entire toroidal plasma column has to be held in by external forces. Such a force could be provided by an external, vertical magnetic field. The strength of this field is expected to scale as $(a/R)B$. Thus, toroidal coils would be needed to provide this equilibrium, although the required field is relatively weak.

Finally, techniques have to be found to start up the system. This is no doubt an area that will need much development. In fact, small-scale experiments to demonstrate components of all the above conjectures are needed. From the overall viewpoint of realizing fusion, though, the possibility of a fusion device with no large electromagnets and effectively no limit on the pressure contained is appealing to contemplate.

Useful discussions with Dr. R. J. Goldston are gratefully acknowledged. This work was supported by the U.S. DOE.

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